Strategic incentives in a subsidized mixed duopoly

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Abstract

This paper investigates the effects of production subsidies in a mixed duopoly in which the owners of firms provide strategic incentives to their managers. When the asymmetric subsidy is introduced to the public firm, it is shown that neither industry output nor welfare can be changed. This means that the optimal level of such subsidy in a mixed duopoly must be zero. Furthermore, unlike previous studies, it is shown that the government should privatize the public firm by arranging for an asymmetric subsidy when there are two firms in a market.

Key words: Mixed oligopoly; Strategic contract; Subsidy; Privatization

JLE classification: D21; H20; L13; L20; L32

1. Introduction

One of the standard results of privatizing public firm is that privatization may lower welfare when there are very few private firms in a mixed oligopoly (see DeFraja and Delbono (1989)).

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White (1996), however, showed that welfare is unchanged if the production subsidies are used before and after privatization. The purpose of this paper is to model principal-agent contract in a mixed duopoly and investigate the effects of different types of subsidies on the behaviors of players and the equilibrium welfare - a step that has not yet been taken.[1]

In this paper, we analyze how different type of subsidies affect the behaviors of strategic players and welfare. Extending Fershtman and Judd (1987), we consider a situation in which a public firm and a private firm are operated by managers (agents), who in turn are strategically motivated by owners of different objectives (principals). When an asymmetric subsidy is arranged for the public firm in a mixed duopoly, this subsidy does not affect welfare. The government can not be in the Stackelberg-leader position.[2] That is because a discriminative subsidy does not affect the behavior of unsubsidized private owner, but that of the subsidized public owner. In a private duopoly, however, the government can affect the behaviors of both owners, and thus increase welfare by choosing the optimum level of the asymmetric subsidy for the ex-public firm under proper incentive schemes. The government should privatize the public firm by administering an asymmetric subsidy in a mixed duopoly. This is in sharp contrast with the standard result that privatization decreases welfare, especially when the market consists of relatively few firms.

This paper is organized as follows. The model is described in Section 2. We present the results obtained in an asymmetrically subsidized mixed and private duopoly in the frame work of a three-stage game.[3] Comparisons among equilibrium outcomes in the different regimes are also

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1 Fershtman and Judd (1987) showed that strategic contract might increase the owners' equilibrium payoffs when the owner and the manager have different objective functions in a duopoly. Extending them into a mixed duopoly, Barros (1995) showed that privatization under optimal incentive schemes decreased welfare. For a recent survey on mixed markets, see B ös (1994).

2 For example, injecting public funds into a weak bank placed under government supervision has been implemented as one option to strengthen the Japanese monetary system.

3 When there are a public firm and a private firm in a market, *i.e.*, before privatization, we call this a mixed duopoly. After privatizing the public firm, two private firms form an ordinal duopoly which we call a private duopoly.

discussed. Our conclusions are presented in Section 3.

2. The model

We consider a situation in which the cost conditions among firms are different. We model such situations by introducing an asymmetric subsidy in a mixed duopoly. Each owner of the firms (principal) hires a manager of the same kind (agent) to produce homogeneous goods. The private owner maximizes the expected net profits, while the public owner is assumed to maximize the expected social welfare, defined as the sum of the consumers' and producers' surpluses, the managers' utilities and the subsidy expenditure.

We examine a three-stage game. In the first stage, the government chooses the level of subsidies. Then in the second stage, the owners of each firm simultaneously determine the incentive schemes for their own manager. In the third stage, managers play a Cournot-Nash duopoly game. We then have a four-player game; for each of the two firms, we have an owner and a manager noting that the government is double-counted as an owner of a public firm and a policy maker considering subsidies. For tractability, we restrict consideration to risk neutrality for all players.

