

The International Transfer Problem
An Essay on the Transfer Paradox and Intergenerational Allocation

HAMADA Kojun
YANAGIHARA Mitsuyoshi

NIIGATA

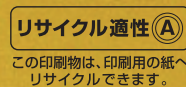
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AIZU Yaichi was born in Niigata in 1881.

He was Professor at Waseda University in Tokyo
from 1931 to 1945.

He was a scholar of oriental art history,
an unrivalled tanka poet and a calligrapher of
outstanding ability. He received the Yomiuri
Literary Prize for his tanka collection, *Aizu Yaichi
Zen Kashu*. He died in 1956.

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Sciences and technology made remarkable progress in the 20th Century, which was at the same time the century of war and upheaval. Wars of a global scale broke out twice. The number of local wars and upheavals is too numerous to be counted.

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HAMADA Kojun and YANAGIHARA Mitsuyoshi

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Preface

The purpose of this book is to provide a wider and deeper perspective on the international transfer problem based on a recent theoretical viewpoint. Since Keynes vs. Ohlin debated over the German war reparations in 1929 after World War I, which is a well-known classical example of debates on international transfers of income, the transfer problem has sparked a lot of public interest and has raised various economic issues in international trade theory. Even in recent decades, a considerable number of articles have tackled various aspects and issues of the transfer problem between a donor country and a recipient country in the context of international trade or capital movement. In this book, we summarize our theoretical contributions to the international transfer problem and present some new conclusions. In particular, throughout this book, we attempt to present some novel insights on the transfer paradox, which is a paradoxical situation wherein the donor of the transfer is enriched in spite of a decrease in income by transfer and/or the recipient is immiserized in spite of an increase in income.

This book includes many results of our existing studies as original works. Almost all these works have already been published in refereed journals or academic bulletins of our universities. Some studies have been presented in international or domestic conferences or seminars. In this book, we would like to introduce the whole picture of our contributions. Chapters 2, 3, 4, 6, 7, and 8 summarize several of our results published in refereed journals; see, respec-

tively, Hamada (2012); Hamada and Yanagihara (2014); Hamada and Yanagihara (2016); Hamada, Shinozaki, and Yanagihara (2017); Hamada, Kaneko, and Yanagihara (2016); and Hamada, Kaneko, and Yanagihara (2017b). Chapter 5 rearranges and translates Hamada's (2016) original work into English, which was published in the academic bulletin of Niigata University.

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Niigata, Japan
January 2019

Kojun Hamada
Mitsuyoshi Yanagihara

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Chapter 1

Introduction

The economic term, “transfer problem,” was first used during a well-known debate between Keynes (1929) and Ohlin (1929) over the issue of the German ability to make reparation payments to the Allies after World War I. This is a well-known classical example of a debate on international transfer of income. Since then, the transfer problem has been a research theme in international economics and, especially, in international trade theory. Even in recent decades, a considerable number of articles have tackled various aspects and issues of the transfer problem between a donor country and a recipient country in the context of international trade or capital movement.¹ Generally speaking, the transfer problem refers to the effect of transfer of income on the donor’s terms of trade. Sometimes, the reversal of capital flows forces countries to go from current account deficit to current account surplus. Among other things, the transfer paradox is the most controversial issue in the transfer problem. This paradox indicates a paradoxical situation wherein the donor of the transfer is enriched in spite of the decrease in income by transfer and/or the recipient is immiserized in spite of the increase in income. Not only academic scholars but also practical policymakers have contributed to the controversy over whether and how such a paradoxical situation arises for many decades.

Leontief (1936) was the first to present a rigorous numerical example and

¹ For a concise survey of the transfer problem, see Ickes (2009) and Krugman and Obstfeld (2006). In addition, for a recent concise survey of the transfer paradox, see Kang and Rasmussen (2016).

handle the transfer problem theoretically. Samuelson (1947, 1952, 1954) showed that the transfer paradox occurs only when the market equilibrium is not Walrasian stable. Balasko (1978, 2014) proved that Samuelson's results can be extended to the case of generalized n -good markets.

In contrast, Gale (1974) showed that the transfer paradox can occur even under stable equilibria if there are more than two countries in the economy. Gale's work showing the three-country transfer paradox was followed by a large number of studies, for example, Chichilnisky (1980, 1983), Geanakoplos and Heal (1983), and Polemarchakis (1983) among others. Yano (1983) defined the weak transfer paradox, which denotes the situation wherein both the donor and recipient are enriched or immiserized by the transfer, and the strong transfer paradox, wherein the donor is enriched but the recipient is immiserized. Yano (1983) showed that both transfer paradoxes can arise depending on the parameters of the model. Bhagwati, Brecher, and Hatta (1983) showed that when there exists an outsider other than the donor and the recipient in the economy, the transfer paradox can arise. Furthermore, if some distortions exist in the economy, the paradox is likely to occur. Bhagwati, Brecher, and Hatta (1983) showed that even in a two-country stable market equilibrium, the transfer paradox arises when some exogenous and endogenous distortions are introduced. A considerable number of studies have examined whether and how the transfer paradox can arise with some distortions.

Furthermore, in a dynamic framework, the transfer paradox can arise even with the Walrasian stability of an international capital market equilibrium under dynamic efficiency. Galor and Polemarchakis (1987) first showed that a permanent lump-sum transfer can bring about the transfer paradox in a steady state being away from the golden rule. Haaparanta (1989) clarified that the transfer paradox can arise, even if the transfer is temporary, when financed by the issuance of public debt in the donor country and/or when the transfer is used for debt relief in the recipient country. Tan (1998) also argued that any transfer from a rich to a poor country does not cause the transfer paradox in a steady state under dynamic efficiency. Cremers and Sen (2008) extended the analysis into the transition path converging to the steady state and showed that the results of Galor and Polemarchakis (1987) can also be applied to the transition path.

This book is organized as follows: The first three chapters focus particularly

on altruism, that is, the altruistic behavior of the donor country and/or recipient country. We examine how the altruistic utilities of the donor and recipient influence the effect of the transfer. The subsequent two chapters deal with two topics related to behavioral economics: consumption externality and aspirations. We investigate how such externality or aspirations change the effect of the transfer. The remaining three chapters are devoted to the relationship between international transfer and public pension or intergenerational redistribution.

Chapters 2 to 4 argue the relationship between altruism and the transfer paradox. Chapter 2 introduces altruistic utility into the usual model with two countries and two goods and examines whether altruism causes the transfer paradox. Unlike existing literature on the subject in which people in the donor country maximize their own utility and do not care about people in the recipient country, we analyze a situation in which consumers of the donor and recipient countries have altruistic utility. We demonstrate that if the Walrasian stability condition is satisfied in the general equilibrium, the transfer paradox can never take place, irrespective of the definition of utility.

Chapter 3 examines the transfer problem between two countries when a donor exhibits altruistic utility toward a recipient in a one-sector overlapping generations (OLG) model. In this chapter, we clarify that whether the transfer enriches the donor with strong altruism depends on the relative size of the marginal propensity to save between the donor and the recipient. We especially demonstrate that if the donor has larger marginal propensity to save than the recipient, donor enrichment never occurs; the donor's altruism never contributes to donor enrichment irrespective of the degree of the donor's altruism. Donor enrichment occurs only if the donor has smaller marginal propensity to save and sufficiently high level of altruism.

Chapter 4 investigates how intergenerational altruism affects the possibility of the transfer paradox occurring in a two-country, one-sector OLG model. We derive the conditions under which the transfer paradox occurs in our model, where a generation within each country has altruism toward the subsequent generation. Contrary to earlier results in the literature, we find that altruism does not enter the conditions under which the transfer paradox occurs in the steady state. Moreover, we show that although altruism affects the condition of the transfer paradox along the transition path, its effect on this condition vanishes as the economy converges to the steady state.

Chapters 5 and 6 deal with two behavioral aspects other than altruism: consumption externality and aspirations. Chapter 5 explores a model in which there exists consumption externality for the donor country and the recipient country in a one-sector OLG model, and it considers the transfer problem between the donor and the recipient. We present some results on how an income transfer between countries changes social welfare of the two countries when people of the donor country have the consumption externality to increase their utility with the increase in the recipient's consumption. In this chapter, first, we clarify the relationship between the relative size of both labor and capital shares and the likelihood of the transfer paradox in each case wherein an income transfer increases or decreases the interest rate. Second, we examine how the indirect effect of the transfer on social welfare is affected by consumption externality.

Chapter 6 examines the transfer problem between two countries when either a donor or a recipient has aspirations based on the standard of living of parents in a one-sector OLG model. Focusing on whether and how aspirations affect the welfare effect of a transfer, we first demonstrate that when the donor forms aspirations, as the degree of the donor's aspirations to their parents increases, a transfer is more likely to cause donor enrichment, while it does not affect the recipient's welfare at all. In contrast, when the recipient forms aspirations, whether the increase in the degree of the recipient's aspirations causes recipient immiserization depends on whether the transfer raises the recipient's consumption. Second, we show that if the donor's or recipient's marginal utility increases with the donor's or recipient's aspirations, the transfer is more likely to cause recipient immiserization, whereas whether donor enrichment occurs depends on the situation. These results claim that the effects of aspirations on the welfare of both countries can be distinguished into two types: the effect caused by aspirations themselves and the effect through the capital market. We clarify that two different effects of aspirations on welfare do not necessarily work in the same direction.

Chapters 7 and 8 consider the intergenerational redistribution within a country and investigate how a domestic redistribution policy such as a public pension plan affects the efficiency of the income transfer between countries. Chapter 7 investigates the transfer problem between two countries in the steady state in a one-sector OLG model and explains how transfers should be shared between the young and old generations of the donor country and allocated across the

generations of the recipient country. Except at the golden rule of capital accumulation, the ratios of the burden and distribution of transfers between the young and old generations affect welfare. We first obtain the result that sharing of the transfer burden in the donor country depends on the relative size of two effects, namely, a direct negative effect and an indirect positive effect. If the former exceeds the latter, it is preferable for the donor country to allocate all the transfer burden to the old generation and vice versa. Second, we show that it is preferable for the recipient country to distribute all the transfers to the young generation.

In Chapter 8, we consider the situation in which both countries adopt a pay-as-you-go (PAYG) pension system. Using a one-sector OLG model, we examine how international transfers affect the welfare levels of a donor with higher marginal propensity to save and a recipient with lower marginal propensity to save. We demonstrate that in a dynamically efficient economy, except at the golden rule, when a per capita PAYG pension contribution of either a donor or recipient increases marginally, the effect of the transfer on the donor's welfare can be reduced, whereas whether the effect of the transfer on the recipient's welfare is reduced is ambiguous. These results imply that the existence of a PAYG pension might hinder the effectiveness of the transfer on the donor's welfare, and the adoption of a PAYG pension system is likely to cause a weak transfer paradox wherein both the donor and recipient immiserize.

This book includes many results of our existing studies as original works. Almost all these works have already been published in refereed journals or academic bulletins of our universities. Some studies have been presented in international or domestic conferences or seminars. In this book, we would like to introduce the whole picture of our contributions. Chapters 2, 3, 4, 6, 7, and 8 summarize several of our results published in refereed journals; see, respectively, Hamada (2012); Hamada and Yanagihara (2014); Hamada and Yanagihara (2016); Hamada, Shinozaki, and Yanagihara (2017); Hamada, Kaneko, and Yanagihara (2016); and Hamada, Kaneko, and Yanagihara (2017b).² Chapter 5 rearranges and translates Hamada's (2016) original work into English, which was published in the academic bulletin of Niigata University.

² Hamada, Kaneko, and Yanagihara (2017a) provide some numerical examples of the inter-generational redistribution and PAYG pension, based on the model presented in Chapters 7 and 8.

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Part I

Altruism

Chapter 2

Altruism and the Transfer Paradox in a Static Model

In this chapter, we examine whether altruism causes the transfer paradox in the model with two countries and two goods. Unlike the existing literature on the subject in which the people in the donor country maximize their own utility and do not care about the people in the recipient country, we analyze a situation in which when the consumers of the donor and recipient countries have altruistic utility. We demonstrate that if the Walrasian stability condition is satisfied in the general equilibrium, the transfer paradox can never take place irrespective of the definition of utility. The result suggests that the motivation for transfer cannot be explained by the donor's enrichment because it is not caused by the introduction of altruism into the model.

Keywords: altruism, transfer paradox, Walrasian stability

JEL classifications: F11, F35

* This chapter is based upon Hamada (2012). We express our gratitude to the editor of the *International Economy* for permitting the reproduction subject to stipulating the permission from the editor in the chapter.

2.1 Introduction

Since Keynes vs. Ohlin debated over the German war reparations after World War I—a well-known classical example of debates on international transfers of income—the transfer problem has sparked a lot of public interest and has raised various economic issues in the international trade theory.¹ An income transfer may improve or worsen the terms of trade of both the transferer and transferee. If the secondary effect of the income transfer on the terms of trade exceeds its direct effect, the welfare of the transferer may improve and/or the welfare of the transferee may worsen against the will of the former. Such a situation is called the “transfer paradox.” Leontief (1936) presented an example of the fact that the change of terms of trade resulting from a transfer causes the transfer paradox for the donor through one of the classical articles that dealt with the transfer paradox in a two-commodity world involving two countries. On the contrary, using the notion of Walrasian stability, Samuelson (1952, 1954) showed that if the general equilibrium is stable, the transfer paradox cannot arise under free trade and as a result, the example of Leontief (1936) is excluded under the stability assumption.

In order for the transfer paradox to take place in a stable equilibrium, some presumptions of Samuelson (1952) need to be modified. The existing literature on the subject has explained why the transfer paradox takes place by mainly extending Samuelson’s model in the following two directions. The first extension is the introduction of the third country into the model. In the model with three countries and two goods, Bhagwati, Brecher, and Hatta (1983) show that there is a possibility of the paradoxes of enriched donor and immiserized recipient arising when there is an outside country in a multilateral world. Second, when free trade is hindered by distortions such as trade barriers, the transfer paradox takes place in a two-country model. Bhagwati, Brecher, and Hatta (1985) demonstrate that when there is exogenous distortion by trade barriers such as tariff and subsidy, a transfer may paradoxically enrich the donor and immiserize the recipient. They also show that if the transfer induces endogenous distortion such as lobbying and rent-seeking, there exists the possibility of

¹ Regarding the Keynes vs. Ohlin controversy, see Keynes (1929) and Ohlin (1929). As an example of a concise survey on the transfer problem, see Brakman and van Marrewijk (1999, Ch.2).

paradoxes of donor and recipient arising. The existing literature has analyzed a variety of distortions. For example, the restriction of the recipient country's behavior in exchange of aid effectively constitutes a distortion (see Kemp and Kojima 1985; Schweinberger 1990; Lahiri and Raimondos-Møller 1995). Administrative cost of transfer, sticky wage and unemployment, and transfer of production factors are another examples.²

In recent years, several articles have attempted to explain the transfer paradox by introducing altruism into the donor's utility in the model. If a donor has altruistic utility, the transfer raises the recipient's utility and the increase in the recipient's utility may in turn raise the donor's utility. If this result is correct, the introduction of altruism into the donor's utility explains the donor's enrichment. As a result, altruism presents the reason behind the donor's voluntary contribution of foreign aid. Kemp and Shimomura (2002) explore the model of voluntary unrequited transfer and show that altruism might be the motivating factor behind the donor's transfer. Contrary to altruism, in the setting in which the welfare of each country is negatively influenced, Kemp and Shimomura (2003) demonstrate that the donor might benefit at the expense of the recipient. Lahiri and Raimondos-Møller (1999) develop a model wherein altruism is the motive for the donor giving aid and trade is distorted by tariffs or quotas. They show that if the donor is sufficiently altruistic, transfer is strictly Pareto improving. In other words, they conclude that the donor's altruism enriches the donor itself. Takarada and Tawada (2003) explore the model wherein the donor has altruistic utility and the donor's government gives aid for attaining political objectives. They demonstrate that it is always optimal for the recipient to accept the transfer and if the donor takes care of the recipient's welfare sufficiently, the donor's enrichment takes place.

However, most of the existing literature has explained the transfer paradox by combining altruism with other distortions such as tariffs or political motives into the model. In the present article, we examine whether altruism itself causes the transfer paradox in a simple two-commodity world with two countries when there is no other distortion. By applying the traditional argument on the stability of the general equilibrium, we demonstrate that even if the donor and/or

² The distortions that cause the transfer paradox have been comprehensively analyzed by Brakman and van Marrewijk (1999).

recipient has or have altruistic utility, the transfer paradox can never take place irrespective of whether utility is defined to include altruism. Altruism does not cause the donor's enrichment. Therefore, our result assures that the benevolent assumption that the donor country has altruistic intentions toward the recipient country cannot elucidate the reason why the donor gives aid voluntarily. As the existing literature has theorized, the distortion that hinders free trade is necessary for the transfer paradox to take place in the model with two countries and two goods.

The remainder of this chapter is organized as follows. Section 2.2 describes the model wherein the donor and/or recipient has or have altruistic utility in the model with two countries and two goods. Section 2.3 analyzes the transfer paradox with regard to the self-utility that excludes altruism and that with regard to the total utility that includes altruism, respectively. We present the main result that the transfer paradox can never take place irrespective of the definition of utility. Section 2.4 concludes this chapter with some remarks.

2.2 The model

Consider a general equilibrium model of international trade in a two-commodity world involving two countries. There are two countries—a donor country (indexed by D) and a recipient country (indexed by R). They trade in two goods—the non-numeraire good (x_1) and the numeraire good (x_2). Without loss of generality, we assume that the donor (recipient) is an exporter (resp. importer) of the non-numeraire good. It is assumed that foreign aid is distributed in lump-sum among consumers. There is no import tariff or export subsidy. $T \geq 0$ denotes the transfer as foreign aid. The donor provides foreign aid of the amount T in terms of the numeraire good to the recipient. p represents the international price of the non-numeraire good, which could be interpreted as a relative price because unity constitutes the domestic price of the numeraire good.

The consumption pair of the representative consumer in country $i = D, R$ (hereafter consumer i) is denoted by (x_1^i, x_2^i) , where x_1^i is the non-numeraire good and x_2^i is the numeraire good. Define the utility that consumer i obtains directly from the consumption of goods by $u^i \equiv u^i(x_1^i, x_2^i)$. We denote u^i as “self-utility.” Self-utility does not include any altruistic part of utility. Con-

sumer D (R) obtains the utility $u^D = u^D(x_1^D, x_2^D)$ (resp. $u^R = u^R(x_1^R, x_2^R)$) by consuming goods. In order to describe altruism for the people in both countries, we define the “total utility” of consumer i by $U^i \equiv U^i(u^i, u^j) = U^i(u^i(x_1^i, x_2^i), u^j(x_1^j, x_2^j))$, $i, j = D, R, j \neq i$. It should be noted that the total utility of consumer i includes the altruistic utility that is raised by the increase in u^j . For simplification of analysis, we assume that self-utility and total utility are continuously differentiable. We define $U_j^i \equiv \partial U^i / \partial u^j$, $i, j = D, R$ and assume, as usual, $U_i^i > 0$. Under differentiability, altruism implies $U_j^i \geq 0$, $j \neq i$.

Denote trade expenditure function by E^i , which is defined as the difference between the expenditure function e^i and revenue function r^i . Thus, the following equations are satisfied:

$$E^D(p, U^D) \equiv e^D(p, U^D) - r^D(p), \quad (2.1)$$

$$E^R(p, U^R) \equiv e^R(p, U^R) - r^R(p). \quad (2.2)$$

We denote the import demand function of the non-numeraire good in country i by m^i . By the above-mentioned assumption that the donor (recipient) is an exporter (importer) of the non-numeraire good, $m^D < 0$ and $m^R > 0$.

The budget constraints in the countries are as follows:

$$E^D(p, U^D) = -T, \quad (2.3)$$

$$E^R(p, U^R) = T. \quad (2.4)$$

The product market-clearing condition is as follows:³

$$m^D(p, U^D) + m^R(p, U^R) = 0. \quad (2.5)$$

Using McKenzie’s lemma, the following equation is satisfied:⁴

$$m^i = E_p^i. \quad (2.6)$$

³ The world market-clearing condition for the numeraire good has been omitted due to Walras’s law.

⁴ The subscript x represents the partial derivative of the functions with respect to x .

2.3 Results

In this section, we examine whether the transfer paradox takes place with regard to the self-utility and total utility by applying the argument of Walrasian stability. First, let us investigate the impact of an increase in the unfettered transfer T upon the self-utility (u^D, u^R) . By totally differentiating (2.3)–(2.5) with respect to (U^D, U^R, p) , which are variables of the model described above, the following equation is obtained:

$$\begin{bmatrix} E_U^D & 0 & m^D \\ 0 & E_U^R & m^R \\ m_U^D & m_U^R & M_p \end{bmatrix} \begin{bmatrix} dU^D \\ dU^R \\ dp \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} dT, \quad (2.7)$$

where $M_p \equiv m_p^D + m_p^R < 0$. Totally differentiating the total utility $U^i \equiv U^i(u^i, u^j)$, $i, j = D, R$, $j \neq i$ with respect to the self-utility (u^i, u^j) , we obtain the following equations.

$$dU^D = U_D^D du^D + U_R^D du^R, \quad (2.8)$$

$$dU^R = U_D^R du^D + U_R^R du^R. \quad (2.9)$$

Substituting (2.8) and (2.9) into (2.7) and arranging it with respect to (du^D, du^R, dp) , we obtain the following equation.

$$\begin{bmatrix} E_U^D U_D^D & E_U^D U_R^D & m^D \\ E_U^R U_D^R & E_U^R U_R^R & m^R \\ m_U^D U_D^D + m_U^R U_D^R & m_U^D U_R^D + m_U^R U_R^R & M_p \end{bmatrix} \begin{bmatrix} du^D \\ du^R \\ dp \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} dT. \quad (2.10)$$

Applying Cramer's rule to (2.10), we obtain the following equation.

$$\begin{aligned}
\frac{du^D}{dT} &= \frac{1}{\Gamma} \begin{vmatrix} -1 & E_U^D U_R^D & m^D \\ 1 & E_U^R U_R^R & m^R \\ 0 & m_U^D U_R^D + m_U^R U_R^R & M_p \end{vmatrix} \\
&= \frac{1}{\Gamma} \left\{ \left[-E_U^R U_R^R M_p + (m_U^D U_R^D + m_U^R U_R^R) m^D \right] \right. \\
&\quad \left. + \left[-E_U^D U_R^D M_p + (m_U^D U_R^D + m_U^R U_R^R) m^R \right] \right\} \\
&= \frac{1}{\Gamma} \left[-(E_U^D U_R^D + E_U^R U_R^R) M_p + (m_U^D U_R^D + m_U^R U_R^R) (m^D + m^R) \right] \\
&= -\frac{M_p (E_U^D U_R^D + E_U^R U_R^R)}{\Gamma}, \tag{2.11}
\end{aligned}$$

where $\Gamma \equiv -E_U^D E_U^R (U_D^D U_R^R - U_R^D U_D^R) \Delta$ and $\Delta \equiv -M_p + m^D m_U^D (E_U^D)^{-1} + m^R m_U^R (E_U^R)^{-1}$. Using a similar procedure, we obtain the impact on the self-utility for recipient R as follows:

$$\frac{du^R}{dT} = \frac{M_p (E_U^D U_D^D + E_U^R U_D^R)}{\Gamma} = -\frac{M_p (E_U^D U_D^D + E_U^R U_D^R)}{E_U^D E_U^R (U_D^D U_R^R - U_R^D U_D^R) \Delta}. \tag{2.12}$$

Samuelson (1952) shows that if the general equilibrium satisfies the Walrasian stability condition, it is not possible for any transfer paradox to occur in the model with two countries and two goods. Now, in our model in which there is altruism, we examine whether the transfer paradox occurs with regard to the self-utility when the Walrasian stability condition is guaranteed. The Walrasian stability condition in the model is summarized in the following lemma.

Lemma 2.1. *The Walrasian stability condition is as follows:*

$$\Delta \equiv -M_p + m^D m_U^D (E_U^D)^{-1} + m^R m_U^R (E_U^R)^{-1} > 0. \tag{2.13}$$

Proof. Denote \dot{p} as the change in the price of the non-numeraire good x_1 over time as a result of an imbalance in the demand and supply of good x_1 . In order to analyze the Walrasian price adjustment process, we consider the budget constraint of each country, (2.3) and (2.4), and the following dynamic adjustment

equation:

$$\dot{p} = \Pi \left(m^D(p, U^D(u^D, u^R)) + m^R(p, U^R(u^D, u^R)) \right). \quad (2.14)$$

Since the function $\Pi(\cdot)$ in (2.14), which is assumed to be continuously differentiable, depends on the world excess demand of good x_1 , we assume that the price of good x_1 is rising if and only if the world excess demand for good x_1 is positive, such that $\Pi(0) = 0$ and $\Pi'(0) > 0$. If we linearize systems (2.3), (2.4), and (2.14) around equilibrium values of price and utility, say $(\bar{p}, \bar{u}^D, \bar{u}^R)$, and use the normalization above, we obtain:

$$m^D(p - \bar{p}) + E_U^D [U_D^D(u^D - \bar{u}^D) + U_R^D(u^R - \bar{u}^R)] = 0, \quad (2.15)$$

$$m^R(p - \bar{p}) + E_U^R [U_D^R(u^D - \bar{u}^D) + U_R^R(u^R - \bar{u}^R)] = 0, \quad (2.16)$$

$$\begin{aligned} \dot{p} = \Pi(0) + \Pi'(0) & \left\{ m_p^D(p - \bar{p}) + m_U^D [U_D^D(u^D - \bar{u}^D) + U_R^D(u^R - \bar{u}^R)] \right. \\ & \left. + m_p^R(p - \bar{p}) + m_U^R [U_D^R(u^D - \bar{u}^D) + U_R^R(u^R - \bar{u}^R)] \right\}. \end{aligned} \quad (2.17)$$

By (2.15) and (2.16), $U_D^D(u^D - \bar{u}^D) + U_R^D(u^R - \bar{u}^R) = -(m^D/E_U^D)(p - \bar{p})$ and $U_D^R(u^D - \bar{u}^D) + U_R^R(u^R - \bar{u}^R) = -(m^R/E_U^R)(p - \bar{p})$ are obtained. Substituting them into (2.17) and using $\Pi(0) = 0$ gives

$$\begin{aligned} \dot{p} &= \Pi'(0) \left[\left(m_p^D - \frac{m^D m_U^D}{E_U^D} \right) + \left(m_p^R - \frac{m^R m_U^R}{E_U^R} \right) \right] (p - \bar{p}) \\ &= -\Pi'(0) \Delta (p - \bar{p}), \end{aligned} \quad (2.18)$$

where $\Delta \equiv -M_p + m^D m_U^D (E_U^D)^{-1} + m^R m_U^R (E_U^R)^{-1}$. For Walrasian stability, we want the price change of good x_1 to be negative if p exceeds the equilibrium price \bar{p} and to be positive if p falls short of the equilibrium price \bar{p} . Walrasian stability thus requires that $\Delta > 0$. \square

By Lemma 2.1, the Walrasian stability condition is represented by the impact of price and total utility on trade expenditure function E^i and it does not depend on self-utility. It should be noted that total utility itself prescribes the stability

condition, while self-utility, which affects total utility, have nothing to do with it. As shown in the proof of Lemma 2.1, when substituting (2.15) and (2.16) into (2.17), total differentiation with respect to the self-utility is completely removed. In other words, the change of total utility includes all the change of self-utility of both countries, as evidently shown in $dU^D = U_D^D(u^D - \bar{u}^D) + U_R^D(u^R - \bar{u}^R)$ and $dU^D = U_D^R(u^D - \bar{u}^D) + U_R^R(u^R - \bar{u}^R)$.

By Lemma 2.1, under the Walrasian stability condition, we obtain the following proposition with regard to the transfer paradox of self-utility.

Proposition 2.1. *Suppose that Walrasian stability and $U_D^D U_R^R > U_R^D U_D^R$ are satisfied. As regarding self-utility, even if the donor and/or the recipient has or have altruistic utility, no transfer paradox occurs in the model with two countries and two goods. That is, if $\Delta > 0$ and $U_D^D U_R^R > U_R^D U_D^R$, then $\partial u^D / \partial T < 0$ and $\partial u^R / \partial T > 0$ are satisfied.*

Proof. Note that $M_p < 0$, $E_U^D > 0$, $E_U^R > 0$, $U_i^i > 0$, $U_j^i \geq 0$. By Lemma 2.1, the Walrasian stability condition is $\Delta > 0$. If $U_D^D U_R^R > U_R^D U_D^R$, by (2.11) and (2.12), the following inequalities are immediately obtained:

$$\frac{du^D}{dT} = \frac{M_p(E_U^D U_R^D + E_U^R U_R^R)}{E_U^D E_U^R (U_D^D U_R^R - U_R^D U_D^R) \Delta} < 0, \quad (2.19)$$

$$\frac{du^R}{dT} = -\frac{M_p(E_U^D U_D^D + E_U^R U_D^R)}{E_U^D E_U^R (U_D^D U_R^R - U_R^D U_D^R) \Delta} > 0. \quad (2.20)$$

□

Proposition 2.1 clarifies the condition in which no transfer paradox takes place with regard to self-utility. If the Walrasian stability condition $\Delta > 0$ is satisfied under the assumption $U_D^D U_R^R > U_R^D U_D^R$, any transfer paradox with regard to self-utility does not occur. Under this assumption, even if there is altruism for the donor and/or recipient, the donor does not raise its self-utility by giving aid and the recipient's self-utility does not fall.

This assumption is not restrictive at all because it seems to be always satisfied in real situations. If, as expected, the impact on the total utility of the self-utility exceeds that of the altruistic part in both countries ($U_D^D > U_R^D$ and $U_R^R > U_D^R$), the assumption $U_D^D U_R^R > U_R^D U_D^R$ is necessarily satisfied and any transfer

paradox does not occur. Even if the donor takes care of the recipient more than oneself ($U_D^D < U_R^D$), transfer paradox does not always occur. As often found, if the recipient takes care of oneself more than the donor, for example, if the recipient is not at all interested in the welfare of the other ($U_D^R = 0$), both the donor's enrichment and the recipient's immiserization are not possible to occur. Although the sufficient condition in which transfer paradox occurs is that both countries take care of others more than themselves, such a situation is quite unrealistic. Therefore, $U_D^D U_R^R > U_R^D U_D^R$ is satisfied in most real situations and if the stability of the equilibrium is guaranteed, no transfer paradox takes place with regard to self-utility.

The assertion of Proposition 2.1 seems surprising at first glance, because it implies that even if people give aid to others, they cannot raise their own welfare through this help-providing action in normal situations. Even if the donor is concerned about the rise in the utility of the recipient, the former cannot become happier by supporting the recipient through the transfer. Proposition 2.1 concludes that in order for the transfer paradox to take place, other distortions—which have already been analyzed in the existing literature—are required to be introduced into the model. Therefore, although Lahiri and Raimondos-Møller (1999) and Takarada and Tawada (2003) explore the model in which altruism is introduced, the transfer paradox never occurs without the distortions such as trade barriers or political objectives which they assume in their models.

We can, of course, consider the situation in which $U_D^D U_R^R > U_R^D U_D^R$ is not assumed and there exists a possibility of the transfer paradox occurring. However, the violation of this assumption is quite unrealistic in the sense that it is based on the assumption that the people in a country emphasize the welfare of the other country more than their own. Therefore, for example, although Kemp and Shimomura (2002) show the possibility of a Pareto-improving transfer, in order for both the countries to benefit from the transfer, it is necessary that the irregular assumption about the impact of altruistic utility is satisfied. If a condition similar to the assumption $U_i^i > U_j^i$ is assumed, the possibility of a Pareto-improving transfer will be excluded. As long as the consumers of both the donor and recipient countries are predominantly concerned about the maximization of their own utility, the transfer necessarily causes a decrease in the donor's welfare and an increase in the recipient's welfare.

Next, we examine whether the transfer paradox takes place with regard to the

total utility. We investigate the impact of an increase in the unfettered transfer T upon U^D and U^R . Substituting (2.11) and (2.12) into (2.8) and (2.9) and arranging them, the following equations are obtained.⁵

$$\begin{aligned}
 \frac{\partial U^D}{\partial T} &= U_D^D \frac{du^D}{dT} + U_R^D \frac{du^R}{dT} \\
 &= U_D^D \frac{M_p(E_U^D U_R^D + E_U^R U_R^R)}{E_U^D E_U^R (U_D^D U_R^R - U_R^D U_D^R) \Delta} - U_R^D \frac{M_p(E_U^D U_D^D + E_U^R U_D^R)}{E_U^D E_U^R (U_D^D U_R^R - U_R^D U_D^R) \Delta} \\
 &= \frac{M_p [U_D^D (E_U^D U_R^D + E_U^R U_R^R) - U_R^D (E_U^D U_D^D + E_U^R U_D^R)]}{E_U^D E_U^R (U_D^D U_R^R - U_R^D U_D^R) \Delta} \\
 &= \frac{M_p E_U^R (U_D^D U_R^R - U_R^D U_D^R)}{E_U^D E_U^R (U_D^D U_R^R - U_R^D U_D^R) \Delta} = \frac{M_p}{\Delta E_U^D}, \tag{2.21}
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial U^R}{\partial T} &= U_D^R \frac{du^D}{dT} + U_R^R \frac{du^R}{dT} \\
 &= U_D^R \frac{M_p(E_U^D U_R^D + E_U^R U_R^R)}{E_U^D E_U^R (U_D^D U_R^R - U_R^D U_D^R) \Delta} - U_R^R \frac{M_p(E_U^D U_D^D + E_U^R U_D^R)}{E_U^D E_U^R (U_D^D U_R^R - U_R^D U_D^R) \Delta} \\
 &= \frac{M_p [U_D^R (E_U^D U_R^D + E_U^R U_R^R) - U_R^R (E_U^D U_D^D + E_U^R U_D^R)]}{E_U^D E_U^R (U_D^D U_R^R - U_R^D U_D^R) \Delta} \\
 &= -\frac{M_p E_U^D (U_D^R U_R^R - U_R^R U_D^D)}{E_U^D E_U^R (U_D^D U_R^R - U_R^D U_D^R) \Delta} = -\frac{M_p}{\Delta E_U^R}. \tag{2.22}
 \end{aligned}$$

Mentioned repeatedly, Samuelson (1952) shows that if the general equilibrium satisfies the Walrasian stability condition, it is not possible for any transfer paradox to occur in the model with two countries and two goods. The following proposition is obtained by applying a similar argument to the total utility, U^i .

Proposition 2.2. *If the Walrasian stability condition is satisfied, it is not possible for any transfer paradox with regard to the total utility to occur in the model with two countries and two goods. That is, $dU^D/dT < 0$ and $dU^R/dT > 0$ are*

⁵ (2.21) and (2.22) can be also obtained by directly applying Cramer's rule to the equation of total differentiation (2.7).

satisfied.

Proof. Walrasian stability requires $\Delta > 0$. As $M_p < 0$ and $E_U^i = e_U^i > 0$ are satisfied, by (2.21) and (2.22), $dU^D/dT = M_p/\Delta E_U^D < 0$ and $dU^R/dT = -M_p/\Delta E_U^R > 0$ are immediately obtained. \square

Proposition 2.2 implies that even if the altruistic utility is introduced into the model, no transfer paradox with regard to total utility occurs in the world in which Walrasian stability is guaranteed. In other words, as regards the total utility level $U^i \equiv U^i(u^i, u^j)$, the donor can never enrich and the recipient can never immiserize by the transfer from the donor to the recipient. From this proposition, the motivation for the donor to give aid cannot be explained by its enrichment as a result of altruism. The existing literature concerning altruism has not analyzed the relationship between the total utility and the transfer paradox in an explicit manner and has been concerned about the impact that altruism has on self-utility excluding the altruistic part. By focusing on not only the self-utility but also total utility, we clarify in this proposition that the transfer paradox with regard to the total utility does not occur as long as the stability is guaranteed.

It should be noted that unlike Proposition 2.1, the proof of Proposition 2.2 does not depend on the assumption $U_D^D U_R^R > U_R^D U_D^R$. Therefore, the result of Proposition 2.2 is satisfied irrespective of how the total utility function $U^i(u^i, u^j)$ depends on u^i and u^j . Moreover, the above result remains to be seen in other general cases in which both utilities have other externalities than altruism. Even when the utility is not altruistic but negatively influenced—for example, when the utility is envious—the Walrasian stability as defined in this chapter guarantees that there is no transfer paradox with regard to the total utility.

2.4 Concluding remarks

In this chapter, we challenged the conventional wisdom that suggests that altruism motivates the donor country to give aid to the recipient. We demonstrated that in the model with two countries and two goods, even if the donor and/or recipient has or have altruistic utility, the transfer paradox can never take place with regard to both the self-utility and total utility. Although the existing litera-

ture that focuses only on selfish utility has emphasized that if Walrasian stability is guaranteed, no transfer paradox can take place, a similar result—of the impossibility of the transfer paradox—is achieved even if the extended utility is allowed to include altruism. As a result, the reason behind the donor country's transfer of economic aid to the recipient cannot be explained by the externality between the utilities of the donor and recipient, such as altruism. Irrespective of the existence of altruism, the donor country must sacrifice its own welfare for the improvement of the welfare of the other country.

The result of the study implies that the contention that altruism can raise the welfare of the people in the donor country is just an illusion. The study suggests that the motivation of compassionate or charity cannot justify the transfer activities by the people in developed countries as economic aid from donor countries to developing countries. Even if the people in developed countries possess a merciful disposition toward the people in poor countries—apart from having the economic abundance necessary for providing economic aid to these people—they cannot drive themselves to help toward the cause of poverty reduction in developing countries.

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Chapter 3

Donor Altruism and the Transfer Paradox in an Overlapping Generations Model

This chapter examines the transfer problem between two countries when a donor exhibits altruistic utility toward a recipient in a one-sector overlapping generations model. We demonstrate that if the donor has a larger marginal propensity to save than the recipient, the donor's altruism never contributes to donor enrichment irrespective of the degree of the donor's altruism. Donor enrichment occurs only if the donor has a smaller marginal propensity to save and a sufficiently high level of altruism. These findings imply that the altruism of a donor toward a recipient does not necessarily explain the motivation to voluntarily provide a transfer.

Keywords: altruism, transfer paradox, overlapping generations model, capital accumulation

JEL classifications: F11, F35, F43, O41

3.1 Introduction

Why do countries provide foreign aid to other countries? One response is that it has long been accepted that a considerable amount of voluntary aid is implemented given the pure goodwill of the nationals concerned. That is, it is well recognized that transfers as an important form of foreign aid are at least partially supported by the altruism of the donor toward the recipient. For example, according to a survey conducted in 1983 by the Japanese Ministry of Foreign Affairs, among those respondents who evaluated Japan's Official Development Assistance (ODA) program positively, the most common justification given was the stability and peace of developing countries (42.3%) followed by humanitarian obligations (32.7%).¹ The results of this survey seemingly support the argument that the presence of altruistic feelings motivates Japan's ODA. Conversely, another, and somewhat more realistic, reason for motivating a country to undertake international transfers arises from strategic considerations. That is, foreign aid is regarded as one means to achieve desirable diplomatic and/or economic policy goals for the assisting country itself. Among other things, Japan's ODA White Paper (Ministry of Foreign Affairs of Japan 2011), which emphasizes the underlying philosophy of the Japanese foreign aid program, concludes that international transfers are not so much an *act of charity* from developed countries to developing countries, but rather a *tool* for the world community to pursue common interests. In short, we can explain the motivation for foreign aid from two perspectives, goodwill (altruism) and own benefit (selfishness), and note that which particular motivation dominates has likely changed over time.

If donor altruism is the main reason for undertaking transfers, introducing donor altruism into the existing transfer problem framework should be able to explain how a highly altruistic donor can improve utility by providing transfers to a recipient. Therefore, this study examines the transfer problem between a donor and a recipient when a donor exhibits altruistic utility toward a recipient in a one-sector overlapping generations (OLG) model and investigates the possibility for the transfer paradox to arise. To achieve this objective, we clarify whether the transfer paradox is likely to occur as the degree of donor altruism

¹ See the Ministry of Foreign Affairs of Japan (1989).

becomes high, as well as how this affects the welfare levels of both countries. In particular, we provide a response to the question of whether the motivation that encourages a donor to undertake an untied voluntary transfer is goodwill or benefit. More specifically, if the donor's altruism enriches the donor in terms of utility, the transfer made by the donor country will be consistent with altruistic feelings by the donor nation and goodwill will be the main motivation for the transfer. Otherwise, altruism may not provide a reason for the donor to undertake the transfer. In this instance, we can recognize the transfer as merely another form of policy instrument for the donor government to achieve some economic or political objective. In other words, the benefit arising from the transfer will be the main motivation for the donor.

In a seminal paper on the transfer problem in a static framework, Samuelson (1952, 1954) showed that under free trade in a two-country framework, when the market equilibrium is Walrasian stable, neither a weak paradox, defined as the situation where both the donor and the recipient are enriched or immiserized (impoverished), nor a strong paradox, defined as the situation where the donor is enriched but the recipient is immiserized, can arise.² In order for the transfer paradox to take place in a stable market equilibrium, several assumptions in Samuelson (1952, 1954) need to be modified. Since his seminal work, a voluminous static framework literature has shown how the transfer paradox arises by relaxing these assumptions. Various distortions have also been introduced into the static framework in order to explain the occurrence of the transfer paradox in a two-country model.³

² The definitions of weak and strong paradox were first given by Yano (1983). We use these definitions throughout this analysis.

³ For instance, by extending the two-country model to a three-country model, Bhagwati, Brecher, and Hatta (1983) have shown the possibility of the transfer paradox when a bystander country other than the donor and recipient countries exists. Further, Bhagwati, Brecher, and Hatta (1985) have demonstrated that the transfer paradox can take place when exogenous distortions of trade barriers, such as tariffs and subsidies, or endogenous distortion, including lobbying and rent-seeking, prevail. The subsequent literature has considered the transfer paradox in a two-country model by introducing various distortions where free trade is hindered, for example, restrictions on the recipient country's behavior in exchange for the transfer (Kemp and Kojima 1985; Schweinberger 1990; Lahiri and Raimondos-Møller 1995), the administrative costs of the transfer (Kemp and Wong 1993), and the transfer of production factors (Neary 1995). For a concise survey of the transfer problem in a static model, see Brakman and van Marrewijk (1998).

When individuals exhibit altruism toward other individuals, they obtain utility not only from their own consumption, but also from the utility levels of the other individuals. In this sense, we can regard altruism as a sort of externality, and this, like many other externalities, may cause economic distortion. In actuality, some studies have suggested that altruism itself could be a source of the transfer paradox, in particular through donor enrichment. For instance, Kemp and Shimomura (2002) employed a voluntary unrequited transfer model and showed that altruism could be the motivation for the donor to provide the transfer. Similarly, Lahiri and Raimondos-Møller (1999) developed a model where altruism is introduced to motivate the transfer and international trade is distorted through tariffs or quotas. These studies have not, however, fully proven that altruism itself brings about the transfer paradox. For example, in Kemp and Shimomura (2002), the transfer paradox is derived under a quite specific type of altruistic utility, such that when the utility level of a country increases, the expenditure of the other country may also increase, despite the transfer. In Lahiri and Raimondos-Møller (1999), the transfer paradox arises not from altruism itself but from other distortions. Bearing these limitations in mind, Hamada (2012) has shown that even if the donor and/or recipient has altruistic utility, the transfer paradox never takes place in a simple two-country, two-commodity static model with no other distortion as long as the Walrasian stability of the equilibrium is guaranteed. This result implies that in a static framework, the benevolent assumption that the individuals of the donor country have altruistic intentions toward the recipient country cannot explain why the donor government voluntarily provides aid.

In contrast, because of capital accumulation and the movement of international capital in a dynamic framework, the transfer paradox can arise even with the Walrasian stability of an international capital market equilibrium under dynamic efficiency. Using a Diamond (1965) type, one-good, two-country OLG model, Galor and Polemarchakis (1987) first argued that a permanent lump-sum transfer can bring about the transfer paradox in a steady state being away from the golden rule. Subsequently, Haaparanta (1989) incorporated public debt into Galor and Polemarchakis (1987) and clarified that the transfer paradox can arise, even if the transfer is temporary, when financed by the issuance of public debt in the donor country and/or when the transfer is used for debt relief in the recipient country. Thus, when the temporary transfer relates to debt

financing and/or relieving, it has the same long-run effect as a permanent lump-sum transfer. Tan (1998) also argued that any transfer from a rich to a poor country does not cause the transfer paradox in a steady state under dynamic efficiency. Utilizing a figure developed by Buiters (1981), Yanagihara (2006) explained graphically how the transfer paradox occurs, even if the economy is dynamically efficient. Cremers and Sen (2008) extended this analysis into the transition path converging to the steady state and showed that the results in Galor and Polemarchakis (1987) can also be applied to the transition path. Overall, in a dynamic framework, there is a possibility for the transfer paradox to occur in the dynamically efficient region.⁴

In this chapter, we clarify how introducing donor altruism into the utility of individuals in the donor country affects the possibility of the transfer paradox under the dynamic efficiency condition in an OLG framework. First, we explore whether the transfer paradox arises under the existence of altruism in a dynamic framework, though this cannot be acknowledged in a static framework. To clarify the difference in the results obtained with and without altruistic utility, we focus on the circumstance where the transfer paradox cannot transpire without altruism. Second, we investigate whether the donor's altruism increases the opportunity for donor enrichment.

We demonstrate that if the donor has a larger marginal propensity to save than the recipient, the donor's altruism never contributes to donor enrichment irrespective of the degree of the donor's altruism. Otherwise, donor enrichment occurs when the donor has sufficiently high altruism. These findings imply that the altruism of a donor toward a recipient does not necessarily explain the motivation to voluntarily provide a transfer. Further, if the donor has a larger marginal propensity to save than the recipient, the altruism of the donor's individuals toward the recipient cannot justify the transfer made by the donor's government. Rather, in this case, even if the donor's altruism is sufficiently

⁴ As an exception other than the OLG model, Djajic, Lahiri, Raimondos-Møller (1999) explored the foreign aid in a one-good, two-period, two-country model of the representative agent to clarify what brings about a welfare improving outcome both for the donor and the recipient countries. They showed that the source for the welfare improving outcome attributes to the difference in the rates of capital return between the two countries. This difference comes from the assumptions of the different production technology and the imperfect capital movement between the countries.

high, the transfer may cause the weak paradox such that both the donor and the recipient are immiserized, this being the Pareto-inferior outcome for both countries. However, if the donor has a relatively smaller marginal propensity to save, the donor's altruism provides the reason why the transfer is made by the donor's government. In this case, and in contrast with the former, if the donor's altruism is sufficiently high, the transfer may account for a Pareto-improving outcome for both countries.

The remainder of this chapter is organized as follows. Section 3.2 describes a one-sector OLG model wherein the individuals of the donor country have altruistic utility. Section 3.3 gives the welfare implications with regard to whether or when the transfer paradox takes place when there is donor altruism. Section 3.4 concludes this chapter with some final remarks.

3.2 The model

Two countries comprise the world economy, a donor and the recipient of an international income transfer, indexed country $i = D$ and R , respectively. These two countries are identical except for their time preferences and the altruism of individuals. Capital is fully mobile between the countries, however, goods and labor are immobile. The (gross) growth rates of the population in both countries are exogenously given, identical, and constant over time: $1 + n \geq 1$.

3.2.1 Individuals

Individuals live for two periods. In each period, both countries are populated by two generations, the young who inelastically supply labor for one unit of time and earn wages, and the old who retire. We assume that the individuals of the donor country exhibit altruism toward the individuals of the recipient country, while the individuals of the recipient country display no altruism. $u_t^i = u^i(c_t^i, d_{t+1}^i)$ denotes the (sub-) utility function that the individuals of country i born in period t (we refer to this as generation t) obtain from their own consumption when young, c_t^i , and when old, d_{t+1}^i . In the model, the altruism of the individuals in country D is described as follows. We assume that the total utility of the donor's individuals depends on both u_t^D and u_t^R , that is, the selfish utility and the altruistic utility obtained from the utility of the recipient's

individuals. Thus, generation t in country D with altruism has the total utility function denoted by $U_t^D \equiv U^D(u_t^D, u_t^R) = U^D(u^D(c_t^D, d_{t+1}^D), u^R(c_t^R, d_{t+1}^R))$. Alternatively, as the individuals in country R do not display any altruism, their total utility is denoted by $u_t^R = u^R(c_t^R, d_{t+1}^R)$. $u^i(\cdot)$ and $U^D(\cdot)$ are assumed to be twice-differentiable, increasing, and quasi-concave in (c_t^i, d_{t+1}^i) , that is, $\partial u^i / \partial c_t^i \equiv u_c^i > 0$, $\partial u^i / \partial d_{t+1}^i \equiv u_d^i > 0$, and $u_{cc}^i u_{dd}^i - (u_{cd}^i)^2 > 0$. Moreover, we assume that $U_D^D \equiv \partial U^D / \partial u^D > U_R^D \equiv \partial U^D / \partial u^R \geq 0$, which implies that the effect of self-utility on total utility is always larger than that of altruistic utility. The larger U_R^D grows, the stronger the altruistic feelings become.

The budget constraints of generation t in their young and old periods in country $i = D, R$ are, respectively:

$$c_t^i + s_t^i = w_t + T^i \quad \text{and} \quad d_{t+1}^i = (1 + r_{t+1})s_t^i, \quad (3.1)$$

where r_{t+1} , w_t , and s_t are the net interest rate in period $t + 1$, wages in period t , and savings in period t , respectively. The net income in period t consists of the wage and a permanent international lump-sum transfer T^i . We convert these two budget constraints into the following lifetime budget constraint:

$$c_t^i + \frac{d_{t+1}^i}{1 + r_{t+1}} = w_t + T^i. \quad (3.2)$$

Individuals in both countries maximize their total utilities subject to their lifetime budget constraints (3.2). The utility maximization problem for the individuals of country i facing the lifetime budget constraint (3.2) can be formulated as:

$$\max_{\{c_t^D, d_{t+1}^D\}} U^D(u^D(c_t^D, d_{t+1}^D), u^R(c_t^R, d_{t+1}^R)), \quad \text{s.t.} \quad c_t^D + \frac{d_{t+1}^D}{1 + r_{t+1}} = w_t - T, \quad (3.3)$$

$$\max_{\{c_t^R, d_{t+1}^R\}} u^R(c_t^R, d_{t+1}^R), \quad \text{s.t.} \quad c_t^R + \frac{d_{t+1}^R}{1 + r_{t+1}} = w_t + T, \quad (3.4)$$

where $T \equiv T^R = -T^D (> 0)$ defines the permanent transfer from the donor to the recipient. It should be noted that when the individuals of country D

determine their levels of consumption, they only care about their own self-utility u^D and take the altruistic utility u^R as given. This is because they cannot control the consumption levels of individuals in another country. That is, $\arg \max U^D = \arg \max u^D$.

The first-order condition gives the optimal consumption bundle, $(c_t^i(w_t + T^i, r_{t+1}), d_{t+1}^i(w_t + T^i, r_{t+1}))$. The savings function is defined by $s_t^i = s^i(w_t + T^i, r_{t+1}) \equiv d_{t+1}^i(w_t + T^i, r_{t+1})/(1 + r_{t+1})$. As in Haaparanta (1989) and Yanagihara (1998, 2006), we assume that the savings function is increasing in both the wage and interest rate, that is, $s_w^i \equiv \partial s_t^i / \partial w_t > 0$ and $s_r^i \equiv \partial s_t^i / \partial r_{t+1} > 0$.

Substituting the optimal consumption bundle into the total utility function, we obtain the indirect utility function:

$$V^D(w_t, r_{t+1}; T) \equiv U^D\left(u^D(c_t^D(w_t - T, r_{t+1}), d_{t+1}^D(w_t - T, r_{t+1})), u^R(c_t^R(w_t + T, r_{t+1}), d_{t+1}^R(w_t + T, r_{t+1}))\right), \quad (3.5)$$

$$V^R(w_t, r_{t+1}; T) \equiv u^R\left(c_t^R(w_t + T, r_{t+1}), d_{t+1}^R(w_t + T, r_{t+1})\right). \quad (3.6)$$

Given that $u_c^i(\partial c_t^i / \partial w_t) + u_d^i(\partial d_{t+1}^i / \partial w_t) = u_c^i > 0$ and $u_c^i = u_d^i / (1 + r_{t+1}) > 0$ hold from the first-order condition, the indirect utility functions of both countries have the following properties:⁵

$$V_w^D = U_D^D u_c^D + U_R^D u_c^R, \quad V_r^D = U_D^D s_t^D u_d^D + U_R^D s_t^R u_d^R, \quad V_T^D = -U_D^D u_c^D + U_R^D u_c^R, \quad (3.7)$$

$$V_w^R = u_c^R, \quad V_r^R = s_t^R u_d^R, \quad \text{and} \quad V_T^R = u_c^R. \quad (3.8)$$

For simplicity, the marginal self-utility of consumption in the young period can be normalized to unity ($u_c^i = 1$), so that $u_d^i = 1/(1 + r_{t+1})$. Likewise, the marginal effect of self-utility on the donor's total utility can be normalized to unity. Moreover, we assume that when evaluating the neighborhood of the equilibrium, the marginal effect of the altruistic utility is less than unity: $U_D^D = 1$ and $U_R^D = \alpha \in [0, 1)$. The value of α indicates the degree of altruism of individuals in the donor country toward individuals in the recipient country.

⁵ We define $V_w^i \equiv \partial V^i / \partial w_t$, $V_r^i \equiv \partial V^i / \partial r_{t+1}$, and $V_T^i \equiv \partial V^i / \partial T$.

Thus, (3.7) and (3.8) are arranged as follows:

$$V_w^D = 1 + \alpha, \quad V_r^D = \frac{s_t^D + \alpha s_t^R}{1 + r_{t+1}}, \quad V_T^D = -(1 - \alpha), \quad (3.9)$$

$$V_w^R = 1, \quad V_r^R = \frac{s_t^R}{1 + r_{t+1}}, \quad \text{and } V_T^R = 1. \quad (3.10)$$

3.2.2 Firms

Firms in both countries produce output under perfect competition. The production function exhibits constant returns to scale in capital and labor, independent of time, and is identical in both countries. We denote the per capita production function by $f(k_t^i)$, where k_t^i represents per capita capital in country i in period t . This is assumed to satisfy the following conditions: (i) $f(k_t^i)$ is continuously differentiable and (ii) $f(k_t^i) > 0$, $f'(k_t^i) > 0$, and $f''(k_t^i) < 0$ for all $k_t^i > 0$. Moreover, we assume the Inada conditions: (iii) $f(0) = 0$ and (iv) $\lim_{k_t^i \rightarrow 0} f'(k_t^i) = \infty$ and $\lim_{k_t^i \rightarrow \infty} f'(k_t^i) = 0$. For simplicity, we assume that capital does not depreciate over time.

Firms maximize their profit in per capita terms denoted by $\pi(k_t^i) \equiv f(k_t^i) - r_t k_t^i - w_t$. Profit maximization requires the equivalence of marginal productivity and the price of each input such that:

$$f'(k_t^i) = r_t \text{ and } f(k_t^i) - f'(k_t^i)k_t^i = w_t. \quad (3.11)$$

From the first equation of (3.11), we obtain the capital demand function represented by $k_t^i(r_t)$, where $k_t^i(r_t) = 1/f'' < 0$. This implies that (per capita) capital demand is decreasing in the interest rate r_t . Similarly, from the second equation of (3.11), $w_t^i(r_t) = -k_t < 0$ is obtained.

3.2.3 Equilibrium

We consider a world capital market equilibrium in period t , which requires the sum of the per capita savings of the young generation in both countries in period t to equal the sum of per capita capital demand in the subsequent period $t + 1$. As capital is perfectly mobile, the interest rates in both countries become

the same, so that $k_{t+1}^D = k_{t+1}^R \equiv k_{t+1}$ holds through factor price equalization. Therefore, given w_t , or equivalently k_t , the capital market equilibrium in period t is expressed as follows:

$$2(1+n)k_{t+1}(r_{t+1}) = s^D(w_t(r_t) - T, r_{t+1}) + s^R(w_t(r_t) + T, r_{t+1}). \quad (3.12)$$

Define the excess demand in the world capital market as $D(w_t, r_{t+1}) \equiv 2(1+n)k_{t+1} - s_t^D - s_t^R$. Then, under the assumption that savings are increasing in the interest rate,

$$\Delta_t \equiv \frac{\partial D(w_t, r_{t+1})}{\partial r_{t+1}} = 2(1+n)k'_{t+1}(r_{t+1}) - s_r^D - s_r^R < 0 \quad (3.13)$$

holds. Therefore, the Walrasian stability condition is satisfied in the capital market equilibrium in each period.

3.3 Donor altruism and the transfer paradox

We focus on the steady-state analysis in order to investigate the effect of the transfer on the donor's and recipient's welfare when the donor has altruistic utility. As usually assumed when analyzing the steady state, following Sibert (1985), we limit our analysis to the case in which the economy is dynamically efficient, that is, $r_t > n$ for all t .

3.3.1 Savings and the interest rate

From (3.12), we immediately obtain the equilibrium condition of the world capital market in the steady state as follows:⁶

$$2(1+n)k(r) = s^D(w(r) - T, r) + s^R(w(r) + T, r). \quad (3.14)$$

In order for the economy to converge monotonically to the steady-state equilibrium, we assume the following dynamic stability condition:

⁶ Henceforth, the variables with no subscript t represent those in the steady state.

$$\begin{aligned}\Gamma \equiv \frac{dD(w(r), r)}{dr} &= 2(1+n)k'(r) - s_r^D - s_r^R - (s_w^D + s_w^R)w'(r) \\ &= \Delta + (s_w^D + s_w^R)k < 0,\end{aligned}\quad (3.15)$$

where $\Delta \equiv 2(1+n)k'(r) - s_r^D - s_r^R$, and $\Delta < 0$ under the Walrasian stability condition. It should be noted that the difference between Γ and Δ is $(s_w^D + s_w^R)k$, as obviously obtained from comparing (3.13) with (3.15). This difference comes from the fact that the change in the interest rate affects wages through the change in factor demand by firms and this brings about the long-run effect through the change in capital accumulation.

By totally differentiating (3.14), we obtain the effect of the transfer on the interest rate as follows:

$$\Gamma dr = (s_w^R - s_w^D)dT. \quad (3.16)$$

With respect to the difference in the marginal propensity to save between the donor and the recipient, we obtain the following lemma. All proofs are in the Appendix.

Lemma 3.1. *Suppose that the time preference of the recipient is higher (lower) than that of the donor. In the steady state,*

- (i) $s^R < (1+n)k < s^D$ ($s^R > (1+n)k > s^D$) and
- (ii) *the transfer increases (decreases) the interest rate.*

It should be noted that Lemma 3.1 holds irrespective of the degree of donor altruism. Part (i) of this lemma implies that the individuals in the country with higher time preference have smaller savings and they therefore need more capital to invest from the other country. In contrast, the individuals in the country with lower time preference have larger savings and can therefore supply their capital to the other country, which is in need of capital. Therefore, the difference in the marginal propensity to save between the donor and the recipient, $(s_w^R - s_w^D)$, determines the direction of international capital movement between the two countries. If $s_w^R < s_w^D$, the donor is the lender of capital and the recipient is the borrower of capital; otherwise, vice versa.

Part (ii) of Lemma 3.1 implies the following. As the recipient with higher

time preference has a lower marginal propensity to save than the donor, the donor becomes the lender of capital and the recipient becomes the borrower of capital. Although the savings of the donor decrease and those of the recipient increase given the transfer from the donor to the recipient, total world capital decreases. As a result, if the marginal propensity to save is higher in the donor, the transfer from the donor to the recipient brings about an increase in the interest rate; otherwise, vice versa.

3.3.2 The effect of the transfer on welfare

We now proceed to investigate the effect on welfare in the steady state, as defined by the representative individual's utility in the country. By totally differentiating the indirect utility functions in the steady state and substituting (A.3.3), (3.10), and $dw = -kdr$ into them, we obtain the following equations.

$$\begin{aligned} dV^D &= V_w^D dw + V_r^D dr + V_T^D dT \\ &= \left(-(1+\alpha)k + \frac{s^D + \alpha s^R}{1+r} \right) dr - (1-\alpha)dT, \end{aligned} \quad (3.17)$$

$$\begin{aligned} dV^R &= V_w^R dw + V_r^R dr + V_T^R dT \\ &= \left(-k + \frac{s^R}{1+r} \right) dr + dT. \end{aligned} \quad (3.18)$$

From (3.17) and (3.18), we find that both the donor's and the recipient's welfare are affected by the changes in both the interest rate and the transfer, as shown in existing studies. By substituting (3.16) into (3.17) and (3.18), we obtain the effects on welfare in both the donor and recipient countries as follows:

$$\begin{aligned} dV^D &= \left\{ \underbrace{\left[(s^D - (1+r)k) + (s^R - (1+r)k)\alpha \right] \frac{s_w^R - s_w^D}{(1+r)\Gamma}}_{\substack{\text{(the indirect effect)} \\ \text{=(the intertemporal terms-of-trade effect)}}} - \underbrace{(1-\alpha)}_{\substack{\text{(the direct effect)} \\ \text{=(income effect)}}} \right\} dT, \\ &\hspace{25em} (3.19) \end{aligned}$$

$$dV^R = \left[\underbrace{(s^R - (1+r)k) \frac{s_w^R - s_w^D}{(1+r)\Gamma}}_{\substack{\text{(the indirect effect)} \\ \text{=(the intertemporal terms-of-trade effect)}}} + \underbrace{1}_{\substack{\text{(the direct effect)} \\ \text{=(income effect)}}} \right] dT. \quad (3.20)$$

The total effect of the transfer on welfare is divided into two parts, a direct effect and an indirect effect. The direct effect is the income effect brought about by the change in the income level of both countries by the transfer itself. This effect is necessarily positive for the recipient country and negative for the donor country, and is shown by the second term in both (3.19) and (3.20). The indirect effect is what we call an intertemporal terms-of-trade effect and is shown by the first term in both (3.19) and (3.20). The indirect effect is the result of the following process: changes in the level of income given the transfer alter the pattern of consumption and savings, which in turn change the level of world capital, unless the marginal propensities to save in both countries are the same. The change in world capital then brings about a permanent change in the interest rate.

Noting that $s^i - (1+r)k = -(r-n)k + (s^i - (1+n)k)$, the indirect intertemporal terms-of-trade effect can be further decomposed into two effects, the capital accumulation effect and the capital movement effect. For example, in (3.20), the former is shown by $-(r-n)k \frac{s_w^R - s_w^D}{(1+r)\Gamma}$, because $-(r-n)k$ indicates how the level of capital moves away from the level in the golden rule: when the economy is dynamically efficient, as assumed, and capital accumulation is promoted, the increase in production given the increase in accumulated capital brings about an increase in the utility of both countries. Alternatively, the capital movement effect is shown by $(s^R - (1+n)k) \frac{s_w^R - s_w^D}{(1+r)\Gamma}$, where $(s^R - (1+n)k)$ represents the direction and amount of capital movement: the country in which this term is positive (negative) becomes the lender (borrower) of capital. If world capital is accumulated and, as a result, the interest rate decreases, the capital movement effect decreases the welfare of the capital lender and increases that of the capital borrower. It should be noted here that the capital accumulation effect works in the same direction as the level of welfare in both countries, whereas the capital movement effect works in the opposite direction to the level of welfare in both countries. Therefore, the difference in the impact of the capital movement effect between the donor and the recipient provides one source of the transfer

paradox.

As an illustrative example, consider the case in which the donor has a larger marginal propensity to save than the recipient, that is, $s_w^R < s_w^D$. Since $\frac{s_w^R - s_w^D}{(1+r)^T} > 0$, the sign of the recipient's indirect effect of (3.20) corresponds to the sign of $(s^R - (1+r)k) = -(r-n)k + (s^R - (1+n)k)$. The first term, $-(r-n)k$, which determines the sign of the capital accumulation effect, is always negative because in a dynamically efficient situation the transfer decreases world per capita capital. The second term, $(s^R - (1+n)k)$, which determines the sign of the capital movement effect, becomes negative because the recipient country is a capital borrower given Lemma 3.1. The intertemporal terms-of-trade effect then consists of both the above effects.

As shown in (3.19) and (3.20), the donor's indirect effect depends not only on $(s^D - (1+r)k)$ but also on $(s^R - (1+r)k)$ because of the existence of altruism, while the recipient's indirect effect depends only on $(s^R - (1+r)k)$. $(s^i - (1+r)k), i = D, R$ is positive (negative) if and only if the capital movement effect is larger (smaller) than the capital accumulation effect. Therefore, if the capital accumulation effect is sufficiently larger than the capital movement effect, the indirect effects for both countries become negative. On the contrary, if the capital accumulation effect is sufficiently small, it is likely that the directions of the donor's and the recipient's indirect effects differ. Thus, $(s^i - (1+r)k)$ determines the sign of the indirect effect.

As the degree of donor altruism becomes higher, the direct income effect, which is negative for the donor country, becomes smaller. In particular, if $\alpha = 1$ at the most extreme, it is nullified. However, the indirect terms-of-trade effect remains, even though the degree of donor altruism increases up to the extreme. Therefore, the indirect effect triggers the occurrence of the transfer paradox.

3.3.3 The transfer paradox

We are now in a position to clarify the conditions for the transfer paradox to occur in the steady state. First, we examine the indirect effect of the transfer on welfare. The following two lemmas summarize the sign of the donor's and the recipient's indirect effects.

Lemma 3.2. *In the steady-state equilibrium,*

(i) *if $s_w^R < s_w^D$ and $s^D \leq (1+r)k$, the donor's indirect effect is always negative*

irrespective of the degree of donor altruism;

(ii) if $s_w^R < s_w^D$ and $s^D > (1+r)k$, there exists a threshold of α , α_0 , such that the donor's indirect effect is positive if $\alpha < \alpha_0$, and negative if $\alpha > \alpha_0$;

(iii) if $s_w^R > s_w^D$, the donor's indirect effect is always positive irrespective of the degree of donor altruism.