2.1. Asymmetric subsidy in a mixed duopoly: Before privatization

We assume that the firms face a linear demand function given by $p=a-X$, where $X=x_1+x_2$ and x_i is the firm i's output for $i=1,2$, letting firm 1 be the public firm and firm 2 be the private firm with identical linear cost functions $C(x_i)=cx_i$ and $a>c>0$. When the government gives the public firm the discriminative subsidy s, the profit function of the unsubsidized private firm is $\pi_2=px_2-cx_2$, while that of the public firm becomes $\pi_1=px_1-(c-s)x_1$, where $c>s$. Given this subsidy level, owners design the wage structure. Following Fershtman and Judd (1987), manager i is motivated by an incentive that is a linear combination of profit and sales revenue, and that is envisioned to receive the opportunity cost of participation. More formally, manager i's compensation is $\beta_i+\alpha_i\pi_i+(1-\alpha_i)R_i$, where α_i, β_i and R_i are firm i's incentive parameters, a fixed transfer, and sales revenue, respectively. We set no restrictions on α_i nor β_i, and allow even negative values.

Starting from the third stage, we solve problems by backward induction. Given (α_i, β_i, s; b), manager 1 of the public firm and manager 2 of the private firm choose the level of outputs (and consequently the effort level) that maximize their own expected utilities net of effort disutilities defined as:

(1) $$U_1 = E\,[\beta_1+px_1-\alpha_1(c-s)x_1-e_1] \geq \overline{U}$$

and

(2) $$U_2 = E\,[\beta_2+px_2-\alpha_2 cx_2-e_2] \geq \overline{U}$$

, where $e_i=b(x_i)^2/2$, $e_i'>0$, $e_i''>0$ with $b>0$, is the manager's disutility and $\overline{U}$ is his reservation utility. We assume interior solutions. Maximizing (1) and (2), and solving them simultaneously in terms of (α_i, β_i, s; b), the set of third-stage Cournot-Nash equilibrium output (x_1*, x_2*) is given by:

(3) $$x_1^*=\frac{a(1+b)-(2+b)(c-s)\alpha_1+c\alpha_2}{3+4b+b^2} \quad \text{and} \quad x_2^*=\frac{a(1+b)+\alpha_1(c-s)-c\alpha_2(2+b)}{3+4b+b^2}$$

, where asterisk " * " over variables and parameters means the equilibrium values and

(4) $$\partial x_i^*/\partial\alpha_i<0, \quad \partial x_j^*/\partial\alpha_i>0, \quad \text{and} \quad \partial X^*/\partial\alpha_i<0 \quad \text{for } i\neq j\,.$$

In the second stage, the public and the private owners simultaneously choose optimal contracts, taking into account the third-stage equilibrium outcomes (3) of the managers' subgame. The public owner maximizes the expected welfare defined as

$$(5) \qquad W = E[\int^{X^*} p(q)dq - c(x_1^* + x_2^*) - (e_1^* + e_2^*)] \ .$$

As the subsidy is included in the welfare as both a component of profit and a state expenditure, those terms are canceled in the welfare expression. The private owner, on the other hand, maximizes expected profit net of the manager's compensation, defined as:

$$(6) \qquad \Pi_2 = E[(p^* - c)x_2^* - (e_2^* + \overline{U})] \ .$$

Solving first-order conditions of (5) and (6) in terms of (s; b), we get the second-stage equilibrium parameters:

$$(7) \qquad \alpha_1^* = \frac{-a(1+5b+4b^2+b^3)+c(2+17b+20b^2+8b^3+b^4)}{(c-s)g} \ ,$$

$$\alpha_2^* = \frac{-ab(2+b)+c(1+14b+17b^2+7b^3+b^4)}{cg}$$

, where $g = (1+12b+16b^2+7b^3+b^4)$.

Note that the incentive parameter of the owner of the private firm (α_2^*) is independent of the level of the discriminative subsidy s. The intuition is that if the owner of the private firm becomes less aggressive (bigger α_2), then the output of the private firm decreases and hence his profit decreases further. For, total output is already increased by the direct effect of an asymmetric subsidy. On the other hand, if he becomes more aggressive (smaller α_2), this action contributes to the bigger increase in total output. These two negative effects induce the owner of the private firm be indifferent of the level of asymmetric subsidy. This means that, even if the government wants to be in a Stackelberg-leader position, he cannot shift the reaction function of the owner of the public firm as far as the owner of the private firm does not respond to the subsidy in the owners' subgame.