Lemma 3.3. *In the steady-state equilibrium, the recipient's indirect effect is independent of the degree of donor altruism. Moreover,*

(i) if $s_w^R < s_w^D$ or if $s_w^R > s_w^D$ and $s^R > (1+r)k$, the recipient's indirect effect is negative;

(ii) if $s_w^R > s_w^D$ and $s^R \leq (1+r)k$, the recipient's indirect effect is positive.

As shown in (3.19), when $s_w^R < s_w^D$, the donor's indirect effect becomes negative when the capital accumulation effect dominates the capital movement effect. Consider when the donor's marginal propensity to save is higher than that of the recipient, but not so high that $s^D \leq (1+r)k$ holds. Then, the transfer from the donor to the recipient decreases world total savings and increases the interest rate. However, this increase in the interest rate does not bring about a sufficiently large positive capital movement effect to dominate the negative capital accumulation effect. In addition, under dynamic efficiency, $r > n$, $s^R < (1+r)k$ necessarily holds. As a result, the donor's indirect effect becomes negative. This case corresponds to Lemma 3.2(i). If the donor has such a large marginal propensity to save that the capital lending is sufficiently large, then there is a possibility for the positive capital movement effect to dominate the negative capital accumulation effect. Even in this case, however, as the altruistic donor considers the negative impact of the indirect effect on the recipient, the greater donor altruism weakens the positive capital movement effect on the donor. This implies that a threshold for the degree of donor altruism exists, such that the sign of the donor's indirect effect reverses. Lemma 3.2(ii) states this case. Contrary to the above two cases, Lemma 3.2(iii) insists that if the donor has a smaller marginal propensity to save than the recipient, the donor's indirect effect is necessarily positive. In this case, the transfer increases world total savings and lowers the interest rate. The former brings about positive capital accumulation effects in both countries and the latter imposes a positive capital movement effect on the capital lender, that is, the donor. In fact, as under dynamic efficiency $s^D + s^R = 2(1+n)k < 2(1+r)k$ holds, the donor's

indirect effect is necessarily positive.

In contrast to the effect on the donor, because the recipient is not at all altruistic towards the donor, it is sufficient only to investigate the indirect effect on the recipient's own utility. As shown in (3.20), the recipient's indirect effect depends only on $(s^R - (1 + r)k)$. Thus, the configuration of s^R and $(1 + r)k$ completely determines the sign of the recipient's indirect effect. Lemma 3.3 implies that if the donor has a larger marginal propensity to save than the recipient, the transfer decreases world capital, which represents the negative capital accumulation effect, and increases the interest rate, which represents the negative capital movement effect on the recipient in the steady state, as with Lemma 3.2(i) and (ii). In fact, $s^R < (1 + n)k < (1 + r)k$ holds under dynamic efficiency regardless of the presence of donor altruism.

This result lies in contrast to that in Lemma 3.2(iii). If, however, the recipient has a larger marginal propensity to save than the donor, even though the capital accumulation effect is positive, the capital movement effect on the recipient becomes negative. Therefore, as has already been pointed out regarding the existing literature, Lemma 3.2 and Lemma 3.3 confirm that the direction of the indirect effect is determined by the configuration of the above two countervailing effects.

Now we examine how the altruism of the donor has an effect on the occurrence of the transfer paradox. As the recipient's welfare does not depend on the degree of donor altruism, we investigate only the possibility for donor enrichment, in the situation where no kind of transfer paradox—strong or weak—occurs without the donor's altruism. We present the following proposition concerning donor enrichment.

Proposition 3.1. *Suppose the situation where no transfer paradox occurs without the donor's altruism. In the steady-state equilibrium,*

- (i) *when the donor has a larger marginal propensity to save than the recipient, donor enrichment never occurs, irrespective of the degree of donor altruism;*
- (ii) *when the donor has a smaller marginal propensity to save than the recipient, donor enrichment occurs if the donor displays sufficient altruism.*

Proposition 3.1 specifies the condition for donor enrichment to occur in the *normal situation*, where no transfer paradox occurs without donor altruism. Proposition 3.1(i) implies that altruism never contributes to bringing about

paradoxical effects on the donor in the normal situation. As shown in (3.19), the direct effect is necessarily negative, even though it is weakened by the introduction of altruism. In addition, because $(s^D - (1+r)k) \frac{s_w^R - s_w^D}{(1+r)I} < 0$ holds in the normal situation, and $s^R < (1+n)k < (1+r)k$ holds given dynamic efficiency, the indirect effect is also negative. It should be noted that when $s_w^R < s_w^D$, altruism only magnifies the negative indirect effect and never compensates for the negative direct effect on the donor itself. In sum, when the donor has a larger saving propensity, the transfer always worsens donor welfare, irrespective of the existence of altruism.

In contrast, Proposition 3.1(ii) asserts that when the donor has a smaller marginal propensity to save than the recipient, donor enrichment occurs if the donor has sufficient altruism toward the recipient. This can be explained as follows. As shown in Lemma 3.2(iii), when $s_w^R > s_w^D$, the donor's indirect effect is always positive irrespective of the degree of donor altruism, whereas the direct effect is negative. When α approaches unity, although the donor's negative direct effect is nullified, the positive indirect effect still exists. As a result, when the donor has a smaller saving propensity, donor altruism leads to improving donor welfare when the degree of donor altruism is sufficiently large. Intuitively, the increase in world capital given the transfer decreases the interest rate, which in turn decreases the return for capital borrowing from the recipient, and this indirect effect dominates the negative direct effect, which becomes weaker as the degree of altruism becomes larger.

The above result suggests that whether the donor's altruism can explain the motivation for the transfer depends on the relative size of the marginal propensity to save between the donor and the recipient. If the donor has a larger marginal propensity to save than the recipient, which we might often expect in real-world situations, the donor's altruism cannot explain the motivation for the donor to undertake the transfer because the transfer never enriches the donor. In contrast, if the donor has a smaller marginal propensity to save than the recipient, the result that the good you do for others is good you do yourself can be acknowledged and the donor's altruism then brings about an improvement in its own welfare.

Next, we extend the argument in the above normal situation to a general situation where the transfer paradox might occur even if there is no donor altruism. Our aim in this extension is to examine how the donor's altruism has an effect

on the occurrence of a strong transfer paradox. Generally speaking, when there is no donor altruism, as supposed by Galor and Polemarchakis (1987), the following four cases can be considered: (a) No transfer paradox occurs, that is, neither the donor's enrichment nor the recipient's immiserization occurs; (b) Only a weak transfer paradox in which the donor is enriched occurs; (c) Another weak transfer paradox in which the recipient is immiserized occurs; (d) The strong transfer paradox occurs, that is, both the donor's enrichment and the recipient's immiserization occur at the same time. Although we have only dealt with Case (a) in Proposition 3.1, we now extend the analysis to cover all four cases.

When we examine the general situation including Cases (a)–(d), we provide the following proposition about the possibility of the donor's paradoxical enrichment.

Proposition 3.2.

(i) *Suppose in the steady-state equilibrium that the donor has a larger marginal propensity to save than the recipient. If the donor has sufficiently high altruism, the transfer never enriches the donor. That is, when $s_w^R < s_w^D$, $dV^D/dT < 0$ is likely to hold as α approaches unity.*

(ii) *Suppose in the steady-state equilibrium that the donor has a smaller marginal propensity to save than the recipient. If the donor has sufficiently high altruism, the transfer necessarily enriches the donor. That is, when $s_w^R > s_w^D$, $dV^D/dT > 0$ is likely to hold as α approaches unity.*

Proposition 3.2 implies that whether the transfer enriches the donor with strong altruism depends on the relative size of the marginal propensity to save between the donor and the recipient. Proposition 3.2(i) shows that if the donor has a large marginal propensity to save, the transfer necessarily reduces the donor's own welfare, even when the donor has sufficiently large altruism towards the recipient. This result seems to be counterintuitive at first glance, as it is widely believed that one of the motivations for international transfers is the principle of reciprocity. However, this result suggests that such a reciprocal view, that charity brings its own reward, does not necessarily hold. In practice, when the transfer is made from the developed country as a donor, which is usually a higher savings country, to the developing country as a recipient, which is usually a lower savings country, the developed country as a donor cannot enjoy

the benefit of its own welfare improvement through the transfer.

The intuition behind this result is essentially the same as that in Proposition 3.1(i). As α approaches unity, the welfare of the donor becomes equal to world welfare, consisting of the sum of the donor's and the recipient's welfare. In such a situation, the direct income effect vanishes and only the indirect effect remains. In the remaining indirect effect, although the capital movement effect also vanishes, the capital accumulation effect remains. As a result, as the transfer decreases world capital in the steady state under dynamic efficiency, the donor's welfare necessarily decreases.

In contrast, if the donor has a smaller marginal propensity to save than the recipient, a different result can be derived. Proposition 3.2(ii) insists that when the donor has a smaller marginal propensity to save, if the donor is highly altruistic, the transfer necessarily improves the donor's own welfare. In this case, the result appears as expected and the principle of reciprocity can explain the motivation for an international transfer by the donor. Stated differently, the old proverb that charity brings its own reward holds for the donor that makes smaller savings, that is, borrows capital. The basic logic of Proposition 3.2 is quite natural: as long as more world capital is accumulated by the transfer from the donor with smaller savings and high altruism to the recipient with larger savings, the donor can obtain a benefit from capital accumulation that exceeds the loss from the reduction in income caused by the transfer. Therefore, although greater donor altruism can be compatible with donor enrichment when the donor's marginal propensity to save is lower than that of the recipient, the donor's greater altruism by itself cannot cause donor enrichment in the opposite case.

In order to delineate the results shown in Proposition 3.2, we present Table 3.1 by classifying the effects of transfers *without* donor altruism into the above four cases. From Table 3.1, we can confirm that when $s_w^R < s_w^D$, in most cases the donor becomes worse off, except where $\alpha < \alpha_1$ in Case (d). This implies that the strong transfer paradox never occurs if the degree of donor altruism is sufficiently large when $s_w^R < s_w^D$. Furthermore, as shown in Table 3.1, the transfer necessarily brings about a decrease in the welfare of either the donor or the recipient. In contrast, when $s_w^R > s_w^D$, the donor becomes better off in most cases except where $\alpha < \alpha_1$ in Case (a). In this case, there exists the possibility of a strong transfer paradox, and the weak paradox under

Table 3.1: The effect of the transfer on welfare

Case (a): $\left(\frac{dV^D}{dT}\Big _{\alpha=0}, \frac{dV^R}{dT}\Big _{\alpha=0}\right) = (-, +)$			Case (b): $\left(\frac{dV^D}{dT}\Big _{\alpha=0}, \frac{dV^R}{dT}\Big _{\alpha=0}\right) = (+, +)$		
	$s_w^R < s_w^D$	$s_w^R > s_w^D$		$s_w^R < s_w^D$	$s_w^R > s_w^D$
Donor	-	- if $\alpha < \alpha_1$ + if $\alpha > \alpha_1$	Donor	N/A	+
Recipient	+	+	Recipient		+

Case (c): $\left(\frac{dV^D}{dT}\Big _{\alpha=0}, \frac{dV^R}{dT}\Big _{\alpha=0}\right) = (-, -)$			Case (d): $\left(\frac{dV^D}{dT}\Big _{\alpha=0}, \frac{dV^R}{dT}\Big _{\alpha=0}\right) = (+, -)$		
	$s_w^R < s_w^D$	$s_w^R > s_w^D$		$s_w^R < s_w^D$	$s_w^R > s_w^D$
Donor	-	N/A	Donor	+ if $\alpha < \alpha_1$ - if $\alpha > \alpha_1$	+
Recipient	-		Recipient	-	-

Note: Define $\alpha_1 \equiv -\Lambda^D/\Lambda^R$. When $s_w^R < s_w^D$, Case (b) cannot occur and when $s_w^R > s_w^D$, Case (c) cannot occur.

which welfare in both countries deteriorates can never occur. It should be noted, however, that as shown in Case (d), the strong transfer paradox occurs in the circumstance *with donor altruism* only if it occurs *without donor altruism* in the first instance. Therefore, the strength of the donor's altruism itself cannot provide any rationale for the strong paradox.

Our result lies in stark contrast to that obtained in the static framework, in which no transfer paradox occurs, even when a donor exhibits altruism towards its recipient. In the static framework, even though the direct income effect of the transfer countervails the indirect terms-of-trade effect, the latter can never dominate the former under Walrasian stability. As long as the Walrasian stability condition holds, the total effect of the transfer necessarily brings about a decrease in the donor's welfare and an increase in the recipient's welfare in the static setting. In contrast, in our dynamic framework, even when the world capital market equilibrium is Walrasian stable in every period, we have shown that the indirect effect, including that for capital accumulation, could exceed the direct effect to bring about the transfer paradox in the steady state. In particular, we demonstrate that whether the degree of donor altruism raises the possibility of the donor's enrichment paradox depends on the difference in the marginal

propensity to save between the donor and the recipient. This discrepancy between the static and dynamic models is mainly attributed to the intertemporal terms-of-trade effect, although whether the transfer paradox arises when there is donor altruism depends on the relative size of the marginal propensity to save in both countries.

3.4 Concluding remarks

This chapter examined the transfer problem between two countries when a donor exhibits altruistic utility toward a recipient in a one-sector overlapping generations model. We showed that if the donor has a larger marginal propensity to save than the recipient, the donor's altruism never contributes to donor enrichment irrespective of the degree of donor altruism; otherwise, the donor's enrichment occurs when the donor has sufficiently large altruism. Furthermore, we suggested that when the degree of donor altruism is sufficiently large, if the donor has a large marginal propensity to save, a Pareto-inferior but no Pareto-improving result is likely to occur. In contrast, given sufficiently large donor altruism, if the donor has a small marginal propensity to save, a Pareto-improving but no Pareto-inferior result is likely to occur. Our result thus suggests that if the donor is a higher saving country compared with the recipient, transfer on the basis of goodwill may bring about a Pareto-inferior outcome for both countries. Therefore, if the donor is a high saving country, the donor's altruism cannot explain the motivation for transfer.

We should note that our results can be interpreted as another kind of paradox. On one hand, it can be generally accepted that the capital lenders are the developed countries, and therefore, *the rich countries*, and vice versa. On the other hand, it is also agreed that *the rich* is willing to give a donation to *the poor* from a deep compassion of the rich for the poor. In our setting where the donor only exhibits altruistic utility toward a recipient, however, the transfer from the rich to the poor *never enriches* the rich itself, while the transfer from the poor to the rich *is possible to enrich* the poor itself. This means that the rich does not have any incentive to make a transfer, and rather, the poor might make a transfer to improve its own welfare. Therefore, our findings portray a novel paradoxical outcome for donation, in addition to the transfer paradox in

a traditional context.⁷

Finally, we conclude this chapter with some possible future extensions. To start with, we limit our analysis to a model in which only the donor exhibits altruism toward a recipient. As a direct extension, and even though understandably complex, this analysis should be extended to a generalized model in which both the donor and the recipient exhibit altruism toward each other. However, even were we to extend the model to this case, we expect the fundamental results in this study to hold. Nonetheless, through this extension we could obtain a more fruitful conclusion about the impact of the different degrees of altruism on donors and recipients. As another extension, we focus only on the steady-state equilibrium, and so future analysis could consider the transition path. Either of these extensions would yield useful insights into the relationship between altruistic behavior and international transfer activities.

Appendix

A.3.1 Proof of Lemma 3.1

(i) In the steady state, the time preference of the recipient is higher (lower) than that of the donor, if and only if the marginal propensity to save of the recipient is lower (higher) than that of the donor. That is, when $s_w^R < s_w^D$ holds for all t , the steady-state savings of the recipient become necessarily smaller than those of the donor, that is, $s^R < s^D$, irrespective of the initial capital level. By combining $s^R < s^D$ with (3.14), we obtain $s^R < (1+n)k < s^D$. In contrast, when $s_w^R > s_w^D$ for all t , the opposite result holds. (ii) By (3.16), we immediately obtain that $dr/dT \gtrless 0$ if and only if $s_w^R \lessgtr s_w^D$. \square

A.3.2 Proof of Lemma 3.2

Denote the donor's indirect effect as $X(\alpha) \frac{s_w^R - s_w^D}{(1+r)\Gamma}$, where $X(\alpha) \equiv (s^R - (1+n)k)\alpha + (s^D - (1+r)k)$. Because of the dynamic stability condition, $\Gamma < 0$,

⁷ We are grateful to Isidoro Mazza for drawing our attention to another paradoxical interpretation of our conclusion.

$s_w^R \leq s_w^D$ if and only if $\frac{s_w^R - s_w^D}{(1+r)\Gamma} \geq 0$. The sign of the donor's indirect effect is determined by the sign of $X(\alpha)$. From Lemma 3.1, $s_w^R \leq s_w^D$ if and only if $s^R \leq (1+n)k \leq s^D$. Under the dynamic efficiency condition, $r > n$, if $s_w^R < s_w^D$, $s^R < (1+r)k$ holds, and if $s_w^R > s_w^D$, $s^D < (1+r)k$ holds. $X(1) = s^D + s^R - 2(1+r)k = 2(n-r)k < 0$ holds by the market-clearing condition and the dynamic efficiency condition. When $s_w^R < s_w^D$, $X(\alpha)$ is linearly decreasing in α because $s^R < (1+r)k$.

(i) If $s^D \leq (1+r)k$, $X(0) \leq 0$ holds. Thus, $X(\alpha) < 0$ for all $\alpha \in (0, 1)$. $X(\alpha) \frac{s_w^R - s_w^D}{(1+r)\Gamma}$ is negative.

(ii) If $s^D > (1+r)k$, $X(0) > 0$ holds. By $X(1) < 0$ and the fact that $X(\alpha)$ is linearly decreasing, we obtain that there exists a threshold of α , α^D , such that the effect is positive if $\alpha < \alpha^D$ and negative otherwise.

(iii) When $s_w^R > s_w^D$, whether $s^R < (1+r)k$ or $s^R > (1+r)k$ is indeterminate. Thus, whether $X(\alpha)$ is linearly increasing or decreasing in α is not determined. However, in this case, $X(0) < 0$ because $s^D < (1+r)k$. By combining $X(0) < 0$ with $X(1) < 0$, we obtain that $X(\alpha) < 0$ for all $\alpha \in (0, 1)$. Thus, $X(\alpha) \frac{s_w^R - s_w^D}{(1+r)\Gamma}$ is positive. \square

A.3.3 Proof of Lemma 3.3

The sign of the recipient's indirect effect $(s^R - (1+r)k) \frac{s_w^R - s_w^D}{(1+r)\Gamma}$ depends on the signs of $(s^R - (1+r)k)$ and $(s_w^R - s_w^D)$, because, like the proof of Lemma 3.2, $s_w^R \leq s_w^D$ if and only if $\frac{s_w^R - s_w^D}{(1+r)\Gamma} \geq 0$ and $s^R \leq (1+n)k \leq s^D$. Under dynamic efficiency, if $s_w^R < s_w^D$, $s^R < (1+r)k$ holds, and if $s_w^R > s_w^D$, $s^D < (1+r)k$ holds. If $s_w^R < s_w^D$, the recipient's indirect effect is negative because $s^R < (1+r)k$. In contrast, when $s_w^R > s_w^D$, the sign of the recipient's indirect effect corresponds with that of $(s^R - (1+r)k)$. \square

A.3.4 Proof of Proposition 3.1

Denote $\Lambda^D \equiv (s^D - (1+r)k) \frac{s_w^R - s_w^D}{(1+r)\Gamma} - 1$ and $\Lambda^R \equiv (s^R - (1+r)k) \frac{s_w^R - s_w^D}{(1+r)\Gamma} + 1$. (3.19) and (3.20) are rewritten as follows:

$$\frac{dV^D}{dT} = \Lambda^D + \alpha \Lambda^R, \quad (\text{A.3.1})$$

$$\frac{dV^R}{dT} = \Lambda^R. \quad (\text{A.3.2})$$

Note that $\Lambda^D = (dV^D/dT)|_{\alpha=0}$. Suppose $\Lambda^D < 0$ and $\Lambda^R > 0$. By (A.3.1), dV^D/dT is linearly increasing with α . By substituting $\alpha = 1$ into (A.3.1) and noting that $s^D + s^R < 2(1+r)k$ under dynamic efficiency, we obtain the following equation.

$$\left. \frac{dV^D}{dT} \right|_{\alpha=1} = [s^D + s^R - 2(1+r)k] \frac{s_w^R - s_w^D}{(1+r)\Gamma} \leq 0 \quad \text{if and only if} \quad s_w^R \leq s_w^D. \quad (\text{A.3.3})$$

By (A.3.3), we obtain that when $s_w^R < s_w^D$, $dV^D/dT < 0$ holds for all $\alpha \in [0, 1]$. In contrast, we obtain that when $s_w^R > s_w^D$, $dV^D/dT < 0$ if $\alpha < \alpha_1 \equiv -\Lambda^D/\Lambda^R$ and $dV^D/dT > 0$ holds if $\alpha > \alpha_1$. \square

A.3.5 Proof of Proposition 3.2

All the cases that we examine in the general situation are denoted as follows: (a) $(\Lambda^D, \Lambda^R) = (-, +)$; (b) $(\Lambda^D, \Lambda^R) = (+, +)$; (c) $(\Lambda^D, \Lambda^R) = (-, -)$; (d) $(\Lambda^D, \Lambda^R) = (+, -)$. However, we can prove that Case (b) cannot occur when $s_w^R < s_w^D$, and Case (c) cannot occur when $s_w^R > s_w^D$. The proofs are by contradiction. Note that $s^D + s^R < 2(1+r)k$ holds under the dynamically efficient condition. Suppose that $\Lambda^D > 0$ and $\Lambda^R > 0$ hold when $s_w^R < s_w^D$. $\Lambda^D > 0$ and $\Lambda^R > 0$ hold if and only if $(s^D - (1+r)k) \frac{s_w^R - s_w^D}{(1+r)\Gamma} > 1 > -(s^R - (1+r)k) \frac{s_w^R - s_w^D}{(1+r)\Gamma}$. Given $\frac{s_w^R - s_w^D}{(1+r)\Gamma} > 0$, $s^D - (1+r)k > -(s^R - (1+r)k)$. This contradicts $s^D + s^R < 2(1+r)k$. Likewise, suppose that $\Lambda^D < 0$ and $\Lambda^R < 0$ when $s_w^R > s_w^D$. $\Lambda^D < 0$ and $\Lambda^R < 0$ if and only if $-(s^R - (1+r)k) \frac{s_w^R - s_w^D}{(1+r)\Gamma} > 1 > (s^D - (1+r)k) \frac{s_w^R - s_w^D}{(1+r)\Gamma}$.

As $\frac{s_w^R - s_w^D}{(1+r)\Gamma} < 0$, $-(s^R - (1+r)k) < s^D - (1+r)k$. This also contradicts $s^D + s^R < 2(1+r)k$.

Similarly to the proof of Proposition 3.1, (A.3.3) is satisfied under $\alpha = 1$ in all the cases. By the continuity of the function dV^D/dT with regard to α , it is satisfied that $dV^D/dT \leq 0$ if and only if $s_w^R \leq s_w^D$ when α is in the neighborhood of unity. \square

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Chapter 4

Intergenerational Altruism and the Transfer Paradox

This chapter investigates how intergenerational altruism affects the possibility of the transfer paradox occurring in a two-country, one-sector overlapping generations model. We derive the conditions under which the transfer paradox occurs in our model where a generation within each country has altruism toward the subsequent generation. Contrary to earlier results in the literature, we find that altruism does not enter the conditions under which the transfer paradox occurs in the steady state. Moreover, we show that although altruism affects the condition of the transfer paradox along the transition path, its effect on this condition vanishes as the economy converges to the steady state.

Keywords: intergenerational altruism, transfer paradox, overlapping generations model

JEL classifications: D64, E22, F11, F35

4.1 Introduction

Ever since the seminal paper of Bernheim and Ray (1987) considered intergenerational altruism in an aggregative growth model, many macroeconomists have been interested in intergenerational altruism itself and the issues that arise within a framework of intergenerational altruism.¹ For example, Bernheim and Ray (1987) examined the properties of equilibrium behavior in an aggregative growth model and analyzed the normative properties of the steady-state equilibrium. Ray (1987) and Hori and Kanaya (1989) described the characteristics of the steady-state equilibrium and investigated the conditions for its existence and uniqueness in a model of nonpaternalistic intergenerational altruism. Moreover, Bernheim (1989) characterized the welfare properties of a dynastic equilibrium within a framework of intergenerational altruism, while Hori (1997) considered dynamic allocation in an altruistic overlapping generations (OLG) economy and pointed out the possibility that an equilibrium path is generally Pareto suboptimal. Following these studies, a considerable number of authors have dealt with various issues related to intergenerational altruism. For example, environmental issues are a significant concern within a framework of intergenerational altruism because future generations suffer from negative pollution externalities (Jouvet, Michel, and Vidal 2000).

This chapter focuses on another issue related to intergenerational altruism, namely the analysis of the transfer problem, which holds a central place in the literature on the theory of international trade in both static and dynamic frameworks. The transfer problem has long attracted the attention of economists since Keynes (1929) pointed out that, in contrast to the general perception, a transfer is likely to reduce the transferer's welfare. Over the past 85 years, the possibility of such a paradoxical result, namely the transfer paradox, occurring has attracted a substantial amount of theoretical attention by international trade researchers. In a static framework in which the transfer problem is considered, it is widely established that some distortions or hindrances to free trade are required for the transfer paradox to occur in a two-country model, such as the exogenous distortions of trade barriers (tariffs or subsidies) or endogenous dis-

¹ Michel, Thibault, and Vidal (2006) have comprehensively surveyed intergenerational altruism in neoclassical growth models.

tortions (rent seeking or the administrative costs of transfer).² By contrast, in a dynamic framework, the existing literature has clarified that because of capital accumulation and international capital movements, the transfer paradox can occur under free trade and dynamic efficiency, even when there is no distortion. For instance, by using an OLG model, Galor and Polemarchakis (1987) argued that a permanent lump-sum transfer can bring about the transfer paradox in the steady-state equilibrium. Haaparanta (1989) proved that a transfer paradox can occur when the temporary transfer is financed by public debt in the donor country and/or is used for debt relief in the recipient country. This occurs because a temporary transfer involving debt-financed debt relief is equivalent to a permanent lump-sum transfer. Further, Cremers and Sen (2008) extended the analysis to the transition to the steady state and proved that the results obtained in Galor and Polemarchakis (1987) could also be applied to the transition. Overall, in a dynamic framework, it is not unusual for the transfer paradox to occur in the dynamically efficient region.

Thus, we have a question: if individuals are intergenerationally altruistic within a country, is the transfer paradox likely to occur in the steady state? Generally, when individuals are altruistic, they take the utility of other individuals into account as a component of their own utility, which implies that altruism could be regarded as a type of externality and, as a result, could cause distortion. Thus, the introduction of altruism into a dynamic model might change the conditions under which the transfer paradox occurs. Although very few studies have examined the transfer paradox with altruism in an OLG model, Hamada and Yanagihara (2014) clarified that the introduction of altruism toward the individuals of the other country in the model affects the likelihood of the transfer paradox in the steady state under dynamic efficiency. They demonstrated that no transfer can enrich a donor as long as the donor is highly altruistic, whereas a transfer may immiserize a recipient if the recipient is highly altruistic. Stated differently, in contrast to conventional wisdom, as individuals become highly altruistic, the transfer is likely to cause a Pareto-inferior outcome for both countries. However, Hamada and Yanagihara (2014) dealt only with the altruism that exists between a donor and a recipient country, not with intergenerational

² For a seminal paper, see Bhagwati, Brecher, and Hatta (1985). Brakman and van Marrewijk (1998) presented a concise survey of the transfer problem in a static model.

altruism within a country.

This study bridges this gap in the body of knowledge by attempting to examine whether and how the condition under which the transfer paradox occurs in the steady state is affected by the introduction of intergenerational altruism into a one-sector OLG model. It demonstrates that although intergenerational altruism amplifies the effect of the transfer on welfare, it never affects whether the transfer paradox occurs. This result on intergenerational altruism is in sharp contrast to that when a donor displays altruism toward a recipient, as already shown in Hamada and Yanagihara (2014). These results indicate that, depending on what kind of altruism one considers, the effect of altruism on the likelihood of the transfer paradox differs. We also present the condition for the transfer paradox to occur on the transition path and demonstrate that the effect of intergenerational altruism on this condition vanishes as the economy converges to the steady state. In sum, intergenerational altruism affects only the effect of the transfer on the welfare of transitional generations, but not in the steady state.

The remainder of this chapter is organized as follows. Section 4.2 describes the two-country, one-sector OLG model wherein each generation in a country has intergenerational altruism for the next generation. Section 4.3 presents the condition under which the transfer paradox occurs in the steady state when intergenerational altruism exists. Section 4.4 investigates the welfare effect of the transfer for the initial young and old generations and transitional generations. Section 4.5 provides concluding remarks.

4.2 The model

We consider a one-sector OLG model with two countries. A donor and a recipient of an international income transfer, indexed by $i = D$ and R , respectively, are identical except for the time preferences of individuals. Between the two countries, capital is fully mobile, but labor is immobile. Time is discrete and the economy lasts forever. The populations of both countries grow equally and exogenously with a population growth rate of $(1 + n) \geq 0$, which is constant over time.

4.2.1 Individuals

In each period, both countries are populated by two generations, the young, who supply one unit of their labor inelastically and earn wages either to consume or to save, and the old, who retire and consume savings accumulated in the young period. All individuals except for the initial old live for both periods. Individuals who are young in period t in country $i = D, R$ choose levels of consumption in their young period t and in their old period $t + 1$, (c_t^i, d_{t+1}^i) , to maximize their utility, subject to the budget constraints in their young and old periods. Henceforth, we call the individuals who are young in period t generation t .

We formalize the intergenerational altruism, defined as the situation in which generation t in a country cares about the next generation $t + 1$ in the country, as follows. The utility of generation t in country i consists of two subutilities. The first is the subutility obtained from consuming goods by themselves, which is often acknowledged in the usual OLG model. We define this subutility of generation t in country i as $u^{i,t}(c_t^i, d_{t+1}^i)$, which is referred to as the self-subutility of generation t in country i . It is assumed that the self-subutility function is twice differentiable, increasing, and quasi-concave in (c_t^i, d_{t+1}^i) . The second subutility is that of the *next* generation, which represents the intergenerational altruism of generation t . We call this an altruistic subutility. As the altruistic subutility of generation t in country i is denoted as $u^{i,t+1}(c_{t+1}^i, d_{t+1}^i)$, we can define the utility of generation t in country i with intergenerational altruism as follows:³

³ We simplify the utility function considerably in order to focus on how the intergenerational altruism affects the welfare impact of the transfer. Although our formalization in which a generation is concerned only about the subsequent one does not seem to be common, apart from the well-known dynasty model of Barro (1974), this simplification is easy to examine and the qualitative results continue to hold even if we consider the dynasty model. Moreover, there are several articles in which parents only care about their children in an OLG framework. Gaumont and Mesnard (2001), Thibault (2004), and Constantinides, Donaldson, and Mehra (2007) developed models in which parents' utility includes the level of bequest to the child. Viaene and Zilcha (2002) and Kunze (2014) assumed that the utility depends on the income level of the child.

$$U^{i,t} \equiv U^{i,t}(u^{i,t}, u^{i,t+1}) = U^{i,t}(u^{i,t}(c_t^i, d_{t+1}^i), u^{i,t+1}(c_{t+1}^i, d_{t+2}^i)). \quad (4.1)$$

We call $U^{i,t}$ the total utility of generation t in country i . It is also assumed that the total utility function is twice differentiable and increasing in $(u^{i,t}, u^{i,t+1})$: $U_t^{i,t} \equiv \partial U^{i,t} / \partial u^{i,t} > 0$ and $U_{t+1}^i \equiv \partial U^{i,t} / \partial u^{i,t+1} \geq 0$. Moreover, we assume that $U_t^{i,t} \geq U_{t+1}^i$, which implies that the effect of the generation's self-subutility on total utility is larger than the effect of its altruistic subutility.

The budget constraints of generation t in their respective young and old periods are as follows:

$$c_t^i + s_t^i = w_t + T^i \quad \text{and} \quad d_{t+1}^i = (1 + r_{t+1})s_t^i, \quad (4.2)$$

where r , w , and s denote the net interest rate, wages, and savings, respectively. T^i denotes a permanent international transfer and satisfies $T^D < 0$, $T^R > 0$, and $T^D + T^R = 0$. By arranging (4.2), the intertemporal budget constraint of generation t is obtained as follows:

$$c_t^i + \frac{1}{1 + r_{t+1}} d_{t+1}^i = w_t + T^i. \quad (4.3)$$

When generation t in country i decides on consumption levels so as to maximize total utility (4.2), they simply maximize their own self-subutility $u^{i,t}$ given the altruistic subutility $u^{i,t+1}$, because they cannot choose the consumption levels of the next generation. In short, the problem of total utility maximization is equivalent to that of the self-subutility maximization for each generation. Thus, the utility maximization problem for generation t in country i is formulated as follows:

$$\max_{\{c_t^i, d_{t+1}^i\}} u^{i,t}(c_t^i, d_{t+1}^i), \quad \text{s.t.} \quad c_t^i + \frac{1}{1 + r_{t+1}} d_{t+1}^i = w_t + T^i. \quad (4.4)$$

The first-order condition for the utility maximization problem (4.4) can be given by: $u_c^{i,t} = (1 + r_{t+1})u_d^{i,t}$, where $u_c^{i,t} \equiv \partial u^{i,t} / \partial c_t^i$ and $u_d^{i,t} \equiv \partial u^{i,t} / \partial d_{t+1}^i$. The second-order condition is satisfied by the quasi-concavity of the self-subutility function. By this first-order condition and the intertempo-

ral budget constraint (4.4), we obtain the optimal consumption bundle, $(c_t^i(w_t + T^i, r_{t+1}), d_{t+1}^i(w_t + T^i, r_{t+1}))$, as well as the savings function, $s_t^i = s^i(w_t + T^i, r_{t+1})$. Here, we assume that the savings function is increasing both in the wage and in the interest rate, that is, $s_w^i \equiv \partial s^i(w_t + T^i, r_{t+1})/\partial w_t > 0$ and $s_r^i \equiv \partial s^i(w_t + T^i, r_{t+1})/\partial r_{t+1} > 0$, which guarantees that consumption is a normal good.

Substituting the optimal levels of consumption and savings into the total utility function, we can obtain the indirect utility function:

$$\begin{aligned} V^{i,t} &= V^{i,t}(w_t + T^i, r_{t+1}, w_{t+1} + T^i, r_{t+2}) \\ &\equiv U^{i,t}(u^{i,t}(c_t^i, d_{t+1}^i), u^{i,t+1}(c_{t+1}^i, d_{t+2}^i)), \end{aligned} \quad (4.5)$$

where $c_t^i = c_t^i(w_t + T^i, r_{t+1})$, $d_{t+1}^i = d_{t+1}^i(w_t + T^i, r_{t+1})$, $c_{t+1}^i = c_{t+1}^i(w_{t+1} + T^i, r_{t+2})$, and $d_{t+2}^i = d_{t+2}^i(w_{t+1} + T^i, r_{t+2})$.

It should be noted that the indirect utility depends only on the wages and the interest rates confronted by the present and the next generation, not on those confronted by all succeeding generations.⁴ As $u_c^{i,t}(\partial c_t^i/\partial w_t) + u_d^{i,t}(\partial d_{t+1}^i/\partial w_t) = u_c^{i,t} > 0$ and $u_c^{i,t}(\partial c_t^i/\partial r_{t+1}) + u_d^{i,t}(\partial d_{t+1}^i/\partial r_{t+1}) = u_d^{i,t} s_t^i$ hold from the first-order condition, the indirect utility function has the following properties:⁵

⁴ In this regard, the indirect utility of our framework differs from that of the previous literature, represented by Blanchard and Fischer (1989, Ch.3) and Michel and Venditti (1997), in which indirect utility depends on the wages and the interest rates of all succeeding generations. This difference arises from different assumptions about whether the total utility of individuals depends on the subutility of the next generation or on their total utility.

⁵ The subscripts of the indirect utility function imply partial differentiation. That is, $V_{w_t}^{i,t} \equiv \partial V^{i,t}/\partial w_t$, $V_{r_{t+1}}^{i,t} \equiv \partial V^{i,t}/\partial r_{t+1}$, $V_{w_{t+1}}^{i,t} \equiv \partial V^{i,t}/\partial w_{t+1}$, $V_{r_{t+2}}^{i,t} \equiv \partial V^{i,t}/\partial r_{t+2}$, and $V_T^{i,t} \equiv \partial V^{i,t}/\partial T^i$.

$$V_{w_t}^{i,t} = U_t^{i,t} \left(u_c^{i,t} \frac{\partial c_t^i}{\partial w_t} + u_d^{i,t} \frac{\partial d_{t+1}^i}{\partial w_t} \right) = U_t^{i,t} u_c^{i,t} > 0, \quad (4.6)$$

$$V_{r_{t+1}}^{i,t} = U_t^{i,t} \left(u_c^{i,t} \frac{\partial c_t^i}{\partial r_{t+1}} + u_d^{i,t} \frac{\partial d_{t+1}^i}{\partial r_{t+1}} \right) = U_t^{i,t} u_d^{i,t} s_t^i > 0, \quad (4.7)$$

$$V_{w_{t+1}}^{i,t} = U_{t+1}^{i,t} \left(u_c^{i,t+1} \frac{\partial c_{t+1}^i}{\partial w_{t+1}} + u_d^{i,t+1} \frac{\partial d_{t+2}^i}{\partial w_{t+1}} \right) = U_{t+1}^{i,t} u_c^{i,t+1} > 0, \quad (4.8)$$

$$V_{r_{t+2}}^{i,t} = U_{t+1}^{i,t} \left(u_c^{i,t+1} \frac{\partial c_{t+1}^i}{\partial r_{t+2}} + u_d^{i,t+1} \frac{\partial d_{t+2}^i}{\partial r_{t+2}} \right) = U_{t+1}^{i,t} u_d^{i,t+1} s_{t+1}^i > 0, \text{ and } \quad (4.9)$$

$$V_T^{i,t} = V_{w_t}^{i,t} + V_{w_{t+1}}^{i,t} = U_t^{i,t} u_c^{i,t} + U_{t+1}^{i,t} u_c^{i,t+1} > 0. \quad (4.10)$$

Without loss of generality, the marginal self-utility of the consumption in the young period can be normalized to unity ($u_c^{i,t} = u_c^{i,t+1} = 1$), so that $u_d^{i,t} = 1/(1 + r_{t+1})$ is obtained. Likewise, it can be assumed that the marginal effect of the generation's self-utility on their total utility is normalized to unity, $U_i^{i,t} \equiv \partial U^{i,t} / \partial u^{i,t} = 1$. Moreover, it is permissible to assume that the marginal effect of the altruistic utility on the total utility evaluated at equilibrium is less than unity, $U_{t+1}^{i,t} \equiv \partial U^{i,t} / \partial u^{i,t+1} = \beta^{i,t} \in [0, 1]$. Here, $\beta^{i,t}$ can be regarded as the degree of intergenerational altruism of generation t in country i for the next generation in the country. As a result, (4.6)–(4.10) can be rewritten as:

$$V_{w_t}^{i,t} = 1 > 0, \quad (4.6')$$

$$V_{r_{t+1}}^{i,t} = \frac{s_t^i}{1 + r_{t+1}} > 0, \quad (4.7')$$

$$V_{w_{t+1}}^{i,t} = \beta^{i,t} > 0, \quad (4.8')$$

$$V_{r_{t+2}}^{i,t} = \beta^{i,t} \frac{s_{t+1}^i}{1 + r_{t+2}} > 0, \text{ and } \quad (4.9')$$

$$V_T^{i,t} = 1 + \beta^{i,t} > 0. \quad (4.10')$$

4.2.2 Firms

Firms in both countries produce their outputs under perfect competition. The aggregate production function $F(K_t^i, L_t^i)$ exhibits constant returns to scale in capital K_t^i and labor L_t^i , independent of time, and is identical in both countries. This aggregate production function can be rewritten as the per capita production function, $f(k_t^i)$, where $k_t^i \equiv K_t^i/L_t^i$ represents the per capita capital in country i in period t . This per capita production function is assumed to satisfy the following conditions: (i) $f(k_t^i)$ is continuously differentiable and (ii) $f(k_t^i) > 0$, $f'(k_t^i) > 0$, and $f''(k_t^i) < 0$ for all $k_t^i > 0$. Moreover, we assume the Inada conditions: (iii) $f(0) = 0$ and (iv) $f'(0) = +\infty$ and $f'(\infty) = 0$.

Firms maximize their profit in per capita terms, denoted by $\pi(k_t^i) \equiv f(k_t^i) - r_t k_t^i - w_t$. Profit maximization, with no capital depreciation, requires the equivalence of the marginal productivity and the price of each input as:

$$f'(k_t^i) = r_t \text{ and } f(k_t^i) - f'(k_t^i)k_t^i = w_t. \quad (4.11)$$

From the first equation of (4.11), we obtain the capital demand function represented by $k_t^i(r_t)$, where $k_t^i(r_t) = 1/f'' < 0$. This implies that the (per capita) demand for capital is decreasing in the interest rate r_t . Similarly, from the second equation of (4.11), $w_t'(r_t) = -k_t < 0$ can be obtained.

4.2.3 Equilibrium

We consider a world capital market equilibrium in period t , which requires the sum of per capita savings of generation t of both countries to be equal to the sum of per capita capital demand in the subsequent period $t + 1$. As capital is perfectly mobile, the interest rates in both countries become the same, so that $k_{t+1}^D = k_{t+1}^R \equiv k_{t+1}$ holds, by factor price equalization. Thus, the capital market equilibrium in period t can be expressed as follows:

$$2(1+n)k_{t+1}(r_{t+1}) = s^D(w_t(r_t) + T^D, r_{t+1}) + s^R(w_t(r_t) + T^R, r_{t+1}). \quad (4.12)$$

The excess demand in the world capital market is defined as $D(w_t, r_{t+1}) \equiv 2(1+n)k_{t+1}(r_{t+1}) - s^D(w_t + T^D, r_{t+1}) - s^R(w_t + T^R, r_{t+1})$. Then, under the

assumption that savings are increasing in the interest rate,

$$\begin{aligned}\Delta_t &\equiv \frac{\partial D(w_t, r_{t+1})}{\partial r_{t+1}} = 2(1+n)k'_{t+1}(r_{t+1}) - s_r^D - s_r^R \\ &= \frac{2(1+n)}{f''} - s_r^D - s_r^R < 0\end{aligned}\quad (4.13)$$

holds. Therefore, the Walrasian stability condition is satisfied in the capital market equilibrium in each period.

4.3 Steady-state analysis

4.3.1 The transfer paradox when there is intergenerational altruism

In this section, we investigate the effect of a permanent transfer on welfare in the steady state when each generation has intergenerational altruism in order to avoid the complexity of the transitional process and present a clear conclusion. The analysis of transitional generations is made in the next section.

We examine the condition under which the transfer paradox occurs in the steady state. The transfer paradox can be acknowledged if the donor is enriched and/or the recipient is immiserized by the transfer. As is usually assumed, we limit the discussion to the case where the economy is dynamically efficient, that is, $r_t > 1 + n$ holds for all t .

From (4.12), we immediately obtain the equilibrium condition of the world capital market in the steady state as follows:⁶

$$2(1+n)k(r) = s^D(w(r) + T^D, r) + s^R(w(r) + T^R, r). \quad (4.14)$$

For the economy to converge monotonically to the steady-state equilibrium, we assume the dynamic stability condition, that is, $0 < dr_{t+1}/dr_t < 1$. This condition is arranged as follows:

⁶ Hereafter, variables with no time subscript indicate steady-state values.

$$\begin{aligned}\Gamma &\equiv \frac{dD(w(r), r)}{dr} = 2(1+n)k'(r) - s_r^D - s_r^R - (s_w^D + s_w^R)w'(r) \\ &= \Delta + (s_w^D + s_w^R)k < 0.\end{aligned}\quad (4.15)$$

It should be noted that by comparing (4.13) with (4.15), when $D(w(r), r)$ is differentiated by r in the steady state, the term $(s_w^D + s_w^R)k$ is added to Δ . This is because the change in the interest rate affects wages through the change in factor demands by firms, and this brings about the long-run effect through the change in capital accumulation. By totally differentiating (4.14), we can obtain the effect of the transfer on the interest rate, as follows:

$$\Gamma dr = s_w^D dT^D + s_w^R dT^R = (s_w^R - s_w^D) dT, \quad (4.16)$$

where $dT^R = -dT^D \equiv dT > 0$.

By totally differentiating the indirect utility functions of the donor and the recipient in the steady state, V^i , $i = D, R$, and noting (4.6')–(4.10') and $dw = -kdr$, we obtain the following equations:

$$\begin{aligned}dV^D &= (1 + \beta^{D,t}) \left(dw + \frac{s^D}{1+r} dr - dT \right) \\ &= (1 + \beta^{D,t}) \left[\left(-k + \frac{s^D}{1+r} \right) \frac{s_w^R - s_w^D}{\Gamma} - 1 \right] dT, \text{ and}\end{aligned}\quad (4.17)$$

$$\begin{aligned}dV^R &= (1 + \beta^{R,t}) \left(dw + \frac{s^R}{1+r} dr + dT \right) \\ &= (1 + \beta^{R,t}) \left[\left(-k + \frac{s^R}{1+r} \right) \frac{s_w^R - s_w^D}{\Gamma} + 1 \right] dT.\end{aligned}\quad (4.18)$$

From (4.17) and (4.18), the condition for the transfer paradox can be stated as follows: (i) The strong paradox occurs if $dV^D > 0$ and $dV^R < 0$; (ii) The weak paradox occurs either if $dV^D > 0$ and $dV^R > 0$, or if $dV^D < 0$ and $dV^R < 0$. In any case, we can immediately obtain the following proposition.

Proposition 4.1. *Intergenerational altruism has no effect on the likelihood of the occurrence of the transfer paradox.*

When individuals have no intergenerational altruism, the effect of the transfer is equivalent to the terms in the square brackets of (4.17) and (4.18), which can be obtained by substituting $\beta^{i,t} = 0$ into (4.17) and (4.18).⁷ This implies that although the existence of intergenerational altruism amplifies the effect on the welfare of the donor and the recipient, it does not at all affect whether the transfer paradox occurs. In fact, both these conditions are independent of the degree of intergenerational altruism, $\beta^{i,t}$. Moreover, the result of Proposition 4.1 holds even when $\beta^{i,t} > 1$, although such a situation is unrealistic because it implies that a generation places greater priority on the other generation than on itself.

The reason why Proposition 4.1 holds is easy to appreciate. At first glance, because intergenerational altruism works as a type of utility externality, it might seem that the introduction of intergenerational altruism into the model would affect the likelihood of the occurrence of the transfer paradox. However, because intergenerational altruism is, by definition, a utility externality that arises only between the generations *within* a donor or recipient country, such a domestic externality has no effect on the welfare of other countries. This is consistent with the result of Hamada and Yanagihara (2014), who examined the transfer paradox in the context of altruism *between* a donor country and a recipient country. These results show that the condition under which the paradox occurs changes if the type of altruism differs.

4.3.2 A generalization to the dynasty model

Based on the result of Proposition 4.1, we can consider a further extension of the transfer problem in an OLG model with intergenerational altruism. For example, by generalizing the above simple setting of intergenerational altruism, we can consider the situation where a generation has concerns about the prosperity of all its descendants, in the sense that a generation has altruism toward all subsequent generations. In this situation, the total utility function of

⁷ The welfare effect without intergenerational altruism has been shown in Galor and Polemarchakis (1987) and Yanagihara (2006).

generation t is denoted by:

$$U^{i,t} = u^{i,t} + \beta u^{i,t+1} + \beta^2 u^{i,t+2} + \dots + \beta^k u^{i,t+k} + \dots, \beta \in [0,1). \quad (4.19)$$

This formalization of utility is the same as in the dynasty model.⁸ Similar to Proposition 4.1, in the steady state, the condition under which the transfer paradox occurs is easy to obtain as follows:

$$dV^D = \frac{1}{1-\beta} \left[\left(-k + \frac{s^D}{r} \right) \frac{s_w^R - s_w^D}{\Gamma} - 1 \right] dT > 0, \quad (4.20)$$

$$dV^R = \frac{1}{1-\beta} \left[\left(-k + \frac{s^R}{r} \right) \frac{s_w^R - s_w^D}{\Gamma} + 1 \right] dT < 0. \quad (4.21)$$

By (4.20) and (4.21), even in the dynasty model, the condition under which the paradox occurs does not depend on the degree of intergenerational altruism β at all. From the above result, we can infer that even in more general situations where a generation has different degrees of concerns about its descendants, the qualitative result of whether the transfer paradox occurs in the steady state does not depend on the strength of intergenerational altruism.

In this section, we focused on the possibility of the transfer paradox arising in the steady-state equilibrium. In the following section, we focus on the transition path to the steady-state equilibrium and investigate the young and old generations in the initial period and the transitional periods, respectively.

4.4 Analysis of transition path

4.4.1 The initial young and old generations

In this subsection, we examine the effects on welfare of the transfer of the initial young and old generations, respectively, by using the similar approach of Cremers and Sen (2008). In the initial period $t = 1$, the amount of capital is

⁸ In the original dynasty model, it is assumed that each generation has concerns about the subsequent one and has recursive utility such that $U^{i,t} = u^{i,t} + \beta U^{i,t+1}$ in our setting. It can be easily verified that this recursive utility is rewritten as (4.19).

given, and the transfer in period $t = 1$ affects neither the wage nor the interest rate; that is, $dw_1 = 0$ and $dr_1 = 0$. However, as the initial old is concerned about the subsequent generation 1, the welfare of the initial old is affected by the change in the interest rate in period 2, r_2 , caused by the change in transfer in period 1 as follows. First, by differentiating the indirect utility function of the initial old with respect to r_2 , we obtain the following equation:

$$\frac{dV^{i0}}{dr_2} = \frac{\partial V^{i0}}{\partial r_2} = \frac{\beta s_1^i}{1 + r_2}. \quad (4.22)$$

We obtain the effect that the transfer has on the interest rate in period 2 by totally differentiating the capital market equilibrium in period 2, (4.12) with respect to r_2 and T , respectively, as follows:

$$\frac{dr_2}{dT} = \frac{s_w^R - s_w^D}{2(1+n)k'(r_2) - s_r^D - s_r^R}. \quad (4.23)$$

Finally, substituting (4.23) into (4.22) gives the welfare effect of the transfer on the initial old:

$$\frac{dV^{D0}}{dT} = \frac{\beta s_1^D (s_w^R - s_w^D)}{(1 + r_2)[2(1+n)k'(r_2) - s_r^D - s_r^R]}, \quad (4.24)$$

$$\frac{dV^{R0}}{dT} = \frac{\beta s_1^R (s_w^R - s_w^D)}{(1 + r_2)[2(1+n)k'(r_2) - s_r^D - s_r^R]}. \quad (4.25)$$

Whether the transfer increases the welfare of each country depends on the relative size of the marginal propensity to save between the donor and the recipient, that is, s_w^D and s_w^R . When $s_w^D > s_w^R$ as is usually assumed, the welfare of the initial old in the donor (recipient) country necessarily worsens (improves).

Next we consider the effect that the transfer has on the welfare of the initial young, that is, the first generation. It should be noted that although generation 1 is not affected by the previous interest rate as $dr_1 = 0$, the transfer and the resulting change in the interest rate affect their welfare. By totally differentiating the indirect utility function of the initial young, we obtain the following

equation:

$$\begin{aligned} dV^{i1} &= \frac{\partial V^{i1}}{\partial r_2} dr_2 + \frac{\partial V^{i1}}{\partial r_3} dr_3 + \frac{dV^{i1}}{dT} dT^i \\ &= \left(\frac{s_1^i}{1+r_2} - \beta k_2 \right) dr_2 + \frac{\beta s_2^i}{1+r_3} dr_3 + (1+\beta) dT^i. \end{aligned} \quad (4.26)$$

Similar to the manner in which we derived dr_2/dT , we obtain dr_t/dT for $t \geq 3$ as follows:

$$\frac{dr_t}{dT} = \frac{s_w^R - s_w^D}{2(1+n)k'(r_t) - s_r^D - s_r^R}. \quad (4.27)$$

By substituting the effect of the transfer on the interest rate, dr_t/dT into (4.26), we obtain the effect that the transfer has on the welfare of the initial young of the donor and the recipient countries, respectively, as follows:

$$\begin{aligned} \frac{dV^{D1}}{dT} &= \frac{[s_1^D - \beta(1+r_2)k_2](s_w^R - s_w^D)}{(1+r_2)[2(1+n)k'(r_2) - s_r^D - s_r^R]} \\ &\quad + \frac{\beta s_2^D (s_w^R - s_w^D)}{(1+r_3)[2(1+n)k'(r_3) - s_r^D - s_r^R]} - (1+\beta), \end{aligned} \quad (4.28)$$

$$\begin{aligned} \frac{dV^{R1}}{dT} &= \frac{[s_1^R - \beta(1+r_2)k_2](s_w^R - s_w^D)}{(1+r_2)[2(1+n)k'(r_2) - s_r^D - s_r^R]} \\ &\quad + \frac{\beta s_2^R (s_w^R - s_w^D)}{(1+r_3)[2(1+n)k'(r_3) - s_r^D - s_r^R]} + (1+\beta). \end{aligned} \quad (4.29)$$

In (4.28) and (4.29), the first term indicates the current indirect effect that is caused by the change in the interest rate in period 2 by the transfer in period 1. The second term is brought about by the intergenerational altruism. The third term is the direct income effect of the transfer, and is negative (positive) for the donor (recipient), respectively.

4.4.2 Transitional generations

Following the recent methodology of Kuhle (2014), we consider the marginal change in the transfer T .⁹ The capital market equilibrium in period t , (4.12), is rewritten by the implicit function $r_{t+1} = \psi(r_t, T)$. By partially differentiating (4.12) with respect to r_t or T , we obtain the following equations:

$$\psi_r \equiv \frac{\partial r_{t+1}}{\partial r_t} = -\frac{(s_w^D + s_w^R)k_t}{2(1+n)k'(r_{t+1}) - s_r^D - s_r^R}, \quad (4.30)$$

$$\psi_T \equiv \frac{\partial r_{t+1}}{\partial T} = \frac{s_w^R - s_w^D}{2(1+n)k'(r_{t+1}) - s_r^D - s_r^R}. \quad (4.31)$$

As we assume that dynamic stability holds on the transitional path as well as in the steady state, $\psi_r \in (0, 1)$ holds. $\psi_T > 0$ holds by (4.27). By the recursive form of r_t , we obtain the effect of T on r_t as follows:

$$\frac{dr_t}{dT} = \sum_{\tau=0}^{t-1} (\psi_r(r, T))^{\tau} \psi_T(r, T) = \frac{1 - (\psi_r(r, T))^t}{1 - \psi_r(r, T)} \psi_T(r, T). \quad (4.32)$$

It should be noted that the indirect utility of generation t depends on interest rates and the transfer in three periods, $(r_t, r_{t+1}, r_{t+2}, T)$, because the total utility depends on the subutility of the subsequent generation, which is affected by (r_{t+1}, r_{t+2}, T) . Therefore, by totally differentiating the indirect utility function and substituting (4.32), we obtain the marginal change in the indirect utility as follows:

⁹ We thank the anonymous referee of the *Quarterly Review of Economics and Finance* who informed us of this study in which transition dynamics can be concisely elucidated.

$$\begin{aligned}
dV^{it} &= \frac{\partial V^{it}}{\partial r_t} dr_t + \frac{\partial V^{it}}{\partial r_{t+1}} dr_{t+1} + \frac{\partial V^{it}}{\partial r_{t+2}} dr_{t+2} + \frac{\partial V^{it}}{\partial T^i} dT^i \\
&= -k_t dr_t + \left(\frac{s_t^i}{1+r_{t+1}} - \beta k_{t+1} \right) dr_{t+1} + \beta \frac{s_{t+1}^i}{1+r_{t+2}} dr_{t+2} + (1+\beta) dT^i \\
&= \left(-k_t + \frac{s_t^i}{1+r_{t+1}} - \beta k_{t+1} + \beta \frac{s_{t+1}^i}{1+r_{t+2}} \right) \sum_{\tau=0}^{t-1} \psi_r^\tau \psi_T dT \\
&\quad + \left(\frac{s_t^i}{1+r_{t+1}} - \beta k_{t+1} \right) \psi_r^t \psi_T dT \\
&\quad + \beta \frac{s_{t+1}^i}{1+r_{t+2}} (\psi_r^t + \psi_r^{t+1}) \psi_T dT + (1+\beta) dT^i. \tag{4.33}
\end{aligned}$$

As we consider marginal parameter changes, the values of the variables are evaluated at the initial steady state where $k_t = k_{t+1} \equiv k$. Finally, by rearranging (4.33) we obtain the effect of the transfer on the welfare of the donor and the recipient, respectively, as follows:

$$\begin{aligned}
\frac{dV^{Dt}}{dT} &= \left\{ \underbrace{(1+\beta)[s^D - (1+r)k] \sum_{\tau=0}^{t-1} \psi_r^\tau}_{\substack{1. \text{ (the accumulation effect)} \\ (+) \text{ or } (-)}} + \underbrace{[s^D - \beta(1+r)k] \psi_r^t}_{\substack{2. \text{ (the current effect)} \\ (+) \text{ or } (-)}} \right. \\
&\quad \left. + \underbrace{\beta s^D (\psi_r^t + \psi_r^{t+1})}_{\substack{3. \text{ (the subsequent effect)} \\ (+)}} \right\} \frac{\psi_T}{1+r} - \underbrace{(1+\beta)}_{\substack{4. \text{ (the income effect)} \\ (-)}}, \tag{4.34}
\end{aligned}$$

$$\begin{aligned}
\frac{dV^{Rt}}{dT} = & \underbrace{\left\{ (1 + \beta)[s^R - (1 + r)k] \sum_{\tau=0}^{t-1} \psi_r^\tau + [s^R - \beta(1 + r)k] \psi_r^t \right\}}_{\substack{1. \text{ (the accumulation effect)} \\ (-)}} \underbrace{+ \underbrace{\beta s^R (\psi_r^t + \psi_r^{t+1})}_{3. \text{ (the subsequent effect)} (+)} \left\} \frac{\psi_T}{1 + r}}_{4. \text{ (the income effect)} (+)} + (1 + \beta) \quad . \quad (4.35)
\end{aligned}$$

From (4.34) and (4.35), we can confirm that the welfare effects of the transfer for both the donor and the recipient are divided into four, depending on the whole sequence of past interest rates, the current and future interest rates, and the transfer itself, that is, $(\{r_\tau\}_{\tau=1}^t, r_{t+1}, r_{t+2}, T)$. We call the first term the accumulation effect, which indicates that permanent transfers made in the past cause the increase in past interest rates from the initial period to the previous period, that is, $\{r_\tau\}_{\tau=1}^t$ and affect capital accumulation up to the present period. This accumulation effect includes all influences of capital accumulation on welfare not only for the present generation but also for the subsequent generation throughout all periods up to the present. Notably, it also contains the effect on the labor income of the present generation of the rise in the interest rate r_t associated with the increase in the transfer in the previous period. The sign of this effect depends on whether a country is the capital borrower or lender. Noting that $s^D > (1 + n)k > s^R$ because the donor (recipient) is the capital lender (borrower) and $r > n$ under dynamic efficiency, $(1 + r)k > s^R$ necessarily holds, while whether $s^D > (1 + r)k$ is satisfied depends on the amount of lending. Thus, the sign of the first term for the recipient is certainly negative, whereas the sign for the donor is indeterminate. If the donor saves enough to satisfy $s^D > (1 + r)k$, the accumulation effect of the donor tends to be positive.

We call the second term the current effect because this effect is the one caused by the current interest rate r_{t+1} . The current effect implies that the transfer of the present generation from the donor to the recipient affects the interest rate in the next period r_{t+1} for the present generation and the change in labor income for the subsequent generation. The sign of the second term also depends on whether the country is a donor or a recipient. Within the parentheses in the

second term, $(s^i - \beta(1+r)k)$ depends on the relative size of intergenerational altruism β . If β is sufficiently small, the second term is positive for both countries, while if β is sufficiently large, that is, the generation has sufficiently high intergenerational altruism, it tends to be negative. Furthermore, the welfare of the present generation is affected by the subsequent interest rate r_{t+2} because each generation is also concerned about the utility of the subsequent generation. We call this third term the subsequent effect. It is worth noting that the subsequent effect is positive for both countries because this term only contains the effect on the interest rate but not the effect on labor income. Thus both countries are affected by the subsequent effect in the same way. The fourth effect indicates the direct income effect because the change in income from the transfer affects utility directly. Evidently, the income effect of the donor (recipient) is negative (positive).

Unlike the steady-state analysis, the effect of the transfer on the welfare of transitional generations is affected by the degree of intergenerational altruism. As β increases, the accumulation effect of the donor (recipient) is positively (negatively) amplified, the positive current effect of the donor shrinks, the positive subsequent effects of both countries increase, and the income effect is negatively (positively) amplified for the donor (recipient). However, we can easily confirm that the second and third terms in (4.34) and (4.35) vanish as t approaches infinity. This result corresponds with the fact that there do not exist the second and third terms in the steady state, which is regarded as the limit value when t approaches infinity, as is already shown in Subsection 4.3.1. The reason is because the accumulation effect by capital accumulation in the past necessarily overwhelms the other two effects that arise only in two periods, the present and the subsequent periods, as time approaches infinity.

4.5 Concluding remarks

In this chapter, we have demonstrated that the condition under which the transfer paradox occurs in the steady state is not affected by the degree of intergenerational altruism. This result is in sharp contrast to the results of previous research, which found that when both a donor and a recipient country have altruism toward each other, the condition under which the paradox occurs changes. We also presented the condition for the transfer paradox to occur on

the transition path. We clarified that although intergenerational altruism affects the condition for the transfer paradox to occur on the transition path, the effect that intergenerational altruism has on this condition vanishes as the economy converges to the steady state.

It seems at first glance that the introduction of intergenerational altruism raises the likelihood of the transfer paradox, in particular the likelihood of the donor's enrichment by the transfer. However, our study concludes that, in a dynamic framework, although the intergenerational altruism within a country amplifies the effect of the transfer, it does not change the condition under which the transfer paradox occurs. This implies that intergenerational altruism does not contribute to explaining the motivation for the donor's voluntary transfer. Combining the result obtained in this chapter, which is based on intergenerational altruism, with the results obtained in previous studies involving altruism between two countries suggests that the likelihood of the transfer paradox occurring depends crucially on the characteristics of altruism assumed. Therefore, it can be inferred from our conclusion that even if another externality is added to the model, the externality itself will not cause any change in the condition influencing the likelihood of the paradox, unless the interdependency is between a donor and a recipient. A possible extension of our research is to investigate whether this inferred conclusion is correct; that is, whether the condition for the occurrence of the transfer paradox is changed when there is another externality.

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Part II

Other Aspects of Behavioral Economics

Chapter 5

Consumption Externality and the Transfer Paradox

In this chapter, we examine the transfer problem between two countries by exploring a model in which there exists consumption externality between a donor and a recipient country by using a one-sector overlapping generations model. We investigate how the income transfer from the donor to the recipient affects both countries' social welfare, when a donor country has consumption externality toward the recipient country such that the donor's utility increases as the recipient's consumption increases. The paradoxical situation, in which a donor enriches and/or a recipient immiserizes despite the fact that the income transfer is made from the donor to the recipient, is called the transfer paradox, and a large number of studies have examined the transfer paradox. In particular, this chapter investigates how the existence of consumption externality affects the likelihood of the transfer paradox by considering the situation in which each generation of the donor and recipient countries have consumption externality with each other. The main contents of this chapter are as follows: First, we consider each situation in which an income transfer increases or decreases the interest rate and clarify the relationship between the likelihood of the transfer paradox and the relative size between labor share and capital share. Second,

we confirm the result of existing literature that when there does not exist any consumption externality, the relative sizes of discount factors between two countries determine whether the indirect effects on the donor and recipient's social welfare are positive or negative. Third, in contrast with the conclusion of existing studies, we clarify how the indirect effects on social welfare manifest when introducing consumption externality. In particular, we provide several results regarding the indirect effect of consumption externality not only when the rates of time preference are equal between the two countries but also when they differ from each other.

Keywords: transfer paradox, consumption externality, overlapping generations model, bandwagon effect, capital accumulation

JEL classifications: D64, E22, F21, F35

5.1 Introduction

The purpose of this chapter is to examine the international transfer problem between two countries by exploring a model in which there exists consumption externality in the utility of each generation of the donor country and recipient country by using a one-sector overlapping generations (OLG) model. We present several theoretical results on how the international income transfer affects the social welfare of both countries when the donor country has consumption externality, such that the donor's utility increases with the increase in the recipient's consumption. The paradoxical situation in which a donor enriches and/or a recipient immiserizes despite the fact that a transfer is made from the donor to the recipient is called the transfer paradox, and a considerable number of studies have tackled various issues regarding the transfer paradox. The possibility that paradoxical situations will occur, such as donor enrichment and/or recipient impoverishment, when a donor gives aid to a recipient has been theoretically pointed out by existing literature regardless of the static general equilibrium model and the dynamic model used. In this chapter, we attempt to

extend the existing framework to incorporate consumption externality and add some additional results to the issues on the transfer paradox that have not been tackled by existing studies yet. We consider the situation in which each generation of the donor and recipient countries has consumption externality in their utilities with the other country, and we present several novel results on how the degree of consumption externality affects the possibility of the occurrence of the transfer paradox.