Now we'll look into the effects of an asymmetric subsidy on the public firm. As the production pressure of the direct effect of the subsidy results in *ceteris paribus* higher net wage of his manager, because $e_i'>0$. Preparing for higher compensation to the manager, the owner of the public firm intends to increase profit by shifting to a more profit-oriented strategy (bigger α_1). In addition to this, cost-consciousness induces the owner of public firm to be less aggressive because its marginal cost was higher than that of the private firm in an unsubsidized mixed duopoly ($x_1^{M*}(0;\, b)>x_2^{M*}(0;\, b)$). In the following analysis, the superscript M and P indicate a mixed duopoly and a private duopoly, and $(A;\, b)$, $(0;\, b)$, and $(S;\, b)$ indicate the cases of with asymmetric subsidy, without subsidies, and with symmetric subsidies, respectively. As a result, total output as well as its output decrease with the increase of the parameter α_1 ((4)), and this offsets the positive effect of the subsidy.

In the first stage, the welfare-maximizing government tries to determine the optimal level of subsidy by substituting (7) for (5), and we find:

$$(8) \qquad W^{*}=\frac{(a-c)^2(1+23b+160b^2+308b^3+263b^4+113b^5+24b^6+2b^7)}{2g^2}$$

, which is independent of s as expected. Therefore, the asymmetric subsidy cannot affect welfare.

By substituting (7) for (3), we have the equilibrium outcomes:

(9) $x_1^{M*}(A;b)=x_1^{M*}(0;b)=\frac{(a-c)(1+6b+5b^2+b^3)}{g}$, $x_2^{M*}(A;b)=x_2^{M*}(0;b)=\frac{(a-c)b(2+b)^2}{g}$,

$X^{M*}(A;b)=X^{M*}(0;b)$, $W^{M*}(A;b)=W^{M*}(0;b)$, $\alpha_2^{M*}(A;b)=\alpha_2^{M*}(0;b)$, $\pi_2^{M*}(A;b)=\pi_2^{M*}(0;b)$,

$\alpha_1^{M*}(A;b)>\alpha_1^{M*}(0;b)$, $\pi_1^{M*}(A;b)>\pi_1^{M*}(0;b)$.

All equilibrium values other than the incentive parameter and the profit of public firm are equivalent to those under without subsidies. So, the optimal level of the asymmetric subsidy must be zero in a mixed duopoly. For calculations of the above equalities, inequalities, and those of following analysis, see Appendix.

Proposition 1: In a mixed duopoly, an asymmetric subsidy for the public firm changes neither industry output nor welfare. The government can not be in the Stackelberg-leader position in this game. Thus, the optimal level of asymmetric subsidy must be zero.

2.2. Asymmetric subsidy in a private duopoly: After privatization

Next, we examine the case when the government privatizes the public firm giving the ex-public firm the asymmetric subsidy. The equilibrium outputs in the managers' subgame are given by (3) from solving (1) and (2) as in *2.1.*. On the other hand, the second-stage maximization problems of the two private owners become:

(10) $\max\ E[p^* x_1^* - (c-s)x_1^* - (e_1^* + \overline{U})]$

and

(6) $\max\ E[(p^* - c)x_2^* - (e_2^* + \overline{U})]$.

Taking into account (3) and solving two first-order conditions of (10) and (6), we have:

(11) $$\alpha_1^* = \frac{-a(1+3b+b^2)+(2+b)(3+10b+6b^2+b^3)c-(4+10b+6b^2+b^3)s}{(c-s)h},$$

$$\alpha_2^* = \frac{-a(1+3b+b^2)+(2+b)((3+10b+6b^2+b^3)c+s)}{ch}$$

, where $h=(5+20b+21b^2+8b^3+b^4)$.

Contrary to (7) in a mixed duopoly, we must note that the government can affect the behavior of both owners when the public firm is privatized. The government strategically chooses the optimal level of the discriminative subsidy by substituting (11) for (5) in the first stage. Its maximization yields:

(12) $$s^* = \frac{(a-c)(1+3b+b^2)^2}{(2+b)k}$$

, where $k=(1+18b+21b^2+8b^3+b^4)$.