The transfer problem between two countries has long been an important issue in international economics. In 1922 after World War I, John Maynard Keynes criticized the payment of huge amount of war reparations imposed on Germany, which was defeated in World War I. At that time, he pointed out that the transfer problem occurred because Germany had to pay reparations with the foreign currencies of victorious countries, and this triggered a well-known “transfer controversy” between Keynes and Bertil Gotthard Ohlin, a famous Swedish international economist, on whether Germany can really pay the war reparations.¹ The occurrence of the transfer problem is basically attributed to the change in the terms of the trade of tradable goods caused by an income transfer from a donor to a recipient. For example, if the positive indirect effect of improving the terms of trade for tradable goods is larger than the negative direct effect of decreasing the income by transfer aid, the donor can paradoxically improve its social welfare by giving the aid to a recipient. On the other hand, if the negative indirect effect of worsening the terms of trade is larger than the positive direct effect of increasing the income by transfer, occurrence of such a paradoxical situation in which the recipient decreases its social welfare despite the increase in income is possible. Since the abovementioned transfer controversy, several economists have been tackling, for a long time, the transfer problem about whether and how such a transfer paradox will occur. Among other things, works by Samuelson (1952, 1954) are the most represen-

¹ For the transfer controversy, see Keynes (1929) and Ohlin (1929). Keynes (1929) argued that it is impossible for Germany to pay the huge amount of war reparations, and there was a possibility that the fiscal crisis will cause a new war. Ohlin (1929) offered a counterargument that Germany had sufficient financial margin to pay such a huge amount of reparations. However, history proved that Keynes’s claim was correct. The controversy between Keynes and Ohlin had a great influence on the subsequent development of international economic theory. Bertil Ohlin was awarded the Nobel Prize in economics in 1977 for path-breaking contribution to the theory of international trade and international capital movements.

tative classical studies. Samuelson considered the transfer problem in a static two-country general equilibrium model and proved that if market equilibrium is Walrasian stable under free trade, the transfer paradox never occurs. Furthermore, he showed that in order for the transfer paradox to arise, some distortions impeding free trade must be introduced in the model.²

In a static two-country free trade economy without any distortions, no transfer paradox occurs, and the donor cannot improve its social welfare by giving aid to the recipient. This implies that there is no motivation for a donor country to implement any income transfer. However, if the donor country's citizens have altruism toward the recipient country's citizens, aid transfer might improve the social welfare of the donor country, owing to just the existence of altruism toward others. If transfer leads to increase in the donor's welfare, existence of the donor's altruism provides one explanation for the motivation of donor countries to give an aid to recipient countries. In practice, several existing studies suggested that introducing the donor's altruism into a model can strengthen the likelihood of the transfer paradox, especially, the donor's enrichment, that is, the improvement of the donor's social welfare after a transfer. Kemp and Shimomura (2002) explored a model with untied voluntary aids and showed that donor altruism can be a motivation for donor countries to provide aid transfer. Lahiri and Raimondos-Møller (1999) introduced altruism into the model as a motivation for aid transfer and investigated the situation in which some economic distortions are generated by tariffs or import quota in international trade. Nevertheless, the above two studies did not sufficiently elucidate whether the existence of altruism itself is the cause of the transfer paradox, especially donor enrichment. Kemp and Shimomura (2002) only stated that under certain specific conditions, the transfer paradox can arise assuming altruistic utility. Even in Lahiri and Raimondos-Møller (1999), the paradox is not caused by altruism itself but by other economic distortions such as tariff or import quota. Eventually, Hamada (2012) elucidated whether the donor's altruism itself motivates the donor to provide aid transfer in a two-country free trade. As a result, it is shown that even if a donor has the altruistic utility to-

² For the survey of the transfer problem in a static model, see Brakman and van Marrewijk (1998). We omit a comprehensive explanation of the existing literature in this chapter because there exist too many prior studies to examine the transfer problem, typically represented by Bhagwati, Brecher, and Hatta (1983, 1985).

ward a recipient, no transfer paradox arises in a static two-country two-good free trade model without any distortions when market equilibrium is Walrasian stable. Therefore, in the framework of the static general equilibrium, Hamada (2012) suggests that even if donor countries have altruistic intention to give recipient countries aid transfer, it does not imply that donor countries can voluntarily support recipient countries.

On the other hand, unlike the conclusion in the static model, when we consider the dynamic model, it is shown that the transfer paradox can occur even if there is no economic distortion. In the dynamic macroeconomic situation, it is well-known that the transfer paradox can occur under dynamic efficiency even if the international capital market satisfies Walrasian stability, because there exist intertemporal capital accumulation and international capital movement. Galor and Polemarchakis (1987) for the first time demonstrated the possibility that a permanent lump-sum income transfer from a donor to a recipient brings about the transfer paradox in the steady state by using a one-sector two-country OLG model. Haaparanta (1989) introduced public bond into the setting of Galor and Polemarchakis (1987) and provided the result that even if income transfer is temporal, the transfer paradox can continue to occur when the transfer is financed by issuing public bonds. In addition, several existing studies, such as Tan (1998), Yanagihara (2006), and Cremers and Sen (2008), pointed out the possibility of the paradox occurring in a dynamic framework.

In addition, in the dynamic model, as analogized from the static model, it seems at first glance that when a donor country has altruistic utility to a recipient country, the higher is the degree of altruism, the better is the donor's social welfare. Stated differently, We, at first glance, guess that as the donor's citizens pay more attention to the improvement of other countries' social welfare, donor countries have stronger incentive to provide aid transfer. However, we proved that such an unfounded inference was not correct. Hamada and Yanagihara (2014) considered the transfer problem between two countries when the donor has altruistic utility to the recipient by using a one-sector OLG model and presented the result that irrespective of the degree of altruism, the donor's altruism does not lead to improvement of the donor's social welfare. The main reason for this result is that as the degree of altruism increases, capital accumulation stagnates and brings about a decrease in social welfare. Therefore, even in the dynamic setting, it cannot be claimed that the donor's altruism should provide

appropriate motivation to the donor country to voluntarily provide aid transfer.

From an economic point of view, altruistic utility is regarded as a type of externality. Hamada and Yanagihara (2014) presented another paradoxical situation: as the degree of a donor's altruism to the recipient increases, donor's social welfare decreases as a result of the transfer. In contrast, Hamada and Yanagihara (2016) considered the situation with intergenerational altruism in which parental generations of the donor country have altruism to child generations of the country in an OLG model, and they demonstrated that the condition for the transfer paradox to occur is determined irrespective of the degree of intergenerational altruism. As previous studies show, the effect on the occurrence of the transfer paradox greatly differs between altruism between two countries and intergenerational altruism in a country. Furthermore, different externalities other than altruism can exist in reality. To find an effective way to implement income transfer, it is quite important both in theory and in policy to investigate whether and how the degree of externalities affects the possibility of the occurrence of the transfer paradox by explicitly incorporating such externalities into the model.³ However, except for the abovementioned articles about altruism, there are very limited existing studies that incorporate externalities into a model when examining international income transfer. Therefore, in this chapter, we extend the existing framework of analysis on the transfer paradox to include consumption externality and investigate how the degree of consumption externality that the donor and recipient citizens have to each other affects the likelihood of the transfer paradox.

Consumption externality is defined as the situation in which utility of an economic entity is directly influenced by the consumption behavior of other enti-

³ Hamada, Kaneko, and Yanagihara (2016, 2017) examined whether and how the difference in domestic policy affects the occurrence of the transfer paradox by international transfer. From the viewpoint of social welfare maximization, Hamada, Kaneko, and Yanagihara (2016) investigated how the transfer should be financed between the young and old generations in the donor country and how the received transfer should be distributed between the young and old generations in the recipient country. Hamada, Kaneko, and Yanagihara (2017) explained the situation in which the difference in the social security system between the donor and recipient countries affects the effect of the transfer on welfare of both countries through the different impacts on intergenerational income transfer within the countries. In particular, they showed that under the pay-as-you-go pension schemes, a marginal increase in the amount of pension contribution is likely to cause the deterioration of a donor's social welfare by transfer.

ties. In a sense, altruism, which has already been modeled in the existing literature, might be regarded as a type of consumption externality, because altruism is defined as the situation in which utility of an economic entity is influenced by utility of other entities, and utility depends on the consumption of goods. As two representative examples of consumption externality, the bandwagon effect and snob effect are often mentioned. According to the bandwagon (snob) effect, as others consume more goods, people obtain higher (lower) utility even when consuming the same amount of goods.⁴ The situation wherein the donor's citizens feel happier as the recipient's citizens consume more can be described by incorporating the bandwagon effect into the donor country's utility. However, there are few studies dealing with such consumption externality within the existing research on international income transfer in dynamic macroeconomics.⁵ Stepping forward from the conventional analytical framework on altruism, we incorporate the so-called bandwagon effect into the model so that the increase in the consumption of both young generations in two countries raises the utility of each generation in a country, and we investigate how the condition for the occurrence of the transfer paradox is affected by this newly added external effect.

The main results of this chapter are as follows: First, we consider both situations in which income transfer increases or decreases the interest rate and clarify whether the transfer paradox can occur or not depending on the difference in the relative sizes between labor share and capital share. Second, we consider the situation in which there exists no consumption externality, and we re-present the results of existing studies. That is, the difference in the sizes of the discount factor between two countries determines whether the indirect effect on social welfare of the donor and recipient is positive or negative. Third, in contrast with the result of existing studies, we consider the situation in which there exists consumption externality and examine how the indirect effect on

⁴ The bandwagon and snob effects are concepts introduced by Leibenstein (1950) in the work titled "Bandwagon, Snob, and Veblen Effects in the Theory of Consumers' Demand."

⁵ Futagami and Shibata (1998) tackled a consumption externality called the "keep up with the Joneses" effect. However, they used a Ramsey model but not an OLG model and did not examine the transfer problem at all. Besides, this type of externality is defined as the effect the difference in the relative size of holding assets between agents has on the consumption level.

welfare of the donor and the recipient is affected by consumption externality. In particular, we consider both cases in which the discount factor rates are equal or different between the two countries and derive the conditions for the transfer paradox to occur in each case.

The remainder of this chapter is organized as follows: In Section 5.2, we describe a two-country one-sector OLG model in which each generation has consumption externality. In Section 5.3, we derive consumption, savings, and social welfare in the steady state equilibrium. In Section 5.4, we present the main results on how consumption externality affects the likelihood of the transfer paradox. Section 5.5 presents the concluding remarks.

5.2 The model

We consider a two-country one-sector OLG model. There exist two countries, a donor country and a recipient country, and an international transfer is made from the donor to the recipient. We denote country $i = D$ and R as the donor and recipient, respectively. Both countries are identical except for the rate of time preference and the degree of consumption externality. The capital is fully mobile across the two countries, but goods and labor are immobile. The population growth rates of both countries are exogenously given, identical, and constant over time. $n \geq 0$ denotes the net population growth rate. Thus, the gross population growth rate is $1 + n \geq 1$. We assume throughout the chapter that the dynamic efficiency is satisfied, that is, $r_t \geq n$.

5.2.1 Individuals

All individuals in each country live for two periods, and there exist two types of generations, the young and the old, in each country in each period. The young generation inelastically supplies one unit of labor and obtains the wage. The old generation retires and consumes within the savings of the young period. We henceforth call the generation that is young in period t as generation t . c_t^i and d_{t+1}^i denote the consumption levels for generation t in country i when they are young in period t and old in period $t + 1$, respectively.

There exists consumption externality in the utility of each generation. Specifically, we consider the consumption externality in which the increase in the con-

sumption of the young period of a generation in a country increases the utility of the same generation in another country. Concretely, the utility of generation t in country i increases with the consumption of the young period of generation t in country $j \neq i$, that is, c_t^j . Stated differently, there exists the bandwagon effect such that the larger is the young-period consumption of generation t in country j , c_t^j , the larger is the utility obtained from the young-period consumption of generation t in country i , c_t^i . For example, we can consider the situation in which when the other country consumes more (less), the degree of satisfaction obtained from that country's own consumption in this period increases (decreases), reflecting the mood of the global economic boom (recession).^{6 7}

We specify the utility function, which is assumed to be log-linear.⁸ The utility function of generation t in country i is as follows:

$$u^{i,t} = u^{i,t}(c_t^i, d_{t+1}^i; c_t^j) = \ln(c_t^i + b_i c_t^j) + \delta_i \ln(d_{t+1}^i). \quad (5.1)$$

The utility of generation t in country i depends on its own consumption levels over two periods, (c_t^i, d_{t+1}^i) , as well as on the young-period consumption level of generation t in country j , c_t^j , due to consumption externality. $b_i \in [0, 1)$ denotes the degree of consumption externality of country i .⁹ $\delta_i \in (0, 1]$ denotes

⁶ Although the situation is similar to the situation with a type of altruism such that one's utility increases as other people's utility increases, the setting of this chapter only focuses on the formulation of consumption externality. Hamada and Yanagihara (2014, 2016) conducted the analysis under the formulation of altruism.

⁷ As another formulation, by associating consumption externality as the concern for economic disparity between countries, we might be able to interpret this type of externality with the situation in which the less is the difference in consumption levels between two countries, the more is the utility from the viewpoint of fairness. In this chapter, however, we do not argue the fairness of income distribution between countries.

⁸ However, in reality, we can perform the following analysis even with general forms of utility function, and we assume the log-linear utility function to simplify the analysis.

⁹ $b_i = 0$ degenerates into the usual setting without consumption externality. $b_i > 1$ implies that the marginal utility from the own consumption in young age, c_t^i , exceeds that from other people's consumption in the young age, c_t^j . We exclude the case of $b_i > 1$ from the analysis because such a situation is unrealistic.

¹⁰ In this chapter, we focus only on the case of $b_i \geq 0$, that is, the case in which there exists the bandwagon effect. If we extend the analysis to the case of $b_i \in (-1, 0]$, we can also examine the snob effect. However, in that case, we have to put the additional assumption,

the discount factor between two periods. b_i and δ_i are always identical in all generations in each country through the whole period, but they differ between countries. Time preferences differ between countries due to the difference in the discount factor, and this results in the difference in marginal propensity to save. This log-linear utility function satisfies the second-order condition for utility maximization.

r_t , w_t , and s_t denote the net interest rate, wage, and savings level in period t , respectively. T^i is a permanent transfer of country i and satisfies $T^D < 0$, $T^R > 0$, and $T^D + T^R = 0$. The budget constraint in the young age is $c_t^i + s_t^i = w_t + T^i$, and the budget constraint in the old age is $d_{t+1}^i = (1 + r_{t+1})s_t^i$. The intertemporal budget constraint is as follows:

$$c_t^i + \frac{1}{1 + r_{t+1}} d_{t+1}^i = w_t + T^i. \quad (5.2)$$

Denoting the constant income transfer in every period as T , $T^R = -T^D = T > 0$. We assume that the income after transfer is strictly positive, that is, $w_t + T^D = w_t - T > 0$.

The utility maximization problem under budget constraints for generation t in country i is as follows:

$$\begin{aligned} \max_{\{c_t^i, d_{t+1}^i\}} u^{i,t}(c_t^i, d_{t+1}^i; c_t^j) &= \ln(c_t^i + b_i c_t^j) + \delta_i \ln(d_{t+1}^i), \\ \text{s.t. } c_t^i + \frac{1}{1 + r_{t+1}} d_{t+1}^i &= w_t + T^i. \end{aligned} \quad (5.3)$$

Generation t in country i maximizes its utility given the consumption level in the young age for generation t in the other country j , c_t^j , which brings about consumption externality. By the first-order condition for utility maximization, we obtain the following equation:¹¹

$$d_{t+1}^i = (1 + r_{t+1})\delta_i(c_t^i + b_i c_t^j). \quad (5.4)$$

$c_t^i + b_i c_t^j > 0$. In any case, as it is almost impossible to imagine the realistic case in which consumption externality is negative, we limit the argument to $b_i \geq 0$.

¹¹ The second-order condition is satisfied because the utility function is log-linear.

Solving the simultaneous equations of budget constraint, (5.2), and the first-order condition, (5.4), we obtain the optimal levels of consumption, (c_t^i, d_{t+1}^i) , as follows:

$$c_t^i = \frac{-\delta_i b_i c_t^j + w_t + T^i}{1 + \delta_i}, \quad (5.5)$$

$$d_{t+1}^i = (1 + r_{t+1})\delta_i \frac{b_i c_t^j + w_t + T^i}{1 + \delta_i}. \quad (5.6)$$

When utility is log-linear, the consumption in young age, c_t^i , does not depend on the interest rate, r_{t+1} , but only the consumption in old age, d_{t+1}^i , depends on r_{t+1} . By (5.5), the optimal consumption pairs of generation t in country i , (c_t^i, d_{t+1}^i) , depend on the consumption in young age of generation t in country j , c_t^j , because, as c_t^j increases, the utility increases and the marginal utility obtained from c_t^i decreases due to consumption externality. Thus, as shown in (5.5), the increase in c_t^j decreases the consumption in young age. In contrast, as shown in (5.6), the decrease in the consumption in young age increases the consumption in old age.

The consumption in young age, (5.5), is regarded as a type of best response function of the consumption in young age for both countries. Therefore, by solving the intersection of both countries' best response functions, we obtain the optimal consumption levels in young age for both countries, (c_t^D, c_t^R) , as a type of Nash equilibrium. Furthermore, by substituting (c_t^D, c_t^R) into (5.6), we obtain the optimal consumption levels in old age for both countries. The optimal consumption levels in both young and old periods for the donor and

recipient countries, $(c_t^D, c_t^R, d_{t+1}^D, d_{t+1}^R)$ are calculated as follows:

$$c_t^D = \frac{(1 + \delta_R)(w_t + T^D) - \delta_D b_D(w_t + T^R)}{X}, \quad (5.7)$$

$$c_t^R = \frac{(1 + \delta_D)(w_t + T^R) - \delta_R b_R(w_t + T^D)}{X}, \quad (5.8)$$

$$d_{t+1}^D = \frac{(1 + r_{t+1})\delta_D \left\{ [1 + \delta_R(1 - b_D b_R)](w_t + T^D) + b_D(w_t + T^R) \right\}}{X}, \quad (5.9)$$

$$d_{t+1}^R = \frac{(1 + r_{t+1})\delta_R \left\{ [1 + \delta_D(1 - b_D b_R)](w_t + T^R) + b_R(w_t + T^D) \right\}}{X}, \quad (5.10)$$

where $X \equiv (1 + \delta_D)(1 + \delta_R) - \delta_D \delta_R b_D b_R > 0$. The consumption in young age, c_t^i , and the savings, $s_t^i = d_{t+1}^i / (1 + r_{t+1})$, do not depend on the interest rate, r_{t+1} .¹² The savings function is defined by $s_t^i = s^i(w_t, T^D, T^R)$. Denoting the partial derivative of the savings function with respect to w_t by $s_w^i \equiv \partial s_t^i / \partial w_t$, we obtain from (5.9) and (5.10) that $s_w^D = \delta_D [1 + b_D + \delta_R(1 - b_D b_R)] / X > 0$ and $s_w^R = \delta_R [1 + b_R + \delta_D(1 - b_D b_R)] / X > 0$. That is, the savings increase with wage.

In Table 5.1, we summarize the partial derivatives on the consumptions both in young and old ages as well as the savings, which are required in the steady-state analysis in Section 5.3. Define $c_w^i \equiv \partial c^i / \partial w_t$, $c_r^i \equiv \partial c^i / \partial r_{t+1}$, $c_{T^D}^i \equiv \partial c^i / \partial T^D$, and $c_{T^R}^i \equiv \partial c^i / \partial T^R$, and also define similar notations on the partial derivatives of d^i and s^i .

By Table 5.1, the size of the marginal propensities to save between the donor and recipient depends on the following: $s_w^D \geq s_w^R \Leftrightarrow \delta_D(1 + b_D) \geq \delta_R(1 + b_R)$. Thus, which marginal propensity to save is larger depends on both the discount factor δ_i and degree of consumption externality b_i of both countries.

¹² This characteristic is obtained under the log-linear utility function.

Table 5.1: Partial derivatives of consumptions and savings

$c_w^D = \frac{1+\delta_R-\delta_D b_D}{X} > 0$	$c_r^D = 0$
$c_w^R = \frac{1+\delta_D-\delta_R b_R}{X} > 0$	$c_r^R = 0$
$d_w^D = \frac{(1+r_{t+1})\delta_D[1+b_D+\delta_R(1-b_D b_R)]}{X} > 0$	$d_r^D = s_t^D$
$d_w^R = \frac{(1+r_{t+1})\delta_R[1+b_R+\delta_D(1-b_D b_R)]}{X} > 0$	$d_r^R = s_t^R$
$s_w^D = \frac{\delta_D[1+b_D+\delta_R(1-b_D b_R)]}{X} > 0$	$s_r^D = 0$
$s_w^R = \frac{\delta_R[1+b_R+\delta_D(1-b_D b_R)]}{X} > 0$	$s_r^R = 0$
$c_{TD}^D = \frac{1+\delta_R}{X} > 0$	$c_{TR}^D = -\frac{\delta_D b_D}{X} < 0$
$c_{TD}^R = -\frac{\delta_R b_R}{X} < 0$	$c_{TR}^R = \frac{1+\delta_D}{X} > 0$
$d_{TD}^D = \frac{(1+r_{t+1})\delta_D[1+\delta_R(1-b_D b_R)]}{X} > 0$	$d_{TR}^D = \frac{(1+r_{t+1})\delta_D b_D}{X} > 0$
$d_{TD}^R = \frac{(1+r_{t+1})\delta_R b_R}{X} > 0$	$d_{TR}^R = \frac{(1+r_{t+1})\delta_R[1+\delta_D(1-b_D b_R)]}{X} > 0$
$s_{TD}^D = \frac{\delta_D[1+\delta_R(1-b_D b_R)]}{X} > 0$	$s_{TR}^D = \frac{\delta_D b_D}{X} > 0$
$s_{TD}^R = \frac{\delta_R b_R}{X} > 0$	$s_{TR}^R = \frac{\delta_R[1+\delta_D(1-b_D b_R)]}{X} > 0$

5.2.2 Firms

In both the donor and the recipient countries, firms supply a good in a perfectly competitive market. Denote the aggregate production function by $F(K_t^i, L_t^i)$, where K_t^i and L_t^i are, respectively, the aggregate capital and labor in country i in period t . The production function has constant returns to scale, is independent of time t , and is identical between two countries. Labor is immobile, but capital is fully mobile. There is no capital depreciation. The per capita production function is denoted by $f(k_t^i) \equiv F(K_t^i/L_t^i, 1)$, where $k_t^i \equiv K_t^i/L_t^i$ is the per capita capital of country i in period t . As the usual assumptions of $f(\cdot)$, we assume that (i) it is continuously differentiable; (ii) $f > 0$, $f' > 0$, and $f'' < 0$ for all $k_t^i > 0$; and (iii) the Inada conditions are satisfied, that is, $f(0) = 0$, $\lim_{k \rightarrow 0} f'(k) = \infty$, and $\lim_{k \rightarrow \infty} f'(k) = 0$.

The firm's profit function is $\pi(k_t^i) \equiv f(k_t^i) - r_t k_t^i - w_t$. The first-order

condition for profit maximization is as follows:

$$f'(k_t^i) = r_t, \quad (5.11)$$

$$f(k_t^i) - f'(k_t^i)k_t^i = w_t. \quad (5.12)$$

By (5.11), we obtain the capital demand function, $k_t^i = k(r_t)$. As $k' = 1/f'' < 0$, per capita capital k_t^i decreases with the interest rate r_t . Moreover, from (5.12), we obtain $w_t = w(r_t)$. As $w_t' = -k_t^i < 0$, the wage w_t decreases with interest rate r_t .

5.2.3 Capital market equilibrium

We consider the international capital market in period t . As capital is fully mobile across two countries, the factor price of capital equalizes between two countries, and, as a result, the per capita capital is identical between them, that is, $k_{t+1}^D = k_{t+1}^R \equiv k_{t+1}$. In the capital market equilibrium, total amount of the per capita savings of generation t in both countries is equal to total amount of the per capita capital demand in period $t + 1$. Thus, the capital market equilibrium satisfies the following equation:

$$2(1+n)k_{t+1}(r_{t+1}) = s^D(w_t, T^D, T^R) + s^R(w_t, T^D, T^R). \quad (5.13)$$

Since per capita capital decreases with the interest rate and savings does not depend on the interest rate, Walrasian stability is necessarily satisfied.

5.3 The steady-state analysis

We remove the time subscript t in the steady-state variables. We assume that dynamic stability is satisfied in the steady state. That is, we assume that the following inequality is satisfied.

$$\Gamma \equiv 2(1+n)k'(r) - (s_w^D + s_w^R)w'(r) = \frac{2(1+n)}{f''} + (s_w^D + s_w^R)k < 0. \quad (5.14)$$

As is already shown, note that $s_w^i > 0$.

Table 5.2: Partial derivatives of savings with respect to b_D and b_R

$s_{b_D}^D = \frac{\delta_D(1+\delta_R)(1+\delta_D-\delta_R b_R)w}{X^2} > 0$	$s_{b_R}^D = -\frac{\delta_D \delta_R b_D(1+\delta_R-\delta_D b_D)w}{X^2} < 0$
$s_{b_D}^R = -\frac{\delta_D \delta_R b_R(1+\delta_D-\delta_R b_R)w}{X^2} < 0$	$s_{b_R}^R = \frac{\delta_R(1+\delta_D)(1+\delta_R-b_D \delta_D)w}{X^2} > 0$

In the following analysis, we investigate the effect of the marginal increase in transfer starting when there is initially no income transfer, that is, $T = 0$.¹³

5.3.1 Savings function

From (5.9) and (5.10), we obtain the savings functions of both countries in the steady state when evaluating the transfer at $T = 0$ as follows:

$$s^D = \frac{\delta_D[1 + b_D + \delta_R(1 - b_D b_R)]w}{X}, \quad (5.15)$$

$$s^R = \frac{\delta_R[1 + b_R + \delta_D(1 - b_D b_R)]w}{X}. \quad (5.16)$$

By (5.15) and (5.16), $s^D \geq s^R \Leftrightarrow \delta_D(1 + b_D) \geq \delta_R(1 + b_R)$. Thus, which of the savings of the donor and the recipient, s^D and s^R , is larger depends on both discount factor δ_i and degree of consumption externality b_i between two countries. Since $s^i = s_{w,w}^i$ holds under the log-linear utility function, the order of the size of the marginal propensity to save necessarily corresponds to that of savings. As is already shown, the savings do not depend on interest rate r .

We summarize the partial derivatives of the savings function in the steady state with respect to (b_D, b_R) in Table 5.2, in which we define $s_{b_j}^i \equiv \partial s^i / \partial b_j$.

¹³ This type of simplification of analysis is usually done without loss of generality in existing studies as a standard way to investigate the effect of the transfer. Moreover, in the first place, we cannot endogenously determine the appropriate level of T in our model.

5.3.2 Indirect utility function

We consider the indirect utility function in the steady state, $V^i(w, r, T^D, T^R)$. The indirect utility function of the donor is obtained by substituting the donor's own consumption levels, $c^D = c^D(w, T^D, T^R)$ and $d^D = d^D(w, r, T^D, T^R)$, and the recipient's consumption level in young age, $c^R = c^R(w, T^D, T^R)$ into the utility, $u^D = \ln(c^D + b_D c^R) + \delta_D \ln(d^D)$. Likewise, the indirect utility function of the recipient is obtained by substituting the donor's own consumption levels, $c^R = c^R(w, T^D, T^R)$ and $d^R = d^R(w, r, T^D, T^R)$, and the recipient's consumption level in young age, $c^D = c^D(w, T^D, T^R)$ into the utility, $u^R = \ln(c^R + b_R c^D) + \delta_R \ln(d^R)$. We denote the indirect utility of the donor and recipient by V^D and V^R , respectively. Totally differentiating V^D and V^R , we obtain the following equation:

$$\begin{aligned} dV^D = \frac{\delta_D}{d^D} \left\{ (1+r) \left[c_w^D dw + c_{T^D}^D dT^D + c_{T^R}^D dT^R \right. \right. \\ \left. \left. + b_D (c_w^R dw + c_{T^D}^R dT^D + c_{T^R}^R dT^R) \right] \right. \\ \left. + d_w^D dw + d_r^D dr + d_{T^D}^D dT^D + d_{T^R}^D dT^R \right\}, \quad (5.17) \end{aligned}$$

$$\begin{aligned} dV^R = \frac{\delta_R}{d^R} \left\{ (1+r) \left[c_w^R dw + c_{T^D}^R dT^D + c_{T^R}^R dT^R \right. \right. \\ \left. \left. + b_R (c_w^D dw + c_{T^D}^D dT^D + c_{T^R}^D dT^R) \right] \right. \\ \left. + d_w^R dw + d_r^R dr + d_{T^D}^R dT^D + d_{T^R}^R dT^R \right\}. \quad (5.18) \end{aligned}$$

Substituting the partial derivatives shown in Table 5.1 into (5.17) and (5.18) and arranging the equations, we obtain the following equations:

$$dV^D = \frac{\delta_D(1 + \delta_D)}{s^D X} \left\{ [1 + b_D + \delta_R(1 - b_D b_R)] dw + [1 + \delta_R(1 - b_D b_R)] dT^D + b_D dT^R \right\} + \frac{\delta_D}{1 + r} dr, \quad (5.19)$$

$$dV^R = \frac{\delta_R(1 + \delta_R)}{s^R X} \left\{ [1 + b_R + \delta_D(1 - b_D b_R)] dw + [1 + \delta_D(1 - b_D b_R)] dT^R + b_R dT^D \right\} + \frac{\delta_R}{1 + r} dr. \quad (5.20)$$

Since $dT^R = -dT^D \equiv dT > 0$ by $T^R = -T^D \equiv T > 0$, the following equations are obtained from (5.19) and (5.20):

$$dV^D = \frac{\delta_D(1 + \delta_D)}{s^D X} \left\{ \underbrace{[1 + b_D + \delta_R(1 - b_D b_R)] dw}_{\text{(the indirect wage effect)}} - \underbrace{[1 - b_D + \delta_R(1 - b_D b_R)] dT}_{\text{(the direct income effect)}} \right\} + \underbrace{\frac{\delta_D}{1 + r} dr}_{\text{(the indirect substitution effect)}} \quad (5.21)$$

$$dV^R = \frac{\delta_R(1 + \delta_R)}{s^R X} \left\{ \underbrace{[1 + b_R + \delta_D(1 - b_D b_R)] dw}_{\text{(the indirect wage effect)}} + \underbrace{[1 - b_R + \delta_D(1 - b_D b_R)] dT}_{\text{(the direct income effect)}} \right\} + \underbrace{\frac{\delta_R}{1 + r} dr}_{\text{(the indirect substitution effect)}} \quad (5.22)$$

From (5.21) and (5.22), the effect that the increase in wage w has on the

social welfare of the donor and recipient is necessarily positive because $1 + b_D + \delta_R(1 - b_D b_R) > 0$ and $1 + b_R + \delta_D(1 - b_D b_R) > 0$. In addition, as the degree of consumption externality, b_D or b_R , increases, the positive effect that wage increase has on social welfare also increases. This is obvious because, as the degree of consumption externality increases, a country benefits from the increase in social welfare by not only the increase in its own wage but also the increase in the other country's consumption caused by the increase in their wages.

Next, we examine the effect that the increase in interest rate r has on the donor and recipient's social welfare. The sign of this effect is necessarily positive. Although the increase in interest rate has no effect on the savings level under the log-linear utility function, it increases the consumption in old age by $d^i = (1 + r)s^i$. Thus, the increase in the interest rate contributes to the increase in social welfare. In addition, this effect of the interest rate on social welfare is independent of the degree of consumption externality, b_D or b_R . Therefore, the terms-of-trade effect caused by the increase in the interest rate is not directly related to the degree of consumption externality. The reason is that under the log-linear utility function, the change in the interest rate does not change the consumption in young age c^D , but only the consumption in old age d^D , and the utility function is additive separable in both consumption levels. As consumption externality affects only the consumption in young age, the terms-of-trade effect does not directly depend on the interest rate.

Finally, we investigate the effect that income transfer T from the donor to the recipient has on the social welfare of both countries, that is, $\partial V^D/\partial T$ and $\partial V^R/\partial T$. From the signs of the coefficient of dT in (5.21) and (5.22), that is, $1 - b_D + \delta_R(1 - b_D b_R) > 0$ and $1 - b_R + \delta_D(1 - b_D b_R) > 0$, we obtained that $\partial V^D/\partial T < 0$ and $\partial V^R/\partial T > 0$. In addition, as the donor's degree of consumption externality b_D increases, the negative impact that the income reduction by giving a transfer has on the donor's welfare decreases because the recipient's income increases whereas the donor's income decreases; moreover, as the externality becomes stronger, the donor receives a more positive effect on its welfare from the increase in the recipient's consumption. Likewise, as the recipient's degree of consumption externality b_R increases, the positive impact that the income increase by receiving a transfer has on the recipient's welfare decreases. This is because, despite the fact that the recipient's income increases,

the recipient cares more about the donor's income reduction if there exists more consumption externality on the donor's consumption, and, as a result, the decrease in the donor's consumption reduces the rate of increase in the recipient's welfare. In any case, the effect of the transfer on social welfare is affected by the sizes of the degree of consumption externality of both countries, b_D and b_R .