Note again that the optimum level of asymmetric subsidy is strictly positive after privatization. Under this optimum subsidy, the SPNE outcomes are:

(13) $$x_1^{P*}(A;b) = \frac{(a-c)(1+9b+6b^2+b^3)}{k}, \quad x_2^{P*}(A;b) = \frac{(a-c)b(2+b)(3+b)}{k},$$

$$\alpha_1^{P*}(A;b) = \frac{(2+b)(-a(1+7b+5b^2+b^3)+c(2+25b+26b^2+9b^3+b^4))}{-a(1+3b+b^2)+c(3+43b+71b^2+43b^3+11b^4+b^5)},$$

$$\alpha_2^{P*}(A;b) = \frac{-ab(3+b)+c(1+21b+22b^2+8b^3+b^4)}{ck},$$

$$W^{P*}(A;b) = \frac{(a-c)^2(1+17b+12b^2+2b^3)}{2k}.$$

As the marginal cost of the public firm was higher than that of the private firm in an unsubsidized mixed duopoly, the owner of ex-public firm becomes more profit-oriented, $\alpha_1^{P*}(A; b)>\alpha_1^{M*}(0; b)$. In contrast, the owner of firm 2 becomes more aggressive knowing that the rival is the profit-maximizer and the direct effect of the subsidy increases total output, $\alpha_2^{P*}(A; b)<\alpha_2^{M*}(0; b)$. These behaviors change the distribution between consumers and producers. By administering the asymmetric subsidy optimally in privatizing the public firm, the firms' total profits increase by sacrificing consumer surplus, $\sum\Pi^{P*}(A; b)>\sum\Pi^{M*}(0; b)$ and $X^{P*}(A; b)<X^{M*}(0; b)$. In the end, welfare after privatization is greater than that before privatization, $W^{P*}(A; b)>W^{M*}(0; b)= W^{M*}(A; b)$. This is in sharp contrast to White's (1996) assertion that welfare is unchanged when symmetric subsidies are used before and after privatization in a model without a principal-agent contract. Furthermore, once asymmetric subsidy for the public firm is removed in privatizing the public firm, welfare decreases, $W^{P*}(0; b)< W^{M*}(A; b)$.

Proposition 2: In a private duopoly under proper incentive contracts, the optimal level of asymmetric subsidy must be strictly positive, and the subsidized ex-public firm is more aggressive than unsubsidized firm. If the optimal asymmetric subsidy is arranged for privatizing the public firm, then welfare increases. On the other hand, if asymmetric subsidy is taken away when privatizing the public firm, welfare decreases.

Finally, we summarize comparisons among equilibrium outcomes, when different types of production subsidies are implemented. Seen from the following,

$$(14) \qquad W^{P*}(0; b)<W^{M*}(0; b)=W^{M*}(A; b)<W^{P*}(A; b)<W^{M*}(S; b)=W^{P*}(S; b),$$

$$X^{P*}(0; b)< X^{P*}(A; b)<X^{M*}(0; b)=X^{M*}(A; b)<X^{M*}(S; b)=X^{P*}(S; b)\,,$$

(i) welfare is the lowest when there is no subsidy after privatization, (ii) before privatization, even if a positive asymmetric subsidy is used for the public firm, welfare is unchanged and it is equivalent to that without subsidy, (iii) when an asymmetric subsidy is used in privatizing the public firm, welfare is higher sacrificing consumers, and (iv) using positive symmetric subsidies before and after privatization does not affect welfare, which is the highest of all.

3. Conclusions

In this paper, we introduce the role of strategic contracts when firms are subsidized in a mixed duopoly. When the government asymmetrically subsidizes the public firm, we find that the equilibrium firm's output and welfare remain unchanged. They are the same as those obtained in an unsubsidized mixed duopoly. The implication is that as only the behavior of the subsidized public firm's owner is affected, positive direct effect of the subsidy is canceled in the equilibrium. Thus the optimal level of the asymmetric subsidy must be zero before privatization. Once the public firm is privatized, however, the asymmetric subsidy increases welfare by affecting the strategic behavior of both owners. Both results contrast with the findings in the existing mixed oligopoly literature.

Appendix

2.1.