We hereinafter call the effects of the three terms in (5.21) and (5.22) as follows. The first term is called the indirect wage effect, which is the effect that wage change dw has on the change of social welfare dV^i . The second term is called the direct income effect, which is the effect that the change in income transfer dT has on the change of social welfare dV^i . The third term is called the indirect substitution effect, which is the effect that the change in the interest rate dr has on the change of social welfare dV^i . The transfer changes the income level of both countries and directly changes social welfare. The direct income effect shown in the second term means that the international transfer changes both countries' income levels and directly affects both countries' social welfare. In addition, since the transfer changes the interest rate, there exist other indirect effects other than the direct effect. The indirect wage effect shown in the first term means that the change in the interest rate changes the wage level through the change in the relative price between production factors and also changes the optimal consumption. On the other hand, the indirect substitution effect shown in the third term is the intertemporal substitution effect wherein the interest rate changes the optimal consumption both in young and old age. The relative sizes of these three effects determine the total effect of the transfer on social welfare.

From the second term of (5.21), the direct income effect of the donor's welfare is necessarily negative because income reduction by the transfer decreases the donor's consumption levels both in young and old age, even if it increases the recipient's consumption level in young age ($T \uparrow \Rightarrow (c_t^D \downarrow, d_{t+1}^D \downarrow, c_t^R \uparrow) \Rightarrow dV^D \downarrow$). The signs of the first and third terms depend on whether the transfer increases or decreases the interest rate. If the transfer decreases the interest rate, the first term as the indirect wage effect is positive ($T \uparrow \Rightarrow r \downarrow \Rightarrow w \uparrow \Rightarrow (c_t^D \uparrow, d_{t+1}^D \uparrow, c_t^R \uparrow) \Rightarrow dV^D \uparrow$). In contrast, if the transfer decreases the interest rate, the third term as the indirect substitution effect is negative, because the consumption in old age is substituted by that in young age ($T \uparrow \Rightarrow r \downarrow$

$\Rightarrow d_{t+1}^D \Downarrow \Rightarrow dV^D \Downarrow$). The same argument can be applied to the recipient's welfare from (5.22). The indirect income effect is necessarily positive because the income increase by transfer increases the recipient's consumption levels both in young and old age, even if it decreases the donor's consumption level in young age ($T \Uparrow \Rightarrow (c_t^R \Uparrow, d_{t+1}^R \Uparrow, c_t^D \Downarrow) \Rightarrow dV^R \Uparrow$). The signs of the first and third terms depend on how the transfer affects the interest rate. If the transfer decreases the interest rate, the indirect wage effect is positive ($T \Uparrow \Rightarrow r \Downarrow \Rightarrow w \Uparrow \Rightarrow (c_t^R \Uparrow, d_{t+1}^R \Uparrow, c_t^D \Uparrow) \Rightarrow dV^R \Uparrow$). In contrast, if the transfer decreases the interest rate, the indirect substitution effect is negative because the consumption in old age is substituted by that in young age ($T \Uparrow \Rightarrow r \Downarrow \Rightarrow d_{t+1}^R \Downarrow \Rightarrow dV^R \Downarrow$).

5.3.3 Capital market equilibrium

From (5.13), we present the capital market equilibrium condition in the steady state as follows:

$$2(1+n)k(r) = s^D(w(r), T^D, T^R) + s^R(w(r), T^D, T^R). \quad (5.23)$$

Substituting the savings functions of the donor and recipient, (5.15) and (5.16) into (5.23), we obtain the steady-state capital level as follows:

$$\begin{aligned} 2(1+n)k &= s^D + s^R = \frac{\delta_D(1+b_D) + \delta_R(1+b_R) + 2\delta_D\delta_R(1-b_Db_R)}{X}w \\ \Leftrightarrow k &= \frac{[\delta_D(1+b_D) + \delta_R(1+b_R) + 2\delta_D\delta_R(1-b_Db_R)]w}{2(1+n)[(1+\delta_D)(1+\delta_R) - \delta_D\delta_Rb_Db_R]}. \end{aligned} \quad (5.24)$$

Partially differentiating the steady-state capital, (5.24), with regard to b_D and b_R , respectively, we obtain the following equations.

$$\frac{\partial k}{\partial b_D} = \frac{\delta_D(1+\delta_D - \delta_Rb_R)(1+\delta_R - \delta_Rb_R)w}{2(1+n)X^2} > 0, \quad (5.25)$$

$$\frac{\partial k}{\partial b_R} = \frac{\delta_R(1+\delta_D - \delta_Db_D)(1+\delta_R - \delta_Db_D)w}{2(1+n)X^2} > 0. \quad (5.26)$$

Here, (5.25) and (5.26) imply that as the degree of consumption externality increases, more capital is accumulated.

By totally differentiating the market equilibrium condition, (5.23), the following equation is obtained.

$$\Gamma dr = s_{TD}^D dT^D + s_{TR}^D dT^R + s_{TD}^R dT^D + s_{TR}^R dT^R \quad (5.27)$$

$$\Leftrightarrow \frac{dr}{dT} = \frac{s_{TR}^D - s_{TD}^D + s_{TR}^R - s_{TD}^R}{\Gamma}. \quad (5.28)$$

$\Gamma < 0$ under the assumption of dynamic stability.

Since $s_{TD}^D = \delta_D[1 + \delta_R(1 - b_D b_R)]/X > 0$, $s_{TR}^D = \delta_D b_D/X > 0$, $s_{TR}^R = \delta_R[1 + \delta_D(1 - b_D b_R)]/X > 0$, and $s_{TD}^R = \delta_R b_R/X > 0$, as shown in Table 5.1, the following inequalities hold:

$$\begin{aligned} s_{TR}^D - s_{TD}^D + s_{TR}^R - s_{TD}^R &= \frac{\delta_R(1 - b_R) - \delta_D(1 - b_D)}{X} \gtrless 0 \\ \Leftrightarrow \delta_R(1 - b_R) &\gtrless \delta_D(1 - b_D). \end{aligned} \quad (5.29)$$

Whether the transfer increases the interest rate depends on the relative sizes of the marginal effects that the transfer has on the donor and recipient's savings.¹⁴ For example, if both the donor and recipient have the same time preference rate, that is, $\delta^D = \delta^R \equiv \delta$, the following equation is satisfied: $s_{TR}^D - s_{TD}^D + s_{TR}^R - s_{TD}^R \gtrless 0 \Leftrightarrow b_D \gtrless b_R$. From (5.28), if the donor's degree of consumption externality is larger (smaller) than that of the recipient, the transfer decreases (increases) the interest rate ($dr/dT \lesseqgtr 0 \Leftrightarrow b_D \gtrless b_R$). On the other hand, when both the donor and recipient have the same degree of consumption externality, that is, $b^D = b^R \equiv b$, then $s_{TR}^D - s_{TD}^D + s_{TR}^R - s_{TD}^R \gtrless 0 \Leftrightarrow \delta_D \lesseqgtr \delta_R$ holds. From (5.28), if the donor's discount factor is smaller (larger) than that of the recipient, the transfer decreases (increases) the interest rate ($dr/dT \lesseqgtr 0 \Leftrightarrow \delta_D \lesseqgtr \delta_R$).

Summarizing the above results, we obtain the following lemma.

Lemma 5.1. *Whether or not the transfer increases the interest rate is deter-*

¹⁴ Note that these relative sizes of the marginal effects on savings differ from those of the marginal propensity to save. The relative sizes of the marginal propensity to save are determined as follows: $s_w^D \gtrless s_w^R \Leftrightarrow \delta_D(1 + b_D) \gtrless \delta_R(1 + b_R)$.

mined by the following inequality: $dr/dT \geq 0 \Leftrightarrow \delta_D(1 - b_D) \geq \delta_R(1 - b_R)$.

5.4 Condition for the transfer paradox to occur

Substituting $dw = -kdr$ into (5.21) and (5.22) and arranging the equations, we obtain the following equations:

$$\frac{dV^D}{dT} = \underbrace{\frac{\delta_D w - (1 + \delta_D)(1 + r)k}{w(1 + r)} \frac{dr}{dT}}_{\text{(the indirect effect)}} - \underbrace{\frac{\delta_D(1 + \delta_D)[1 - b_D + \delta_R(1 - b_D b_R)]}{s^D X}}_{\text{(the direct effect)}}, \quad (5.30)$$

$$\frac{dV^R}{dT} = \underbrace{\frac{\delta_R w - (1 + \delta_R)(1 + r)k}{w(1 + r)} \frac{dr}{dT}}_{\text{(the indirect effect)}} + \underbrace{\frac{\delta_R(1 + \delta_R)[1 - b_R + \delta_D(1 - b_D b_R)]}{s^R X}}_{\text{(the direct effect)}}. \quad (5.31)$$

The first terms of the right-hand side in (5.30) and (5.31) are the indirect effect of the transfer, and the second terms are the direct effect. Denote the indirect effects of the donor and recipient as $A_i \equiv \frac{\delta_i w - (1 + \delta_i)(1 + r)k}{w(1 + r)} \times (dr/dT)$, and define $B_i \equiv \frac{\delta_i(1 + \delta_i)[1 - b_i + \delta_j(1 - b_i b_j)]}{s^i X} = \frac{(1 + \delta_i)[1 - b_i + \delta_j(1 - b_D b_R)]}{[1 + b_i + \delta_j(1 - b_D b_R)]w} > 0, i, j = D, R, j \neq i$. The donor's direct effect is denoted by $-B_D < 0$, and the recipient's direct effect is denoted by $B_R > 0$. Obviously, the donor's (the recipient's) direct effect is negative (positive).

First, we investigate in detail the sign of the indirect effect, A_i , which is determined by the signs of $Y(\delta_i) \equiv \delta_i w - (1 + \delta_i)(1 + r)k$ and dr/dT . When $\delta_i = 0$, $Y(0) = -(1 + r)k < 0$, and when $\delta_i = 1$, $Y(1) = w - 2(1 + r)k$. Thus, when $w < 2(1 + r)k$, $Y(\delta_i) < 0$ for all $\delta_i \in (0, 1]$, and when $w > 2(1 + r)k$, there uniquely exists the threshold of the discount factor, $\bar{\delta}_i \equiv \frac{(1 + r)k}{w - (1 + r)k}$, such that if $\delta_i < \bar{\delta}_i$, $Y(\delta_i) < 0$, and if $\delta_i > \bar{\delta}_i$, $Y(\delta_i) > 0$. From the profit maximization of firms, (5.11) and (5.12), $w + rk = f$ is satisfied, which implies that under the perfectly competitive market, all the total output by firms is distributed by labor share w and capital share rk . Hence, $w > 2(1 + r)k$ implies that labor share is more than twice the capital share.

From (5.30) and (5.31), we can provide conditions for the occurrence of the

transfer paradox in the donor or recipient country. We derive the following two propositions on the conditions for donor enrichment, in which the donor improves its own welfare after transferring, and/or recipient impoverishment, in which the recipient exacerbates its own welfare.

Proposition 5.1. *Suppose that the transfer increases the interest rate.*

(i) *When labor share is less than twice the capital share, donor enrichment never occurs, whereas there is a possibility of the occurrence of recipient impoverishment.*

(ii) *Consider the case in which labor share is more than twice the capital share. If the donor's discount factor is sufficiently small, donor enrichment never occurs, whereas if it is sufficiently large, there is a possibility for donor enrichment to occur. If the recipient's discount factor is sufficiently small, there is a possibility for recipient impoverishment to occur, whereas if it is sufficiently large, recipient impoverishment never occurs.*

Proposition 5.2. *Suppose that the transfer decreases the interest rate.*

(i) *When labor share is less than twice the capital share, there is a possibility for donor enrichment to occur, whereas recipient impoverishment never occurs.*

(ii) *Consider the case in which labor share is more than twice the capital share. If the donor's discount factor is sufficiently small, there is a possibility for donor enrichment to occur, whereas if it is sufficiently large, donor enrichment never occurs. If the recipient's discount factor is sufficiently small, recipient impoverishment never occurs, whereas if it is sufficiently large, there is a possibility for recipient impoverishment to occur.*

We can express Proposition 5.1 in a mathematically strict manner as follows:

(i) if $\delta_D(1 - b_D) > \delta_R(1 - b_R)$ and $w \leq 2(1 + r)k$, it is held that $A_D < 0$ and $A_R < 0$, which imply that the signs of the indirect effect of the donor and recipient, respectively, are both negative. (ii) When $\delta_D(1 - b_D) > \delta_R(1 - b_R)$ and $w > 2(1 + r)k$, if $\delta_D < \bar{\delta}_D$, $A_D < 0$, and if $\delta_D > \bar{\delta}_D$, $A_D > 0$. Similarly, if $\delta_R < \bar{\delta}_R$, $A_R < 0$, and if $\delta_R > \bar{\delta}_R$, $A_R > 0$. When the signs of the direct effect and indirect effect are different from each other, there is a possibility for the paradox to occur. Likewise, we can express Proposition 5.2 in a mathematically strict manner as follows: (i) if $\delta_D(1 - b_D) < \delta_R(1 - b_R)$ and $w \leq 2(1 + r)k$, it is held that $A_D > 0$ and $A_R > 0$, which imply that the signs of the indirect effect of the donor and recipient, respectively, are both positive. (ii) When

$\delta_D(1 - b_D) < \delta_R(1 - b_R)$ and $w > 2(1 + r)k$, if $\delta_D < \bar{\delta}_D$, $A_D > 0$, and if $\delta_D > \bar{\delta}_D$, $A_D < 0$. Similarly, if $\delta_R < \bar{\delta}_R$, $A_R > 0$, and if $\delta_R > \bar{\delta}_R$, $A_R < 0$. We skip the proofs of Propositions 5.1 and 5.2 because we can easily prove them from the explanation in the text.

We give an explanation on the economic intuition of Proposition 5.1. First, the situation in which labor share is less (much more) than twice the capital share implies that the contribution of capital to production is relatively larger (smaller) than that of labor. When the transfer brings about a decrease in the donor's savings exceeding the increase in the recipient's savings, the interest rate increases because the total capital to be supplied decreases. The indirect effect that the increase in the interest rate has on social welfare consists of the following two different effects. On one hand, the increase in the interest rate increases the consumption in old age, thanks to the increased savings, and improves social welfare. On the other hand, the increase in the interest rate due to decrease of total capital brings about a wage decrease, because labor is the substitutable production factor of capital, and a wage decrease always reduces social welfare. In Proposition 5.1(i), when labor share is relatively small, the latter negative effect by wage decrease is larger than the former positive effect, and the whole indirect effect negatively affects social welfare. In Proposition 5.1(ii), contrary to the case of (i), when labor share is relatively large, the latter negative effect is smaller than the first positive effect. Under the above situations, which of the positive and negative effects is larger ultimately depends on the size of the discount factor between countries, because the first positive effect occurs in old age, whereas the second negative effect occurs in young age. As a result, whether the sign of the indirect effect becomes positive or negative depends on the size of the discount factor. Relatively large (small) discount factor implies that consumers place more weight on old-age (young-age) consumption on their utility. This fact leads to the result that the first positive effect is larger (smaller) than the second negative effect, and the total indirect effect becomes positive (negative). We skip the economic intuition on why Proposition 5.2 holds because we can explain it in the same manner as we did in Proposition 5.1.

Next, we investigate in more detail how the degree of consumption externality affects the conditions for the transfer paradox to occur. First, we consider how consumption externality affects the indirect effect. As already mentioned,

Table 5.3: Partial derivatives of B_D and B_R with respect to b_D and b_R

$\frac{\partial B_D}{\partial b_D} = -\frac{2(1+\delta_D)(1+\delta_R)}{[1+b_D+\delta_R(1-b_D b_R)]^2 w} < 0$	$\frac{\partial B_D}{\partial b_R} = -\frac{2\delta_R(1+\delta_D)b_D^2}{[1+b_D+\delta_R(1-b_D b_R)]^2 w} < 0$
$\frac{\partial B_R}{\partial b_D} = -\frac{2\delta_D(1+\delta_R)b_R^2}{[1+b_R+\delta_D(1-b_D b_R)]^2 w} < 0$	$\frac{\partial B_R}{\partial b_R} = -\frac{2(1+\delta_D)(1+\delta_R)}{[1+b_R+\delta_D(1-b_D b_R)]^2 w} < 0$

the donor's direct effect is $-B_D < 0$, and the recipient's direct effect is $B_R > 0$. We summarize the partial derivatives of B_D and B_R with respect to the donor and recipient's degrees of consumption externality, b_D and b_R , in Table 5.3.

From Table 5.3, the partial derivative of the donor's direct effect, $-B_D$, is $-\partial B_D/\partial b_D > 0$. Therefore, as the degree of the donor's consumption externality b_D increases, the absolute value of the negative direct effect decreases. Stated differently, the more consumption externality the donor has on the recipient's consumption, the less is the size of the negative direct effect. However, the direct effect never disappears. On the other hand, as the degree of the recipient's consumption externality b_R increases, the absolute value of the negative direct effect decreases, because $-\partial B_D/\partial b_R > 0$. Thus, the negative direct effect also shrinks by the recipient's consumption externality. However, in the same way as mentioned above, the direct effect never disappears. Likewise, as the degree of the recipient's consumption externality b_R increases, the positive direct effect B_R decreases, because $\partial B_R/\partial b_R < 0$. As the degree of the donor's consumption externality b_D increases, the positive direct effect decreases, because $\partial B_R/\partial b_D < 0$. Stated differently, the more is the consumption externality, the less is the positive direct effect, because the recipient cares more about the decrease in the donor's consumption. In any case, irrespective of the degree of consumption externality, the direct effect never disappears. In sum, if consumption externality strengthens, the negative direct effect for the donor shrinks, while the positive direct effect for the recipient also shrinks.

Second, we consider the relationship between the indirect effect and consumption externality. As a benchmark, we consider the situation in which there is no consumption externality, that is, $b_D = b_R = 0$. This case has already been investigated in existing studies including in the seminal papers by Galor and Polemarchakis (1987) and Cremers and Sen (2008). When there exists no

consumption externality, we obtain the following proposition on the indirect effect of the transfer.

Proposition 5.3. *Suppose that there exists no consumption externality, that is, $b_D = b_R = 0$.*

(i) *If the discount factor is identical between two countries, no indirect effect exists. That is, if $\delta_D = \delta_R$, $A_D = A_R = 0$.*

(ii) *If the donor's discount factor is larger than that of the recipient, the recipient's indirect effect becomes negative, and the donor's indirect effect can take either positive or negative signs. When the golden rule of capital accumulation is satisfied, the donor's indirect effect becomes positive. That is, if $\delta_D > \delta_R$, $A_R < 0$, and when $r = n$, $A_D > 0$.*

(iii) *If the donor's discount factor is smaller than that of the recipient, the donor's indirect effect is positive, and the recipient's indirect effect can take either positive or negative signs. When the golden rule of capital accumulation is satisfied, the recipient's indirect effect becomes negative. That is, if $\delta_D < \delta_R$, $A_D > 0$, and when $r = n$, $A_R < 0$.*

Proof. By Lemma 5.1, when $b_D = b_R = 0$, $dr/dT \gtrless 0$ if and only if $\delta_D \gtrless \delta_R$.

(i) If $\delta_D = \delta_R$, $A_D = A_R = 0$ because $dr/dT = 0$.

(ii) If $\delta_D > \delta_R$, $dr/dT > 0$. Substituting the capital level of (5.24) into $Y(\delta_i) \equiv \delta_i w - (1 + \delta_i)(1 + r)k$, we obtain $Y(\delta_i) = [\delta_i - \frac{(1+r)(1+\delta_i)(\delta_D + \delta_R + 2\delta_D\delta_R)}{2(1+n)(1+\delta_D)(1+\delta_R)}]w$.

Since $r \geq n$ under dynamic stability, $Y(\delta_i) \leq \frac{(\delta_i - \delta_j)w}{2(1+\delta_j)}$. If $\delta_D > \delta_R$, then $\frac{(\delta_D - \delta_R)w}{2(1+\delta_R)} > 0$, $\frac{(\delta_R - \delta_D)w}{2(1+\delta_D)} < 0$, and, as a result, $Y(\delta_R) < 0$. Hence, the recipient's indirect effect is negative, that is, $A_R = [Y(\delta_R)/w(1+r)] \times (dr/dT) < 0$. The sign of the donor's indirect effect is generally indeterminate, but since $Y(\delta_D) = \frac{(\delta_D - \delta_R)w}{2(1+\delta_R)} > 0$, when $r = n$, we obtain $A_D = [Y(\delta_D)/w(1+r)] \times (dr/dT) > 0$.

(iii) If $\delta_D < \delta_R$, $dr/dT < 0$. As with the proof of (ii), under dynamic stability, that is, $r \geq n$, we obtain $Y(\delta_i) \leq \frac{(\delta_i - \delta_j)w}{2(1+\delta_j)}$. If $\delta_D < \delta_R$, $\frac{(\delta_D - \delta_R)w}{2(1+\delta_R)} < 0$, $\frac{(\delta_R - \delta_D)w}{2(1+\delta_D)} > 0$, and as a result, $Y(\delta_D) < 0$. Hence, the donor's indirect effect is positive, that is, $A_D = [Y(\delta_D)/w(1+r)] \times (dr/dT) > 0$. The sign of the recipient's indirect effect is generally indeterminate, but since $Y(\delta_R) = \frac{(\delta_R - \delta_D)w}{2(1+\delta_D)} > 0$, when $r = n$, we obtain $A_R = [Y(\delta_R)/w(1+r)] \times (dr/dT) < 0$. \square

Proposition 5.3 claims that when there exists no consumption externality, the difference in discount factors between two countries, that is, the difference in time preference rates, causes the transfer paradox. As shown in Proposition 5.3(i), when time preferences are identical between two countries, both countries are completely identical in the model, and the transfer brings about no change in capital accumulation at all. If there is no change in capital accumulation, the interest rate also does not change, and there exists no indirect effect. In contrast, Propositions 5.3(ii) and 5.3(iii) demonstrate that the difference in time preferences between two countries yields an indirect effect. As shown in Proposition 5.3(ii), when the donor's discount factor is larger than that of the recipient due to its low time preference rate, the transfer generates an indirect effect such that capital accumulation decreases and the interest rate increases. Under dynamic efficiency, this indirect effect necessarily exacerbates the recipient's welfare. On the other hand, although whether the donor's welfare improves or not by the transfer depends on the situation, the donor's indirect effect is positive under the golden rule. Thus, this case, suggests that there is a possibility that both paradoxes, donor enrichment and recipient impoverishment, will occur at the same time. In contrast, as shown in Proposition 5.3(iii), when the donor's discount factor is smaller than that of the recipient due to its high time preference rate, the transfer generates an indirect effect such that capital accumulation increases and the interest rate decreases. Under dynamic efficiency, this indirect effect necessarily improves the donor's welfare. On the other hand, although whether the recipient's welfare improves or not depends on the situation, the recipient's indirect effect is negative under the golden rule. Thus, this case also suggests that there is a possibility that both transfer paradoxes will occur.

Then, we consider the case in which there exists consumption externality for both countries, and we examine how the possibility for the paradox to occur is affected by the existence of consumption externality. We obtain the following proposition on the indirect effect in this case.

Proposition 5.4. *Suppose that the discount factors of both countries are identical, that is, $\delta_D = \delta_R \equiv \delta$.*

- (i) *If the degree of consumption externality is the same between both countries, no indirect effect exists. That is, if $b_D = b_R$, $A_D = A_R = 0$.*
- (ii) *If the donor's degree of consumption externality is smaller than that of the*

recipient, both the indirect effects of the donor and recipient are negative. That is, if $b_D < b_R$, $A_D < 0$ and $A_R < 0$.

(iii) If the donor's degree of consumption externality is larger than that of the recipient, both the indirect effects of the donor and recipient are positive. That is, if $b_D > b_R$, $A_D > 0$ and $A_R > 0$.

Proof. By Lemma 5.1, when $\delta_D = \delta_R \equiv \delta$, $dr/dT \geq 0$ if and only if $b_D \leq b_R$.

(i) If $b_D = b_R$, since $dr/dT = 0$, it is immediately obtained that $A_D = A_R = 0$.

(ii) If $b_D < b_R$, $dr/dT > 0$. Substituting the capital level of (5.24) into $Y(\delta) \equiv \delta w - (1+\delta)(1+r)k$ yields $Y(\delta) = \left\{ \delta - \frac{(1+\delta)(1+r)[\delta(1+b_D)+\delta(1+b_R)+2\delta^2(1-b_D b_R)]}{2(1+n)[(1+\delta)^2 - \delta^2 b_D b_R]} \right\} w$.

Since $r \geq n$ under dynamic efficiency, $Y(\delta) \leq \frac{\delta[2\delta b_D b_R - (1+\delta)(b_D + b_R)]}{2[(1+\delta)^2 - \delta^2 b_D b_R]} w$.

Since $2\delta b_D b_R - (1+\delta)(b_D + b_R) < 0$ for arbitrary values of $\delta \in (0, 1]$ and $b_i \in [0, 1]$, $Y(\delta) < 0$ is satisfied. Therefore, both the indirect effects of the donor and recipient satisfy $A_D = [Y(\delta)/w(1+r)] \times (dr/dT) < 0$ and $A_R = [Y(\delta)/w(1+r)] \times (dr/dT) < 0$, respectively.

(iii) If $b_D > b_R$, $dr/dT < 0$. As shown in the proof of (ii), since $Y(\delta) < 0$ under dynamic efficiency, both the indirect effects of the donor and recipient satisfy $A_D = [Y(\delta)/w(1+r)] \times (dr/dT) > 0$ and $A_R = [Y(\delta)/w(1+r)] \times (dr/dT) > 0$, respectively. \square

Proposition 5.4 claims that even when the time preference rates are the same between two countries, if the degrees of consumption externality differ between them, the indirect effect exists. In particular, as shown in Proposition 5.4(iii), when the donor's consumption externality is larger than that of the recipient, both the indirect effects are positive, and this implies that donor enrichment is likely to occur. This result suggests that when the donor has consumption externality on its utility, there is a possibility that a Pareto-improving outcome, such that both countries' welfare improves, occurs as a result of the transfer. In contrast, as the opposite case, Proposition 5.4(ii) shows that when the donor has a smaller degree of consumption externality than the recipient, both the indirect effects of the donor and recipient are negative, which implies that recipient impoverishment is likely to occur. This result suggests that when the recipient has consumption externality on its utility, there is a possibility that a Pareto-inferior outcome, such that both countries' welfare deteriorates, occurs.¹⁵ In sum, we

¹⁵ In this study, we analyze the general case in which there exists consumption externality in

can confirm from Propositions 5.4(ii) and 5.4(iii) that the donor's consumption externality is likely to lead to the paradox of donor enrichment, and the recipient's consumption externality is likely to lead to the paradox of recipient immiserization.

The reason why Proposition 5.4 holds is as follows: Even if the time preference rates are identical between two countries, the difference in the degrees of consumption externality brings about changes in the savings level of both countries and also the accumulated capital level. In case (ii), in which the donor's consumption externality is smaller than that of the recipient, a decreasing amount of the donor's savings exceeds an increasing amount of the recipient's savings and causes reduction in the accumulated capital and an increase in the interest rate. The increase in the interest rate causes both the positive effect of the increase in the old-age consumption and the negative effect of the decrease in the wage in young age. As capital is insufficient under dynamic efficiency, the negative effect of the increase in the interest rate exceeds the positive effect. When the time preference rates are identical for both countries, the increase in the interest rate caused by the transfer exacerbates social welfare. On the other hand, in case (iii), in which the donor's consumption externality is larger than that of the recipient, an increasing amount of the recipient's savings exceeds a decreasing amount of the donor's savings, and capital accumulation decreases the interest rate. As is explained in case (ii), under dynamic efficiency, the positive effect of the decrease in the interest rate exceeds the negative effect. Thus, the decrease in the interest rate by transfer improves the social welfare of both countries.

Finally, as a more general case, we consider the situation in which the discount factors are different between two countries, that is, $\delta_D \neq \delta_R$. When time preference rates differ between countries and there exists consumption externality, we obtain the following proposition on the indirect effect.¹⁶

both countries. However, the result of Proposition 5.4 can be interpreted by applying it to a simpler case. For example, Proposition 5.4(ii) includes the case in which only the recipient has consumption externality on its utility ($b_D = 0$ and $b_R > 0$) as a special case, and Proposition 5.4(iii) includes the case in which only the donor has consumption externality on its utility ($b_D > 0$ and $b_R = 0$) as a special case. Thus, our results of this proposition contain those obtained when there exists consumption externality only in one country.

¹⁶ For the proof, see Appendix A.5.1.

Proposition 5.5. *Suppose that the discount factors differ between two countries but the degrees of consumption externality are identical for both countries, that is, $\delta_D \neq \delta_R$ and $b_D = b_R \equiv b$.*

(i) When the donor has a higher discount factor than the recipient, the recipient's indirect effect is necessarily negative, and there exists a threshold of the degree of consumption externality such that when it exceeds a certain level, the donor's indirect effect becomes negative. That is, when $\delta_D > \delta_R$, $A_R < 0 \forall b$, and there exists a threshold $\bar{b}_D \in (0, 1)$ such that if $b > \bar{b}_D$, $A_D < 0$.

(ii) When the donor has a smaller discount factor than the recipient, the donor's indirect effect is necessarily positive, and there exists a threshold of the degree of consumption externality such that when it exceeds a certain level, the recipient's indirect effect becomes positive. That is, when $\delta_D < \delta_R$, $A_D > 0 \forall b$, and there exists a threshold $\bar{b}_R \in (0, 1)$ such that if $b > \bar{b}_R$, $A_R > 0$.

Proposition 5.5 claims that when the discount factors differ between both countries, if the degree of consumption externality is sufficiently large, the signs of the indirect effects for both countries are determined, and, notably, the sign of the donor's indirect effect is the same as that of the recipient's indirect effect. In Proposition 5.5(i), when the donor has a larger discount factor than the recipient, the indirect effects for both countries are negative. This implies that the donor's welfare necessarily deteriorates, and there is a possibility that the recipient impoverishment paradox occurs. If such a paradox occurs, the aid transfer can bring about a Pareto-inferior result. In contrast, when the donor has a smaller discount factor than the recipient, the signs of the indirect effects for both countries are positive. This necessarily results in the improvement of the recipient's welfare, and there is a possibility that the donor enrichment paradox occurs. In such a case, the aid transfer can bring about a Pareto-improving result. In any case, Proposition 5.5 demonstrates that depending on the relative sizes of the discount factors for the donor and recipient, opposite results are obtained regarding the impact that consumption externality has on the indirect effect of transfer on social welfare. Stated differently, in the situation with consumption externality, this proposition shows that the relative sizes of the discount factors between two countries determine whether consumption externality has a desirable influence on social welfare when a transfer is made.¹⁷

¹⁷ To obtain more generalized results than those of Proposition 5.5, we should have examined

5.5 Concluding remarks

In this chapter, by exploring a model in which both the donor and recipient countries have consumption externality in their utility in a one-sector OLG model, we examined the international transfer problem between the donor and the recipient. We consider the situation in which the generation in the donor country has consumption externality such that its utility increases with the increase in the recipient's consumption, and we presented several new results on how consumption externality influences the effect of the transfer on the social welfare of the donor and recipient and, especially, on how it affects the likelihood of the transfer paradox. The main conclusions of this study are as follows: First, in Proposition 5.1, we demonstrated that when the transfer increases the interest rate, (i) if labor share is less than twice the capital share, donor enrichment never occurs while recipient impoverishment might occur; (ii) if labor share is sufficiently larger than the capital share, the sign of the effect of the donor and recipient's welfare on the transfer depends on the relative size of the discount factors for both countries. Second, contrary to the above situation, we presented the result when the transfer decreases the interest rate in Proposition 5.2. Third, as already shown in existing studies, we confirmed in Proposition 5.3 that when there exists no consumption externality, the relative size of discount factors between two countries determines the signs of the indirect effect of the transfer on the social welfare of the donor and recipient. Fourth, in contrast with the results of existing studies, we considered the situation in which there exists consumption externality, and we examined in Proposition 5.4, how consumption externality affects the indirect effect on the social welfare of both countries when the discount factors are identical between two countries. Fifth, in Proposition 5.5, we also presented the result on the impact that consump-

the most general situation in which both the discount factor and degree of consumption externality are not identical for both countries. However, the basic conclusion is the same as Proposition 5.5, although only the calculation becomes complicated because we have to deal with two variables, (δ_D, δ_R) and (b_D, b_R) , simultaneously. Even in the most general case, the basic result is that the signs of the indirect effect depend on the relative sizes of the discount factors between two countries, and when the degree of consumption externality of each country, b_i , is sufficiently large, the sign of the indirect effect is determined and signs of the indirect effects for both countries become the same. We omit the analysis of the most general case on consumption externality.

tion externality has on the indirect effect on the social welfare of both countries when the discount factors are different from each other.