When there is no subsidies, the profit and thus the manager's maximization problem of public firm are $\pi_1=px_1-cx_1$ and max $E[\beta_1+px_1-\alpha_1cx_1-e_1]$. As in *2.1.*, solving this two-stage game by backward induction, we have the following SPNE outcomes:

$\alpha_1^{M*}(0; b)=\{-a(1+5b+4b^2+b^3)+c(2+b)(1+8b+6b^2+b^3)\}/cg,$

$\alpha_2^{M*}(0; b)=\alpha_2^{M*}(A; b)=(-ab(2+b)+c(1+14b+17b^2+7b^3+b^4))/cg,$

$\pi_1^{M*}(0; b)=(a-c)^2b(1+6b+5b^2+b^3)(3+8b+5b^2+b^3)/2g^2,$

$\pi_2^{M*}(0;b)=\pi_2^{M*}(A;b)=(a\text{-}c)^2b^2(2+b)^3(2+4b+b^2)/2g^2$,

$X^{M*}(0;b)=X^{M*}(A;b)=(a\text{-}c)(1+10b+9b^2+2b^3)/g$,

$W^{M*}(0;b)=W^{M*}(A;b)=(1+23b+160b^2+308b^3+263b^4+113b^5+24b^6+2b^7)(a\text{-}c)^2/2g^2$.

Then we compare outcomes as follows:

$\alpha_1^{M*}(A;b)-\alpha_1^{M*}(0;b)=\{s\alpha_1^{M*}(0;b)\}/c(c\text{-}s)g>0$ if $\alpha_1^{M*}(0;b)>0$ and

$\pi_1^{M*}(A;b)-\pi_1^{M*}(0;b)=(a\text{-}c)s(1+6b+5b^2+b^3)/g>0$.

2.2.

When an asymmetric subsidy is arranged in a private duopoly, we have SPNE outcomes $\alpha_i^{P*}(A;b)$, $x_i^{P*}(A;b)$ and $X^{P*}(A;b)$ by substituting (12) for (11) and (3). Those enable comparisons among different regimes:

$\alpha_2^{P*}(A;b)-\alpha_2^{M*}(0;b)=-(a\text{-}c)b/cgk<0$, $x_1^{P*}(A;b)-x_2^{P*}(A;b)=(a\text{-}c)(1+3b+b^2)/k$,

$X^{P*}(A;b)-X^{M*}(0;b)=-(a\text{-}c)b(1+3b+b^2)/gk<0$, where $k=(1+18b+21b^2+8b^3+b^4)>0$, and

$\Sigma\Pi^{P*}(A;b)-\Sigma\Pi^{M*}(0;b)=-(a\text{-}c)^2v/2(2+b)g^2k^2>0$,

where$v=(2+82b+1340b^2+11036b^3+49673b^4+134786b^5+237601b^6+285710b^7+241619b^8+146156b^9+635588b^{10}+19720b^{11}+4262b^{12}+610b^{13}+52b^{14}+2b^{15})$.

When two firms are given subsidies symmetrically, firm's profit and thus each manager's utility are $\pi_i=px_i-(c-s)x_i$ and (1), respectively. So, the SPNE outcomes are derived as in *2.1.* and 2.2. for two regimes. Then we compare welfare as follows;

$W^{M*}(S;b)=W^{P*}(S;b)=(a\text{-}c)^2/(2+b)$, $X^{M*}(S;b)-X^{M*}(0;b)=b(a\text{-}c)(3+4b+b^2)/g(2+b)>0$,

$W^{M*}(S;b)-W^{M*}(0;b)=(1+b)(1+7b+5b^2+b^3)(a\text{-}c)^2/2(2+b)g^2>0$,

$W^{M*}(A;b)-W^{P*}(0;b)=(1+b)^3(1+10b+48b^2+66b^3+384b^4+10b^5+b^6)(a\text{-}c)^2/2(5+5b+b^2)^2g^2>0$,

$W^{P*}(A;b)-W^{M*}(A;b)=b^2(a\text{-}c)^2(1+b)/2g^2k>0$, $W^{P*}(A;b)-W^{M*}(S;b)=-b(a\text{-}c)^2(1+b)/2(2+b)k<0$,

$W^{M*}(0;b)-W^{P*}(0;b)=(1+b)^3(1+10b+48b^2+66b^3+384b^4+10b^5+b^6)(a\text{-}c)^2/2(5+5b+b^2)^2g^2>0$.

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