One main message of this study obtained from the above conclusion is as follows: When the donor's consumption externality is larger than that of the recipient, the international transfer between the two countries brings about a positive indirect effect for both the donor and recipient countries, and there is a possibility that the transfer improves the social welfare of both countries with consumption externality. Thus, if the donor country has more concerns about the recipient's consumption level than the recipient does, it can justify giving aid to the other country because the transfer improves its own welfare. Moreover, since the negative direct effect of the donor also decreases as the degree of consumption externality becomes higher, our result suggests that increasing the donor's interest in the recipient's consumption level leads to realizing a Pareto-efficient transfer aid for both countries. In this sense, raising the awareness and enlightenment of donor countries toward recipient countries is a very important activity in promoting official development assistance.

Finally, we finish our study by discussing future extension of our framework. First, our study is closely related to existing researches that tackled the transfer problem between two countries with altruism. Unlike the result of this study, in which there exists consumption externality, existing studies showed in the model with altruism that even if the degree of altruism is sufficiently high, any transfer paradox, especially donor enrichment paradox, never occurs. In this study, we introduced consumption externality into a model and pointed out the possibility that the transfer paradox can occur. However, although our study just pointed out the possibility of the paradox, we need to scrutinize the relative sizes between the direct effect and indirect effect to investigate whether or not the transfer paradox actually arises. After that, we need to compare the difference in results between the model with altruism and that with consumption externality. Elucidating why results differ between different models is a future subject to be studied. In addition, in analyzing the bilateral income transfer with interdependence between a donor country and a recipient country, identifying what type of model setting is more appropriate is an issue to be clarified from both theoretical and empirical perspectives. Second, basically when considering the international transfer in a dynamic framework thus far, only untied aids have been considered for simplification of analysis. However, in the real world

of the international aid transfer, there is relatively low proportion of untied aid or grant aid, whereas, in contrast, tied aid has become widespread around the world. It is well-known that tied aid causes some distortions on resource allocation. Another extensive direction is to examine how aid transfer affects capital accumulation dynamically as well as affects the social welfare of both countries when tied aids generate a distortion.

Appendix

A.5.1 Proof of Proposition 5.5

From Lemma 5.1, $dr/dT \gtrless 0 \Leftrightarrow \delta_D \gtrless \delta_R$ when $b_D = b_R$.

(i) When $\delta_D > \delta_R$, $dr/dT > 0$. Substituting the capital level (5.24) into $Y(\delta_i) \equiv \delta_i w - (1 + \delta_i)(1 + r)k$, we obtain the following equation.

$$Y(\delta_i) = \frac{2[(1+\delta_D)(1+\delta_R)-\delta_D\delta_Rb^2]\delta_i-(1+\delta_i)(1+b)[(\delta_D+\delta_R)+2\delta_D\delta_R(1-b)]}{2[(1+\delta_D)(1+\delta_R)-\delta_D\delta_Rb^2]}w. \quad (\text{A.5.1})$$

Since $r \geq n$ under dynamic efficiency, $Y(\delta_D) \leq \frac{Z_D(b)w}{2[(1+\delta_D)(1+\delta_R)-\delta_D\delta_Rb^2]}$, where $Z_D(b) \equiv 2\delta_D\delta_Rb^2 - (1 + \delta_D)(\delta_D + \delta_R)b + (1 + \delta_D)(\delta_D - \delta_R)$. $Z_D(b)$ is a quadratic function of b , whose coefficient of b^2 is strictly positive. Since $Z_D(0) = (1 + \delta_D)(\delta_D - \delta_R) > 0$ and $Z_D(1) = -2\delta_R < 0$, there necessarily exists $\bar{b}_D \equiv \frac{(1+\delta_D)(\delta_D+\delta_R)-\sqrt{(\delta_D+\delta_R)^2+\delta_D(\delta_D-3\delta_R)^2}}{4\delta_D\delta_R} \in (0, 1)$ such that if $b < \bar{b}_D$, $Z_D(b) > 0$, and if $b > \bar{b}_D$, $Z_D(b) < 0$. Since $Z_D(b) < 0$ when $b > \bar{b}_D$, $Y(\delta_D)$ is always negative, and it is also satisfied that $A_D = [Y(\delta_D)/w(1+r)] \times (dr/dT) < 0$. As regards the recipient, since $r \geq n$ under the dynamic efficiency, it is held that $Y(\delta_R) \leq \frac{Z_R(b)w}{2[(1+\delta_D)(1+\delta_R)-\delta_D\delta_Rb^2]}$, where $Z_R(b) \equiv 2\delta_D\delta_Rb^2 - (1 + \delta_R)(\delta_D + \delta_R)b + (1 + \delta_R)(\delta_R - \delta_D)$. $Z_R(b)$ is also a quadratic function of b , whose coefficient of b^2 is strictly positive. Since $Z_R(0) = (1 + \delta_R)(\delta_R - \delta_D) < 0$ and $Z_R(1) = -2\delta_D < 0$, $Z_R(b)$ is always negative irrespective of the value of b . Hence, by $Y(\delta_R) < 0$, it is satisfied that $A_R = [Y(\delta_R)/w(1+r)] \times (dr/dT) < 0$.

(ii) The proof follows the same procedure as the above proof of part (i). If $\delta_D <$

δ_R , $dr/dT < 0$. Under $r \geq n$, $Y(\delta_D) \leq \frac{Z_D(b)w}{2[(1+\delta_D)(1+\delta_R)-\delta_D\delta_Rb^2]}$. Since $Z_D(0) = (1+\delta_D)(\delta_D-\delta_R) < 0$ and $Z_D(1) = -2\delta_R < 0$, $Z_D(b)$ is always negative irrespective of the value of b . Hence, it is satisfied that $A_D = [Y(\delta_D)/w(1+r)] \times (dr/dT) > 0$. Likewise, $Y(\delta_R) \leq \frac{Z_R(b)w}{2[(1+\delta_D)(1+\delta_R)-\delta_D\delta_Rb^2]}$. Since $Z_R(0) = (1+\delta_R)(\delta_R-\delta_D) > 0$ and $Z_R(1) = -2\delta_D < 0$, there necessarily exists $\bar{b}_R \equiv \frac{(1+\delta_R)(\delta_D+\delta_R)-\sqrt{(\delta_D+\delta_R)^2+\delta_R(\delta_R-3\delta_D)^2}}{4\delta_D\delta_R} \in (0,1)$ such that if $b < \bar{b}_R$, $Z_R(b) > 0$, and if $b > \bar{b}_R$, $Z_R(b) < 0$. Since $Z_R(b) < 0$ when $b > \bar{b}_R$, it is satisfied that $A_R = [Y(\delta_R)/w(1+r)] \times (dr/dT) > 0$. \square

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Chapter 6

Aspirations and the Transfer Paradox

This chapter examines the transfer problem between two countries when either the donor or the recipient has aspirations, based on parents' standards of living, in a one-sector overlapping generations model. Focusing on whether and how aspirations impact the welfare effect of a transfer, we demonstrate the following results. First, when the donor forms aspirations, as the degree of his/her aspirations to their parents increases, a transfer is more likely to cause donor enrichment. However, this does not affect the recipient's welfare at all. In contrast, when the recipient forms aspirations, whether the increase in the degree of these aspirations causes immiserization depends on whether the transfer raises the recipient's consumption. Second, we show that if the donor's or recipient's marginal utility increases with their respective aspirations, the transfer is more likely to cause recipient immiserization. However, whether donor enrichment occurs depends on the situation. These results imply that there are two types of effects that aspirations can have on the welfare of both countries: effects caused by the aspirations, and effects that occur through the capital market. Furthermore, we find that these two effects on welfare do not necessarily work in the same direction.

Keywords: aspirations, transfer paradox, overlapping generations model, capital accumulation

JEL classifications: D91, E21, F35, F43

6.1 Introduction

In this chapter, we examine whether and how donor or recipient aspirations impact the welfare effect of an international transfer in a dynamic setting. In spite of the fact that intergenerational aspirations can affect capital accumulation in a dynamic setting and that capital accumulation can cause the transfer paradox, few studies have examined the transfer problem when there are aspirations in an overlapping generations (OLG) model. Therefore, this chapter examines the transfer problem involving two countries when either the donor or the recipient has aspirations, based on parents' standards of living, in a one-sector OLG model.

Ever since Keynes (1929) pointed out that a transfer is likely to diminish the transferer's welfare, and Samuelson (1952, 1954) demonstrated that no transfer paradox can occur in a Walrasian stable market equilibrium in two countries, the transfer problem has attracted the attention of many economists engaged in the theory of international trade. In static frameworks, the existing literature has demonstrated that in order for the transfer paradox to occur in a two-country model, distortions are required that restrict free trade. These might include exogenous distortions caused by trade barriers (tariffs or subsidies) or endogenous distortions (rent seeking or administrative costs of transfers).¹ In contrast, in a dynamic framework, and especially in an OLG model, the existing literature has shown that the transfer paradox can take place in a two-country model, without any distortions, even in a dynamically efficient region, because capital accumulated over time affects the welfare of both countries. Ever since the seminal work of Galor and Polemarchakis (1987), who used an OLG model to show that a permanent lump-sum transfer can cause the transfer paradox in

¹ The seminal articles in this context are those of Bhagwati, Brecher, and Hatta (1983, 1985). Brakman and van Marrewijk (1998) present a concise survey of the transfer problem in a static model.

a steady-state equilibrium, numerous studies have noted the possibility of the transfer paradox occurring in a dynamic framework, unlike the case of a static framework. Haaparanta (1989) proved that a transfer paradox can occur when a temporary transfer is financed by public debt in the donor country and/or is used for debt relief in the recipient country. Cremers and Sen (2008) extended the analysis to the transition to a steady state, and demonstrated that the results of Galor and Polemarchakis (1987) can also be applied to such a transition.²

However, in spite of its importance, few studies have addressed the problem of how intergenerational influences in a country, such as aspirations based on parents' living standards, impact the effectiveness of the international transfer. In reality, many macroeconomists have been investigating how such aspirations affect economic growth. Aspirations are an important notion in describing intergenerational behavior. Although modern economics has evolved under the conventional assumption that economic agents are perfectly rational, recent studies have revealed that agents are influenced by various cognitive biases and inherited preferences, which contradicts the assumption of perfect rationality. In particular, since Duesenberry (1949) demonstrated that "keeping up with the neighboring Joneses" drives consumption aspirations, a considerable number of studies have noted the existence of habit formation and aspirations. For example, using direct individual data, Stutzer (2004) showed empirically that higher income aspirations reduce people's utility, and in an experimental study, McBride (2010) found clear evidence that aspirations affect happiness.

Of the few studies to examine aspiration formation in an OLG model, De la Croix and Michel (1999) were the first to incorporate aspiration formation in an OLG model to clarify the conditions required for optimal growth. Then, De la Croix (1996, 2001), De la Croix and Michel (2001), and Artige, Camacho, and De la Croix (2004) investigated the stability condition in an OLG model that included aspirations. Barnett and Bhattacharya (2008) examined how the existence of rejuvenated individuals, being older agents who derive utility from keeping their consumption levels comparable to those of current younger persons, affects economic growth in a standard OLG model. Alonso-Carrera, Caballe, and Raurich (2007) explored a model incorporating both aspirations and

² For other articles that have examined the transfer problem in an OLG model, see Tan (1998) and Yanagihara (2006).

habit formation and demonstrated that aspirations and habit formation have opposite effects on the bequest motive. However, few studies have used an OLG model to investigate how the existence of aspirations affects an international economy in terms of, for example, trade patterns and the transfer problem.

In contrast, several studies have dealt with habit formation in a two-country continuous-time Ramsey model. Since Ryder and Heal (1973) presented a model dealing with consumption habits to clarify the characteristics of optimal paths, several studies have examined the bilateral relationship in a continuous-time Ramsey model with habit formation. Focusing on the effect of habit formation on trade, Mansoorian (1993) used the model of Ryder and Heal (1973) to show that the Harberger–Laursen–Metzler effect holds, where a deterioration in terms-of-trade causes a decrease in savings due to a decrease in real income. Carroll, Overland, and Weil (2000) showed that a growth model in which an individual cares about the habit stock determined by past consumption can explain why savings and growth are positively correlated across countries. Ikeda and Gombi (2009) extended the model with habit persistence to a two-country world economy, and showed that an increase in fiscal spending in one country can benefit the country and harm others, owing to reversed intertemporal terms-of-trade effects. In particular, as the most important contribution in a Ramsey model, Gombi and Ikeda (2003) demonstrated that the transfer paradox can occur in a free-trade, dynamically stable economy, using a two-country model with habit-forming consumers. Although there is an essential difference between the setting of a Ramsey model and that of an OLG model, their study addresses the same problem we do here. Both elucidate how the intertemporal dependence of preferences, such as habit formation or aspirations, impacts the welfare effect of a transfer.

Aspirations in an OLG model differ from habit formation in a continuous-time Ramsey model. Aspirations based on parents' standards of living are an exogenous preference, and generations cannot change the consumption levels inherited from the preceding generation. In contrast, habit formation is based on an individual's previous consumption. In this case, individuals can determine their consumption level, taking into account whether the initial consumption levels they chose subsequently affect their utility. Thus, habit formation in a Ramsey model is referred to as "rational" because a representative individual foresees what will happen in the future perfectly after choosing the initial

consumption. Moreover, we cannot examine the intergenerational influence in a continuous-time Ramsey model because the model does not include different generations; representative individuals live forever.

Therefore, we adopt the OLG model to consider the intergenerational influence explicitly. Then, we investigate how aspirations inherited from parents impact the effectiveness of an international transfer from a donor to a recipient on the welfare of each country through the change in capital accumulation. Considering the situation in which each generation in a country inherits aspirations from parents, we aim to fill the existing gap between the Ramsey model and the OLG model. In particular, we examine whether the degree of aspirations alters the likelihood of the transfer paradox occurring, where the donor is enriched and/or the recipient is immiserized as a result of the transfer.

This study also sheds new light on the conditions necessary for the transfer paradox to occur when there is an intergenerational externality on preferences. Here, we incorporate aspirations into an individual's preference. In the theoretical framework dealing with the transfer problem, several studies have investigated the conditions necessary for the paradox to occur. Hamada and Yanagihara (2014) explored a model in which the donor has an altruistic utility to the recipient, and demonstrated that, contrary to the first impression, as the donor becomes highly altruistic, donor enrichment is not likely to occur. Hamada and Yanagihara (2016) considered a situation in which an individual has intergenerational altruism, and showed that the condition for the transfer paradox to occur is independent of the degree of this altruism. Although aspirations differ from altruism, our study has a common interest with the afore-mentioned literature on how the externality of consumption or utility fosters or hinders the effect of a transfer on the donor's or the recipient's welfare.

Our results are as follows. First, in the case where a donor with an aspiration preference has a higher marginal propensity to save than does the recipient, donor enrichment is more likely to occur in the steady state as the donor's degree of aspirations increases. However, there is no effect on the recipient's welfare. When the recipient has an aspiration preference, whether recipient immiserization occurs depends on whether the transfer raises the consumption of the recipient. Second, when the effect of the donor's (recipient's) aspirations on his/her marginal utility increases, whether the transfer paradox is more likely to occur depends on the savings propensities in the two countries. We also inves-

tigate the dynamic stability condition in the model incorporating aspirations, and show that no additional condition is required when the dynamic stability condition is satisfied in the standard OLG model without aspirations.

The remainder of this chapter is organized as follows. Section 6.2 describes the one-sector OLG model, in which each generation of the donor country or recipient country has aspirations based on the consumption level of the preceding generation. Here, we confirm whether the dynamic stability conditions are satisfied in the model with aspirations. Sections 6.3 and 6.4 present the main results on the relationship between aspirations and the transfer paradox. Section 6.3 examines the effect of aspirations on a transfer when the donor has aspirations, and Section 6.4 does the same in the case of the recipient having aspirations. Lastly, Section 6.5 concludes this chapter.

6.2 The model

We consider a one-sector OLG model with two countries. Time is discrete and starts from the initial period, $t = 0$. There exist two countries in the world economy, namely a donor country and a recipient country, indexed by country as $i = D$ and R , respectively, and an international income transfer. The transfer is made from country D to country R . These two countries are identical except for the time preferences of individuals and the existence of aspiration preferences. Capital is fully mobile between the two countries, but labor is immobile. The populations of both countries grow equally, with a gross population growth rate of $(1 + n) \geq 1$, which is given exogenously and constant over time.

6.2.1 Individuals

In each country, all individuals are identical and live for three periods: infant, young, and old. A new generation is born in each period and each individual has offspring in the second (that is, young) period. The infant individuals neither work nor consume in their first period and only the young and the old generations engage in economic activities during the last two periods. Thus, there are two active generations who make economic decisions in both countries; the young, who supply one unit of labor inelastically and earn wages, and the old, who retire and consume savings accumulated in their youth. We de-

note the young generation in period t as generation t . When generation t is an infant in period $(t - 1)$, they are affected by the living standard of their parent's generation, and form aspirations.

Then, c_t^i and d_{t+1}^i denote the consumption level that generation t in country $i = D, R$ choose in their young period t and in their old period $(t + 1)$. Each generation maximizes their utility, subject to the budget constraints in their respective young and old periods. As in De la Croix (1996), the infant generation in period $(t - 1)$ inherits a certain level of aspirations based on the standard of living achieved by their parents (that is, the young generation in period $(t - 1)$). In order to describe the aspiration preference, we assume that the utility of generation t in country i depends also on the young consumption level of the country's parent generation; that is, c_{t-1}^i . More specifically, the larger c_{t-1}^i becomes, the greater the marginal utility of generation t becomes, with respect to the consumption of the young. This assumption implies that generation t inherits their aspirations with regard to their standard of living from their parents.³ We define the aspirations, a_t^i , inherited by generation t in country i as follows:

$$a_t^i \equiv c_{t-1}^i \quad \forall t. \quad (6.1)$$

The intertemporal utility of generation t in country i is given by

$$u^i(c_t^i, a_t^i) + v^i(d_{t+1}^i). \quad (6.2)$$

Here we assume that u^i and v^i are twice differentiable, $u_c^i, v_d^i > 0$, $u_a^i < 0$, $u_{cc}^i, u_{aa}^i, v_{dd}^i < 0$, and $u_{ca}^i > 0$. Here, $u_{ca}^i > 0$ implies that an increase in aspirations increases the marginal utility of consumption. Moreover, we add

³ However, in the opposite situation, we can also define other aspirations, in which the larger c_{t-1}^i becomes, the less the marginal utility of generation t becomes with respect to the consumption of the young. However, we exclude this unusual negative externality, in which any increase in the parents' consumption decreases the present generation's satisfaction from consumption.

the following usual assumptions:

$$\lim_{c \rightarrow 0} u_c^i(c, a) = \infty, \quad \lim_{d \rightarrow 0} v_d^i(d) = \infty, \quad (6.3)$$

$$u_{cc}^i u_{aa}^i > (u_{ca}^i)^2, \quad (6.4)$$

$$u_c^i + u_a^i > 0, \quad u_{ca}^i + u_{cc}^i < 0. \quad (6.5)$$

Here, (6.3) and (6.4) guarantee the interior solution and the strict quasi-concavity of u^i , respectively. Then, (6.5) is usually satisfied because it implies that the impact on the utility or marginal utility of the consumption of the young is larger than that of their parents' consumption.

The budget constraints in the young and old periods for generation t in country i are given as follows:

$$c_t^i + s_t^i = w_t + T^i \quad \text{and} \quad d_{t+1}^i = (1 + r_{t+1})s_t^i, \quad (6.6)$$

respectively, where r , w , and s denote the net interest rate, wage rates, and savings, respectively. In addition, T^i denotes a permanent transfer from/to country i . We denote the positive amount of the transfer received by the recipient as T ; that is, $T \equiv T^R = -T^D > 0$. However, throughout the chapter, we focus only on the infinitesimal change of the transfer from the situation without the transfer, as in prior studies. The intertemporal budget constraint can be rewritten as follows:

$$c_t^i + \frac{1}{1 + r_{t+1}} d_{t+1}^i = w_t + T^i. \quad (6.7)$$

$w_t + T^i > 0$ for all t , because we assume that the transfer is infinitesimal.

Given the aspiration level a_t^i , generation t in country i solves the following utility maximization problem:

$$\max_{\{c_t^i, d_{t+1}^i\}} u^i(c_t^i, a_t^i) + v(d_{t+1}^i) \quad \text{s.t.} \quad c_t^i + \frac{1}{1 + r_{t+1}} d_{t+1}^i = w_t + T^i. \quad (6.8)$$

The first-order condition is given by

$$u_c^i = (1 + r_{t+1})v_d^i. \quad (6.9)$$

The second-order condition is satisfied by the assumptions, $u_{cc}^i < 0$ and $v_{dd}^i < 0$.

The optimal consumption bundle depends on $w_t + T^i$, r_{t+1} , and a_t^i , yielding the following demand functions: $c_t^i = c^i(w_t + T^i, r_{t+1}; a_t^i)$ and $d_{t+1}^i = d^i(w_t + T^i, r_{t+1}; a_t^i)$. With regard to the demand functions, we obtain the following properties:⁴

$$c_w^i = \frac{(1 + r_{t+1})^2 v_{dd}^i}{u_{cc}^i + (1 + r_{t+1})^2 v_{dd}^i} \in (0, 1), \quad d_w^i = \frac{(1 + r_{t+1})u_{cc}^i}{u_{cc}^i + (1 + r_{t+1})^2 v_{dd}^i} > 0, \quad (6.10)$$

$$c_r^i = \frac{v_d^i + v_{dd}^i d_{t+1}^i}{u_{cc}^i + (1 + r_{t+1})^2 v_{dd}^i}, \quad d_r^i = \frac{-(1 + r_{t+1})u_c^i + u_{cc}^i d_{t+1}^i}{(1 + r_{t+1})[u_{cc}^i + (1 + r_{t+1})^2 v_{dd}^i]} > 0, \quad (6.11)$$

$$c_a^i = -\frac{u_{ca}^i}{u_{cc}^i + (1 + r_{t+1})^2 v_{dd}^i} \in (0, 1), \quad d_a^i = \frac{(1 + r_{t+1})u_{ca}^i}{u_{cc}^i + (1 + r_{t+1})^2 v_{dd}^i} < 0. \quad (6.12)$$

The signs follow from the assumptions on the utility function. Similarly, the savings function can be obtained as follows: $s_t^i = s^i(w_t + T^i, r_{t+1}; a_t^i)$. The

⁴ The subscripts of consumption and savings denote the partial derivatives; that is: $c_w^i \equiv \partial c_t^i / \partial w_t$, $c_r^i \equiv \partial c_t^i / \partial r_{t+1}$, and $c_a^i \equiv \partial c_t^i / \partial a_t^i$.

properties of the savings function satisfy the following equations:

$$s_w^i = \frac{u_{cc}^i}{u_{cc}^i + (1 + r_{t+1})^2 v_{dd}^i} \in (0, 1), \quad (6.13)$$

$$s_r^i = -\frac{v_d^i + v_{dd}^i d_{t+1}^i}{u_{cc}^i + (1 + r_{t+1})^2 v_{dd}^i}, \quad (6.14)$$

$$s_a^i = \frac{u_{ca}^i}{u_{cc}^i + (1 + r_{t+1})^2 v_{dd}^i} \in (-1, 0). \quad (6.15)$$

To guarantee that consumptions are normal goods, we assume that the savings function is not decreasing in the interest rate; that is, $s_r^i \geq 0$.

In many real situations, the donor and the recipient are developed and the developing countries, respectively. Moreover, it has been observed that the developed countries have a higher marginal propensity than do developing countries. Furthermore, in our setting of the OLG model, the country with the higher marginal propensity necessarily has larger (smaller) savings than otherwise in a steady state, which results in it being a capital lender (borrower). In order to exclude this unusual case from the analysis, where the capital borrower becomes the donor, we apply the following assumption throughout the analysis:

Assumption 1 The donor has a higher marginal propensity to save than that of the recipient; that is, $s_w^D > s_w^R$.

Finally, substituting the above consumption functions into the utility function, we obtain the indirect utility function, as follows:

$$\begin{aligned} V^i(w_t + T^i, r_{t+1}; a_t^i) &\equiv u^i(c^i(w_t + T^i, r_{t+1}; a_t^i), a_t^i) \\ &\quad + v^i(d^i(w_t + T^i, r_{t+1}; a_t^i)). \end{aligned} \quad (6.16)$$

Because $u_c^i c_w^i + v_d^i d_w^i = u_c^i > 0$, $u_c^i c_r^i + v_d^i d_r^i = v_d^i s_t^i > 0$, and $u_c^i c_a^i + v_d^i d_a^i = 0$ hold from the first-order condition, the indirect utility function has

the following properties:

$$V_w^i \equiv \frac{\partial V^i}{\partial w_t} = u_c^i > 0, \quad (6.17)$$

$$V_r^i \equiv \frac{\partial V^i}{\partial r_{t+1}} = v_d^i s_t^i > 0, \quad (6.18)$$

$$V_a^i \equiv \frac{\partial V^i}{\partial a_t^i} = u_a^i < 0. \quad (6.19)$$

6.2.2 Firms

Firms in both countries produce their output using the inputs of labor and capital under perfect competition. The production function exhibits constant returns to scale, independent of time, and is identical in both countries. Capital does not depreciate. The per capita production function is $f(k_t^i)$, where k_t^i represents the per capita capital in country i in period t . We assume that the per capita production function satisfies the following conditions: (i) $f(k_t^i)$ is continuously differentiable; (ii) $f(k_t^i) > 0$, $f'(k_t^i) > 0$, and $f''(k_t^i) < 0$ for all $k_t^i > 0$; and (iii) $f(0) = 0$, $\lim_{k_t \rightarrow 0} f'(k_t) = \infty$, and $\lim_{k_t \rightarrow \infty} f'(k_t) = 0$ (the Inada conditions).

Firms maximize their profit in per capita terms, denoted by $\pi(k_t^i) \equiv f(k_t^i) - r_t k_t^i - w_t$. Profit maximization requires the equivalence of the marginal productivity and the price of each input:

$$f'(k_t^i) = r_t \quad \text{and} \quad f(k_t^i) - f'(k_t^i)k_t^i = w_t. \quad (6.20)$$

From (6.20), the interest rate and the wage rate can be represented as a function of k_t^i . Because capital is perfectly mobile, these become the same in both countries by factor price equalization, so that $k_{t+1}^D = k_{t+1}^R \equiv k_{t+1}$ holds. In summary, we obtain $r_t = r(k_t)$ and $w_t = w(k_t)$, where $r'(k_t) = f'' < 0$ and $w'(k_t) = -k f'' > 0$.

6.2.3 Capital market equilibrium

The world capital market equilibrium in period t requires the sum of savings of generation t of both countries to be equal to the sum of the capital demand of both countries in the subsequent period $t + 1$. Therefore, the capital market equilibrium in per capita terms in period t can be expressed as follows:

$$2(1+n)k_{t+1} = s^D(w(k_t) - T, r(k_{t+1}); a_t^D) + s^R(w(k_t) + T, r(k_{t+1}); a_t^R). \quad (6.21)$$

We define the excess demand in the world capital market in period t as $D(k_t, k_{t+1}, a_t^D, a_t^R, T) \equiv 2(1+n)k_{t+1} - s^D - s^R$. The Walrasian stability condition in period t is given by

$$\Delta_t \equiv \frac{\partial D(k_t, k_{t+1}, a_t^D, a_t^R, T)}{\partial k_{t+1}} = 2(1+n) - (s_r^D + s_r^R)f'' > 0, \quad \forall t. \quad (6.22)$$

Note that (6.22) necessarily holds under the assumption of $s_r^i \geq 0$. Therefore, the Walrasian stability condition is satisfied in the capital market equilibrium in each period.

Finally, we assume the dynamic efficiency condition, $r_t \geq n$ for all t .

6.2.4 Dynamic stability

In the model in which individuals of country i , either the donor country or the recipient country, have aspirations, a_t^i and capital k_t are state variables. Thus, we need to check whether the steady-state equilibrium is dynamically stable. Here we consider the case in which only the individuals of the donor country form aspirations, and we limit the argument to the case in which the dynamic stability with monotonic convergence holds.⁵ The sufficient condition for dynamic stability is that the two eigenvalues of the difference equation system with respect to the two state variables, k_t and a_t^D , are between zero and one.⁶

⁵ We can easily check the condition in which only the recipient country forms aspirations in the same way.

⁶ De la Croix and Michel (1999), the seminal work exploring the model of aspirations in an OLG model, point out the possibility of a Neimark–Sacker bifurcation of the steady

By the definition of the aspirations and the budget constraint in the young period, the following equation is satisfied:

$$a_{t+1}^D = w(k_t) - T - s^D(w(k_t), r(k_{t+1}); a_t^D). \quad (6.23)$$

The behavior of the state variables, k_t and a_t , are stipulated by the capital market equilibrium in period t , (6.21), and the recurrence formula for the aspirations of the donor, (6.23).

By totally differentiating (6.21) and (6.23) with respect to k_{t+1} , a_{t+1}^D , k_t , and a_t^D , and evaluating the equations at $T = 0$, because we focus on the infinitesimal change of a transfer from the situation with no transfer, we obtain the following recurrence formula with respect to k_t and a_t^D :

$$\begin{bmatrix} dk_{t+1} \\ da_{t+1}^D \end{bmatrix} = A \begin{bmatrix} dk_t \\ da_t^D \end{bmatrix},$$

where $A \equiv \frac{1}{\Delta_t} \begin{bmatrix} -(s_w^D + s_w^R)k_t f'' & s_a^D \\ [(s_w^D + s_w^R)s_r^D f'' - \Delta_t(1 - s_w^D)]k_t f'' & -(s_r^D f'' + \Delta_t)s_a^D \end{bmatrix}.$ (6.24)

Therefore, the sufficient condition for dynamic stability is that both eigenvalues of the matrix A are strictly between 0 and 1. Now, we define the sufficient condition for dynamic stability to be satisfied when no aspiration exists as $\Gamma \equiv \Delta + (s_w^D + s_w^R)k f''$. By tedious calculation, we derive the sufficient condition for dynamic stability around the neighborhood of the steady state when only the donor forms aspirations, as follows:

$$\Gamma_D \equiv \Gamma + s_a^D \left\{ \Delta + [s_r^D + (1 + s_w^R)k] f'' \right\} > 0. \quad (6.25)$$

state. This bifurcation arises when the eigenvalues of the matrix A in (6.24) in our model are complex conjugates with modulus unity. Then, they show that this bifurcation arises when the degree of aspirations is sufficiently high, without any assumptions on dynamic stability. To avoid the analytical complexity, we rule out the irregular bifurcation case here by assuming that the eigenvalues of the matrix are real numbers or are less than modulus unity, because monotonic convergence to a steady state is usually assumed when the transfer problem is examined in an OLG model.

Similarly, we derive the sufficient condition for dynamic stability when only the recipient forms aspirations, as follows:

$$\Gamma_R \equiv \Gamma + s_a^R \left\{ \Delta + [s_r^R + (1 + s_w^D)k] f'' \right\} > 0. \quad (6.26)$$

In fact, because $s_a^i \in (-1, 0)$, which is satisfied under the assumption on the utility function, (6.25) and (6.26) are guaranteed if $\Gamma > 0$. Throughout the chapter, we assume $\Gamma > 0$. See Appendix A.6.1 for the derivation of the sufficient condition.

In the situation in which the steady state is dynamically stable, we can examine the impact of aspiration formation on transfers when either the donor or the recipient forms aspirations.⁷ In the following section, we explain the results arising from the relationship between the transfer paradox and aspiration formation.

6.3 The donor's aspirations and the transfer paradox

In the following two sections, we examine the effect of the transfer on social welfare when aspirations are formed either in the donor or in the recipient country in the steady state. First, in Section 6.3, we consider the case in which only individuals in the donor country form aspirations. Then, in Section 6.4, we consider the case in which only individuals in the recipient country form aspirations.

The world capital market equilibrium in the steady state and the donor's aspirations are represented by the following equations:

$$2(1 + n)k = s^D(w(k) - T, r(k); a^D) + s^R(w(k) + T, r(k)), \quad (6.27)$$

$$a^D = w(k) - T - s^D(w(k) - T, r(k); a^D), \quad (6.28)$$

respectively, where transfer T is considered. By totally differentiating (6.27)

⁷ However, when both donor and recipient countries form aspirations, some additional assumptions are required in order to satisfy dynamic stability, even under $s_a^i \in (-1, 0)$.

and (6.28), we obtain the following equation:

$$\begin{bmatrix} \Gamma & -s_a^D \\ [k(1 - s_w^D) + s_r^D]f'' & 1 + s_a^D \end{bmatrix} \begin{bmatrix} dk \\ da^D \end{bmatrix} = - \begin{bmatrix} s_w^D - s_w^R \\ 1 - s_w^D \end{bmatrix} dT. \quad (6.29)$$

Note that the determinant of the matrix of the left-hand side of (6.29) is Γ_D , which shows that $\Gamma_D > 0$ when the dynamic stability condition is satisfied. By solving (6.29) with respect to dk/dT and da^D/dT , we obtain the following equation:

$$\begin{bmatrix} \frac{dk}{dT} \\ \frac{da^D}{dT} \end{bmatrix} = \frac{1}{\Gamma_D} \begin{bmatrix} -(s_w^D - s_w^R)(1 + s_a^D) - s_a^D(1 - s_w^D) \\ (s_w^D - s_w^R)[k(1 - s_w^D) + s_r^D]f'' - \Gamma(1 - s_w^D) \end{bmatrix}. \quad (6.30)$$

By (6.30), the effects of the transfer on per capita capital can be divided into two terms. The first term, $-(s_w^D - s_w^R)(1 + s_a^D)$, represents the effect on the capital market. The term $-(s_w^D - s_w^R)$ denotes the effect on capital accumulation brought about by the difference in the marginal propensity to save between a donor and a recipient. In addition, the term $(1 + s_a^D)$ refers to the contraction of the capital accumulation as a result of inherited aspirations. If the donor has a higher savings propensity than the recipient, the transfer directly decreases the world capital in the steady state. On the other hand, the transfer decreases the consumption when young, which weakens the influence of the aspirations. This, in turn, indirectly increases the world capital in the steady state, which contributes to contracting the above effect on the capital market. The sign of the term depends on the configuration of the marginal saving propensities.

The second term, $-s_a^D(1 - s_w^D)$, can be considered to be the effect of the aspirations on capital accumulation. This refers to the effect of changing the amount of consumption when young as a result of the aspiration preferences of the donor country.⁸ This sign is positive, by assumption, because the transfer from the donor weakens the effect that the donor's aspirations induce greater consumption when young. Thus, the decrease in the donor's savings becomes somewhat smaller.

Similarly, the effect of the transfer on aspirations can be decomposed into two parts. Note that since $a^D = c^D$ in the steady state, the effect of the transfer

⁸ Note that $(1 - s_w^D)$ is the marginal propensity to consume when young.

on aspirations, da^D/dT , is the same as the effect of the transfer on the consumption level of the young in the previous generation. The first term is the effect on aspirations through the capital market, which can be viewed as an indirect effect. The change in the amount of world capital, which originated in the difference in the marginal propensity to save between the donor and the recipient, first affects the interest rate. Then, the change in the interest rate causes the substitution of consumption between the young and the old. Furthermore, it changes the wage rate in a different direction. Both constitute the effect on the world capital level and lead to a change in consumption in the steady state. As with the effect on the capital market with regard to the capital level, the sign of this effect on aspirations through the capital market depends on the difference in savings propensities. The second term is the direct effect, which is caused directly by the income change from the transfer. Because the income of the donor decreases, the consumption of the young also decreases. This, in turn, weakens the influence of aspirations. The effect on consumption is negative for the donor and positive for the recipient.

The following lemma follows immediately from (6.30).

Lemma 6.1. *Consider the situation in which the donor has aspirations. If the effect on the capital market is larger (smaller) than the effect of the aspirations, then the world capital decreases (increases) as a result of the transfer. That is, if and only if $(s_w^D - s_w^R)(1 + s_a^D) \geq -s_a^D(1 - s_w^D)$, $dk/dT \geq 0$.*

Proof. The result follows immediately from (6.30). □

By Lemma 6.1, if $s_w^D < s_w^R$, both effects—the effect on the capital market and the effect of the aspirations—would be positive. In this case, the transfer necessarily would promote capital accumulation. However, in Assumption 1, we assume that the donor has a larger marginal propensity to save than that of the recipient ($s_w^D > s_w^R$). Thus, the direction of the change in the level of the world capital cannot be determined. The effect on the capital market is negative because the transfer from the donor decreases the world's total savings, which leads to a decrease in world capital. Since the effect of the aspiration preferences is necessarily positive, these two effects conflict.

Next, we present a lemma for the effect of the transfer on aspirations (equivalently, on consumption), as follows.

Lemma 6.2. *Consider the situation in which the donor has aspirations. If the effect through the capital market is positive, the transfer might make the donor's consumption increase. On the other hand, if it is negative, the transfer necessarily makes the donor's consumption decrease.*

Proof. It is clear that if $-(s_w^D - s_w^R)[k(1 - s_w^D) + s_r^D]f'' > 0$, then the sign of da^D/dT is indeterminate; otherwise, $da^D/dT < 0$. \square

Lemma 6.2 claims that under Assumption 1, the transfer necessarily decreases the world capital level and decreases the consumption (that is, the aspirations) of the young. This corresponds to the fact that the effect on aspirations through capital accumulation, denoted by the first term of the second row of the right-hand side of (6.30), is negative. Since the second term, as the direct effect of the transfer, is also negative, the transfer necessarily decreases the donor's consumption.

Next, we examine the effect of the transfer on the welfare levels. From (6.16), the indirect utility functions for the donor and the recipient in the steady state can be represented by $V^D = V^D(w(r) - T, r; a^D)$ and $V^R = V^R(w(r) + T, r)$, respectively. By totally differentiating the indirect utility function, and rearranging, we obtain the following equation:

$$\begin{bmatrix} dV^D \\ dV^R \end{bmatrix} = \begin{bmatrix} \underbrace{v_d^D [s^D - (1+r)k] f'' dk}_{\text{(the capital accumulation effect)}} + \underbrace{u_a^D da^D}_{\text{(the aspiration effect)}} - \underbrace{u_c^D dT}_{\text{(the income effect)}} \\ \underbrace{v_d^R [s^R - (1+r)k] f'' dk}_{\text{(the capital accumulation effect)}} + \underbrace{u_c^R dT}_{\text{(the income effect)}} \end{bmatrix} \quad (6.31)$$

$$= \begin{bmatrix} v_d^D [s^D - (1+r)k] f'' \frac{dk}{dT} + u_a^D \frac{da^D}{dT} - u_c^D \\ v_d^R [s^R - (1+r)k] f'' \frac{dk}{dT} + u_c^R \end{bmatrix} dT. \quad (6.32)$$

By (6.32), we can divide all the effects on the welfare of the donor into three parts: the capital accumulation effect, the aspiration effect, and the income effect. On the other hand, the first and the third effects can be viewed as the effects on the welfare of the recipient.

Note that $u_a^i \in (-u_c^i, 0)$ by the assumption on the utility function, (6.5), and $v_d^i = u_c^i/(1+r)$ by the first-order condition, (6.9). By substituting $v_d^i = u_c^i/(1+r)$,

dk/dT , and da^D/dT into (6.32), we finally obtain the following equations for the effect of the transfer on the welfare of both the donor and the recipient:

$$\begin{aligned}
 \frac{1}{u_c^D} \frac{dV^D}{dT} &= \frac{\left(k - \frac{s^D}{1+r}\right) f'' \left[(s_w^D - s_w^R) + s_a^D (1 - s_w^R) \right] + u_a^D \left\{ (s_w^D - s_w^R) \left[k(1 - s_w^D) + s_r^D \right] f'' - \Gamma(1 - s_w^D) \right\}}{\Gamma_D} \\
 &\quad \text{(the indirect effect)} \\
 &= \text{(the capital accumulation effect)} + \text{(the aspiration effect)} \\
 &\quad - \underbrace{1}_{\text{(the direct effect)}}, \\
 &\quad \text{(the income effect)} \\
 \frac{1}{u_c^R} \frac{dV^R}{dT} &= \frac{\left(k - \frac{s^R}{1+r}\right) f'' \left[(s_w^D - s_w^R) + s_a^D (1 - s_w^R) \right]}{\Gamma_D} + \underbrace{1}_{\text{(the direct effect)}}. \tag{6.33} \\
 &\quad \text{(the indirect effect)} \quad \text{(the income effect)} \\
 &= \text{(the capital accumulation effect)} \quad \text{(the income effect)}
 \end{aligned}$$

Under Assumption 1, the savings level in the steady state satisfies $s^R < (1 + n)k < s^D$. Combining these inequalities and the dynamic efficiency condition leads to a relation for the international capital movement, $(1 + r)k > s^R$. Note that the configuration of $(1 + r)k$ and s^D is ambiguous. As indicated in Galor and Polemarchakis (1987), Haaparanta (1989), and Yanagihara (1998, 2006), the configuration of $(1 + r)k$ and s^D can cause a paradoxical result on the welfare of the donor. More specifically, if $(1 + r)k < (>)s^D$, the indirect effect of the transfer on the welfare of the donor becomes positive (negative). When the donor's savings are sufficiently large and if the world capital accumulation decreases, then even if the decrease lowers the welfare, the return from capital lending becomes so large that it dominates the loss in welfare. As a result, the indirect effect becomes positive. In particular, if it further dominates the negative income (direct) effect, the welfare of the donor improves as a result of the transfer. As shown in the following, this configuration plays a critical role in determining the effect on welfare brought about by aspirations.

Before investigating the total effect on welfare, we examine the indirect effect of the transfer on the donor's welfare. By (6.33), we obtain the following lemma.

Lemma 6.3. *Suppose the donor has aspirations, as in Assumption 1.*

(a) *As the degree of the donor's aspirations increases, the indirect effect of a*

transfer on the donor's welfare increases.

(b) *As the effect of the donor's aspirations on the marginal utility of the young generation's consumption increases, the indirect effect of a transfer on the donor's welfare decreases (increases) if the indirect effect on the donor's welfare is positive (negative), with no aspirations (that is, if $(1+r)k < (>)s^D$).*

Proof. (a) In (6.33), $(s_w^D - s_w^R)[k(1 - s_w^D) + s_r^D]f'' - \Gamma(1 - s_w^D) < 0$ holds. Because $u_a^D < 0$, as the absolute value of u_a^D increases, the indirect effect on the donor's welfare necessarily increases.

(b) From Assumption 1 and $r \geq n$, $(1+r)k - s^R > 0$ holds. In addition, because $s_w^D - s_w^R > 0 > s_a^D(1 - s_w^D)$, if $(1+r)k < (>)s^D$, the indirect effect of the donor decreases (increases) as the absolute value of s_a^D increases. By (6.15), as the magnitude of s_a^D increases, so does u_{ca}^D . \square

Similarly, we examine the indirect effect of the transfer on the recipient's welfare. By (6.33), we obtain the following lemma.

Lemma 6.4. *Suppose that the donor has aspirations, as in Assumption 1.*

(a) *The degree of the donor's aspirations has no indirect effect on the recipient's welfare as a result of the transfer.*

(b) *As the effect of the donor's aspirations on the marginal utility of the young generation's consumption increases, the indirect effect of a transfer on the recipient's welfare increases.*

Proof. (a) Because there is no aspiration effect for the indirect effect of the recipient, the indirect effect does not depend on u_a^D .

(b) From Assumption 1 and $r \geq n$, $(1+r)k > s^R$ holds. In addition, because $s_w^D - s_w^R > 0 > s_a^D(1 - s_w^D)$, the indirect effect of the recipient increases as the absolute value of s_a^D , or u_{ca}^D , increases. \square

The implications of Lemmas 6.3 and 6.4 are summarized as follows. Only in the indirect effect of the donor is there an aspiration effect. When the donor has aspirations, the pathway that affects the donor's welfare is given by $T \uparrow \rightarrow a^D \downarrow \rightarrow u^D \uparrow$. Stated more precisely, the reduction of the donor's income caused by the donor's transfer decreases the consumption of the young. By $u_a^D < 0$, the decrease in the aspiration level causes an increase in utility because of the utility reduction effect, accompanied by the aspiration preference being mitigated by the transfer. As a result, weakening the donor's aspirations

leads to a welfare improvement in the donor country. On the other hand, there is a capital accumulation effect as the indirect effect for both the donor and the recipient, given by $T \uparrow \rightarrow (s^D - s^R) \downarrow \rightarrow k \downarrow$. More specifically, the reduction of the donor's income caused by the transfer itself decreases the donor's savings and increases the recipient's savings in each period. Since $s_w^D > s_w^R$, the world's total savings and, therefore, the world's accumulated capital, decreases. This reduction in capital works to lower the welfare of both countries when the economy is dynamically efficient. Therefore, as long as the savings of the donor or the international capital lending are not above a certain level, the indirect effect is negative. In this case, because the donor's aspirations become weaker as a result of the reduction in consumption, the negative indirect effect is mitigated. However, if the international capital lending is sufficiently large, then the increase in the interest rate brought about by the reduction in capital increases the return from such lending by the donor country. Therefore, in this case, the decrease in capital might increase the indirect effect on the donor's welfare, as the exporter (importer) of capital. Note that as the (absolute) value of s_a^D becomes large, this effect weakens. That is, the positive indirect effect is mitigated in this case.

The indirect effect on the recipient's welfare, regardless of the amount of savings, is necessarily negative because of the reduction in world capital in a dynamic efficient region. Therefore, through the world capital market, the indirect effect (capital accumulation effect) increases, or the negative indirect effect is necessarily mitigated by the donor's aspirations. Thus, it is clear that there is no aspiration effect on the recipient.

Finally, by Lemmas 6.3 and 6.4, we demonstrate the following proposition for the transfer paradox when the donor inherits aspirations.

Proposition 6.1. *Suppose that the donor has aspirations, as in Assumption 1.*

(a) *If the degree of the donor's aspirations is sufficiently large, donor enrichment becomes possible as a result of a transfer. On the other hand, recipient immiserization as a result of a transfer is independent of the degree of the donor's aspirations.*

(b) *As the effect of the donor's aspirations on marginal utility increases, donor enrichment is less (more) likely to occur as a result of a transfer if $(1 + r)k < (>)s^D$, and recipient immiserization is less likely to occur as a result of a transfer.*

Proof. The proof follows immediately from Lemmas 6.3 and 6.4. \square

Proposition 6.1 claims that the relative magnitude between the direct income effect and the indirect effects, as discussed in Lemmas 6.3 and 6.4, determines the likelihood of the transfer paradox. At first glance, it seems that the aspiration preference of the donor may be a kind of negative externality that leads to a further reduction in the donor's welfare as a result of a transfer. However, the increase in the degree of the donor's aspirations tends to cause donor enrichment. This is because the negative effect caused by the aspirations themselves are weakened by the reduction in world capital as a result of a transfer. In contrast, because aspirations are formed in the utility of the donor, the degree of the donor's aspirations is unrelated to recipient immiserization.

On the other hand, when the donor's aspirations cause a large decrease in the donor's savings (when the absolute value of the effect of the donor's aspiration on marginal utility is large), the transfer lowers the donor's welfare if the return from international lending is sufficiently large to dominate the welfare loss from the capital accumulation. In contrast, this necessarily improves the recipient's welfare. In summary, whether the transfer paradox occurs is determined by which of the above three effects (namely, the capital accumulation effect, the aspiration effect, and the income effect) dominates. Note that the effect of the donor's aspirations on marginal utility works in different directions, depending on the amount of the donor's savings (that is, the sign of $[(1+r)k - s^D]$). Proposition 6.1 implies that if there is no effect of the donor's aspirations on marginal utility, the donor's aspirations foster donor enrichment, but have no influence on the likelihood of recipient immiserization. The larger the donor's aspirations are, the more likely the Pareto-improving result is to occur. However, if there is an effect on marginal utility, this side-effect causes the different results for both countries' welfare.

From the above proposition, it is suggested that when people in a donor country are more conscious of the living standard of their parents' generation, the policy authorities engaged in international transfers are more likely to prefer giving a transfer to a recipient. This depends on the two differing effects that donor's aspirations have on their utility. If the donor's aspirations increase their utility sufficiently, the authorities prefer giving a transfer, because it raises the possibility of donor enrichment. However, the donor's aspirations also have an effect on marginal utility. This indirect effect affects whether the transfer

authorities prefer giving a transfer, and the sign of the effect depends on the relative size of the donor's savings.

Thus far, we have presented the above proposition using general functional forms in order to investigate the common impact of aspirations on welfare. However, we also need to check whether the transfer paradox actually arises under a specific functional form. Now, we show a simulation in which, under specific utility and production functions, the transfer paradox occurs. In particular, donor enrichment occurs as the degree of the donor's aspirations increases, as shown in Proposition 6.1(a). Consider a log-linear utility function and a Cobb–Douglas production function, such as $V_t^i = \ln(c_t^i - \delta^i a_t^i) + \beta^i \ln d_{t+1}^i$ and $y_t^i = A k_t^\gamma$, respectively, where β^i , δ^i , A , and γ denote the subjective discount rate, the degree of aspirations, total factor productivity, and the capital share, respectively. Since only the donor has aspirations in this section, $\delta^D > 0$ and $\delta^R = 0$. We specify $\beta^D = 0.5$, $\beta^R = 0.0001$, $\gamma = 0.9$, $A = 20$, and $n = 0.097$. In this case, the dynamic efficiency conditions are satisfied.

In this specification, we obtain the result that a higher δ^D means donor enrichment is likely to occur. When $\delta^D = 0.001$, the marginal increase in the transfer from 0 to $\Delta T = 10^{-24}$ causes donor enrichment because the donor's welfare increases by $\Delta V^D \approx 8.88 \times 10^{-15} > 0$. When $\delta^D = 0.1$, the donor's welfare increases by $\Delta V^D \approx 1.07 \times 10^{-14} > 0$.⁹ Comparing the difference in donor's welfare when $\delta^D = 0.001$ and 0.1, we obtain that $\Delta V^D|_{\delta^D=0.1} - \Delta V^D|_{\delta^D=0.001} \approx 1.776 \times 10^{-15} > 0$. The positive sign of the difference in welfare shows that as the degree of the donor's aspirations becomes higher, the donor's welfare is likely to increase, which supports the claim of Proposition 6.1(a), in this simulation.

We can present a similar simulation for recipient immiserization. Here, we specify $\beta^D = 0.4$, $\beta^R = 0.01$, $\gamma = 0.7$, $A = 20$, and $n = 0.097$ under the above log-linear utility function and Cobb–Douglas production function. When $\delta^D = 0.1$ ($\delta^D = 0.13$), the marginal increase in transfer worsens the recipient's welfare because the increase from 0 to $\Delta T = 10^{-8}$ decreases the recipient's

⁹ When $\delta^D = 0.001$ ($\delta^D = 0.1$), the increments of capital and aspirations by the marginal increase in the transfer are $\Delta k \approx 4.74 \times 10^{-20}$ (2.67×10^{-20}) > 0 and $a^D \approx 1.86 \times 10^{-19}$ (1.15×10^{-19}) > 0 , respectively, and the eigenvalues of matrix A_2 in (A.6.1) are 0.899 and 0.001 (0.899 and 0.001), respectively. Thus, in both cases, the economy monotonically converges to the steady state.

welfare by $\Delta V^R \approx -3.80 \times 10^{-9}$ (-4.00×10^{-9}) < 0 .¹⁰ Thus, in both cases, recipient immiserization occurs. Moreover, since the difference in the recipient's welfare in both cases is $\Delta V^R|_{\delta^D=0.13} - \Delta V^R|_{\delta^D=0.1} \approx -2.056 \times 10^{-10} < 0$, the recipient's welfare is likely to deteriorate as the degree of the donor's aspirations rises.

6.4 The recipient's aspirations and the transfer paradox

The equilibrium conditions consisting of the world capital market equilibrium and the identity of the recipient's aspirations in the steady state is given by

$$2(1+n)k = s^D(w(r) - T, r) + s^R(w(r) + T, r; a^R), \quad (6.34)$$

$$a^R = w(r) + T - s^R(w(r) + T, r; a^R). \quad (6.35)$$

Total differentiation of (6.34) and (6.35) gives

$$\begin{bmatrix} \Gamma & -s_a^R \\ [k(1-s_w^R) + s_r^R]f'' & 1 + s_a^R \end{bmatrix} \begin{bmatrix} dk \\ da^R \end{bmatrix} = \begin{bmatrix} -(s_w^D - s_w^R) \\ 1 - s_w^R \end{bmatrix} dT. \quad (6.36)$$

Note that the determinant of the 2×2 matrix of the left-hand side of (6.36) is Γ_R , the sign of which is positive ($\Gamma_R > 0$) because the dynamic stability condition is satisfied. By solving (6.36) with respect to dk/dT and da^R/dT , we obtain the following equations:

$$\begin{bmatrix} \frac{dk}{dT} \\ \frac{da^R}{dT} \end{bmatrix} = \frac{1}{\Gamma_R} \begin{bmatrix} -(s_w^D - s_w^R)(1 + s_a^R) + s_a^R(1 - s_w^R) \\ (s_w^D - s_w^R)[k(1 - s_w^R) + s_r^R]f'' + \Gamma(1 - s_w^R) \end{bmatrix}. \quad (6.37)$$

The effects in (6.37) can be divided into two separate effects, similarly to Section 6.3. Therefore, the following lemma is obtained.

¹⁰ When $\delta^D = 0.1$ ($\delta^D = 0.13$), the marginal increase in the transfer decreases capital and aspirations by $\Delta k \approx -3.87 \times 10^{-9}$ (-3.77×10^{-9}) < 0 and $a^D \approx -2.33 \times 10^{-8}$ (-2.34×10^{-8}) < 0 , respectively. Since the eigenvalues of matrix A_2 in (A.6.1) are 0.673 and 0.109 (0.661 and 0.148) when $\delta^D = 0.1$ ($\delta^D = 0.13$), the stability condition is guaranteed.

Lemma 6.5. *Suppose that the recipient has aspirations, as in Assumption 1. The capital stock necessarily decreases as a result of a transfer.*

Proof. The results follow immediately from (6.37). \square

Note that the condition in Lemma 6.5 is different from that in Lemma 6.1. The sign of the aspiration effect on capital stock is positive when a donor has aspirations, while it is negative when a recipient does so. Because the recipient's aspirations accelerate the decrease in world capital, the transfer decreases the world's total savings. Therefore, the effect on capital accumulation is necessarily negative.

With regard to the effect of the transfer on aspirations, we present the following lemma.

Lemma 6.6. *Suppose that the recipient has aspirations, as in Assumption 1. The transfer increases (decreases) the recipient's consumption if the effect through the capital market is dominated by (dominates) the effect on consumption.*

Next, we examine the effect of the transfer on welfare. Similarly to the procedure in the previous section, by totally differentiating the indirect utility functions of the donor and the recipient, given by $V^D = V^D(w(r) + T^D, r)$ and $V^R = V^R(w(r) + T^R, r; a^R)$, we obtain

$$\begin{bmatrix} dV^D \\ dV^R \end{bmatrix} = \begin{bmatrix} \underbrace{v_d^D [s^D - (1+r)k] f'' dk}_{\text{(the capital accumulation effect)}} & \underbrace{-u_c^D dT}_{\text{(the income effect)}} \\ \underbrace{v_d^R [s^R - (1+r)k] f'' dk}_{\text{(the capital accumulation effect)}} & \underbrace{u_a^R da^R}_{\text{(the aspiration effect)}} + \underbrace{u_c^R dT}_{\text{(the income effect)}} \end{bmatrix} \quad (6.38)$$

$$= \begin{bmatrix} v_d^D [s^D - (1+r)k] f'' \frac{dk}{dT} - u_c^D \\ v_d^R [s^R - (1+r)k] f'' \frac{dk}{dT} + u_a^R \frac{da^R}{dT} + u_c^R \end{bmatrix} dT. \quad (6.39)$$

The difference between (6.39) and (6.32) lies in where the aspiration effect appears. We obtain the following equations for the effect of the transfer on the

welfare of the donor and the recipient:

$$\begin{aligned}
 \frac{1}{u_c^D} \frac{dV^D}{dT} &= \underbrace{\frac{(k - \frac{s^D}{1+r})f''[(s_w^D - s_w^R) - s_a^R(1 - s_w^D)]}{\Gamma_R}}_{\substack{\text{(the indirect effect)} \\ \text{= (the capital accumulation effect)}}} - \underbrace{1}_{\substack{\text{(the direct effect)} \\ \text{= (the income effect)}}}, \\
 \frac{1}{u_c^R} \frac{dV^R}{dT} &= \underbrace{\frac{(k - \frac{s^R}{1+r})f''[(s_w^D - s_w^R) - s_a^R(1 - s_w^D)] + u_a^R \left\{ (s_w^D - s_w^R) [k(1 - s_w^R) + s_r^R] f'' + \Gamma(1 - s_w^R) \right\}}{\Gamma_R}}_{\substack{\text{(the indirect effect)} \\ \text{= (the capital accumulation effect) + (the aspiration effect)}}} \\
 &\quad + \underbrace{1}_{\substack{\text{(the direct effect)} \\ \text{= (the income effect)}}}. \tag{6.40}
 \end{aligned}$$

Now, we examine the indirect effect of the transfer on the donor's and on the recipient's welfare. By (6.40), we obtain the following lemma.

Lemma 6.7. *Suppose that the recipient has aspirations, as in Assumption 1.*

- (a) *The degree of the recipient's aspirations has no indirect effect on the donor's welfare as a result of a transfer.*
- (b) *As the effect of the recipient's aspirations on marginal utility increases, the indirect effect of a transfer on the donor's welfare increases (decreases) if the indirect effect on the donor's welfare is positive (negative) with no aspirations (that is, if $(1 + r)k < (>)s^D$).*

Proof. The proof is similar to that of Lemma 6.4. □

Lemma 6.8. *Suppose that the recipient has aspirations, as in Assumption 1.*

- (a) *Whether the increase in the degree of the recipient's aspirations has an indirect effect on the increase in the recipient's welfare depends on whether the transfer increases the recipient's consumption.*
- (b) *As the effect of the recipient's aspirations on marginal utility increases, the recipient's welfare necessarily decreases.*

Proof. The proof is similar to that of Lemma 6.1. □

Finally, from Lemmas 6.7 and 6.8, we demonstrate the following proposition with regard to the transfer paradox when the recipient has aspirations.

Proposition 6.2. *Suppose that the recipient has aspirations, as in Assumption 1.*

- (a) *Donor enrichment is independent of the degree of the recipient's aspirations, and whether recipient immiserization occurs depends on whether the transfer increases the recipient's consumption.*
- (b) *As the effect of the recipient's aspirations on marginal utility increases, donor enrichment is more (less) likely to occur as a result of a transfer if $(1 + r)k < (>)s^D$, and recipient immiserization is more likely to occur as a result of a transfer.*

Proof. The proof follows immediately from Propositions 6.7 and 6.8. □

Proposition 6.2 claims that when the recipient has aspirations, whether a high degree of these aspirations causes recipient immiserization depends on the situation. This result contrast to that of Proposition 6.1, which states that when the donor has aspirations, if the degree of the donor's aspirations is sufficiently large, it is possible for donor enrichment to occur. Whether the aspiration level equal to the young generation's consumption increases as a result of the transfer cannot be determined when the recipient has aspirations. As another claim of Proposition 6.2, when the recipient's aspirations have a significant impact on the decrease in the recipient's savings, the transfer is preferable for the donor, because the capital accumulation effect of the donor improves. However, the transfer is not preferable for the recipient, because the recipient's indirect effect deteriorates. Therefore, Proposition 6.2 suggests that if the capital accumulation effect is sufficiently larger than the other effects, this can cause the strong transfer paradox, where both donor enrichment and recipient immiserization occur as a result of the transfer.

From the viewpoint of transfer policy decisions, Proposition 6.2 makes a different claim to that of Proposition 6.1. When people in a recipient country are conscious about the living standard of their parent generation, there is only an indirect effect on the donor's preference for a transfer, unlike the case of the donor's aspirations. Whether the transfer authorities prefer giving a transfer depends only on the indirect effect that recipient's aspirations have on their marginal utility, and the sign of this effect depends on the relative size of the donor's savings. Thus, in this case, if the donor's savings are sufficiently large, the transfer authorities prefer giving a transfer because the likelihood of donor

enrichment increases.

Now, we present a simulation in which the transfer paradox occurs. Consider a constant elasticity of substitution (CES) utility function and a Cobb–Douglas production function, such as $V_t^i = \frac{1}{1-(1/\sigma)} [(c_t^i - \delta^i a_t^i)^{1-(1/\sigma)} + \beta^i (d_{t+1}^i)^{1-(1/\sigma)}]$ and $y_t^i = Ak_t^\gamma$, respectively, where σ denotes the elasticity of substitution. Since only the recipient has aspirations in this section, $\delta^D = 0$ and $\delta^R > 0$. We specify $\beta^D = 0.2$, $\beta^R = 0.01$, $\gamma = 0.7$, $\sigma = 2$, $A = 100$, and $n = 0.097$. In addition, the dynamic efficiency conditions are satisfied. In these parameters, we obtain the result that donor enrichment and recipient immiserization occur simultaneously. When $\delta^D = 0$ and $\delta^R = 0.001$, the marginal increase in the transfer from 0 to $\Delta T = 10^{-22}$ increases the donor's welfare and decreases the recipient's welfare by $\Delta V^D \approx 4.26 \times 10^{-14} > 0$ and $\Delta V^R \approx -1.26 \times 10^{-12} < 0$, respectively.¹¹ Thus, both transfer paradoxes arise in this example.

6.5 Concluding remarks

In this chapter, we examined the transfer problem between two countries using a one-sector OLG model, when either the donor or the recipient has aspirations. After confirming whether the dynamic stability conditions are satisfied with regard to two state variables, namely the capital and the aspiration level, equal to the preceding young consumption, the results were as follows. First, in the case where a donor has a higher marginal propensity to save than that of a recipient, we demonstrated that when the donor has aspirations, donor enrichment is more likely to occur. However, there is no effect on the recipient's welfare as the degree of the donor's aspirations increases [Proposition 6.1(a)]. When the recipient has aspirations, whether recipient immiserization occurs depends on whether the transfer increases the recipient's consumption [Proposition 6.2(a)]. Second, when the effect of the donor's or recipient's aspirations on their marginal utility increases, whether donor enrichment is more likely to occur depends on the configuration of the savings propensities between the two countries, while recipient immiserization is more likely to occur [Propositions 6.1(b) and 6.2(b)].

¹¹ Since the eigenvalues of matrix A_2 in (A.6.1) are 0.595 and 5.42×10^{-6} , the stability condition is guaranteed.

Our results suggest the following: First, a high degree of donor aspirations can provide a motivation for a voluntary transfer because it is more likely to cause donor enrichment. Aspiration preference is usually regarded as a negative externality on utility that hinders capital accumulation through excessive consumption and a lack of saving. However, the transfer from an aspiration-inherited donor to a recipient partially improves capital accumulation effectively. Therefore, the strong donor aspirations are a positive externality for capital accumulation as a result of the transfer, which can increase the possibility of improving the donor's welfare. Second, we have shown that there are two types of effects that aspirations have on the welfare of both countries: the effect caused by the aspirations themselves, and the effect through the capital market. The former is the effect that inherited aspirations have on the utility level. The latter is the effect that they have on the level of marginal utility. These two effects do not necessarily work in the same direction. Our results emphasize the importance of clearly distinguishing between the two different ways in which aspirations affect welfare.

Finally, we end this chapter by suggesting possible future extensions to this research. We limit our analysis to a model in which either a donor or a recipient has aspirations. First, a direct extension would include a generalized model in which both a donor and a recipient have aspirations, although an increase in the complexity of the analysis is to be expected. However, even in the latter case, the fundamental results presented here are expected to hold. That said, such an extension would yield more comprehensive conclusions on the different degrees of donor and recipient aspirations. As a second extension, the analysis can deal with a transition path, although we focus only on the steady-state equilibrium here. Introducing aspirations into the OLG model has a significant impact on the transition process leading to the steady state. A third interesting challenge would be the endogenization of aspirations and a bequest motive. If aspirations are formed endogenously, the Pareto-improving transfer might be able to be achieved through a spontaneous adjustment of the past consumption level or a bequest by the countries. Fourthly, the optimal level of the transfer is of interest. Although we focused only on the marginal effect of the transfer, it is also important to derive the level of the optimal transfer endogenously from the viewpoint of policy issues. These extensions will be addressed in future research.

Appendix

A.6.1 The sufficient condition for dynamic stability

We present again the recurrence formula with respect to k_t and a_t^D , (6.24), as follows:¹²

$$\begin{bmatrix} dk_{t+1} \\ da_{t+1}^D \end{bmatrix} = A \begin{bmatrix} dk_t \\ da_t^D \end{bmatrix} = \frac{1}{\Delta} \times A_2 \begin{bmatrix} dk_t \\ da_t^D \end{bmatrix},$$

where $A_2 \equiv \begin{bmatrix} -(s_w^D + s_w^R)k_t f'' & s_a^D \\ [(s_w^D + s_w^R)s_r^D f'' - \Delta(1 - s_w^D)]k_t f'' & -(s_r^D f'' + \Delta)s_a^D \end{bmatrix}.$

(A.6.1)

Define the eigenvalues of matrix A_2 as $\lambda \equiv \{\lambda_1, \lambda_2\}$. Therefore, note that the eigenvalues of matrix $(1/\Delta) \times A_2$ are given by $\lambda/\Delta \equiv \{\lambda_1/\Delta, \lambda_2/\Delta\}$. The eigenvalues of A_2 are obtained by solving the characteristic polynomial, $\det(A_2 - \lambda I) = 0$. For a 2×2 square matrix, the following equation is satisfied:

$$\det(A_2 - \lambda I) = 0 \Leftrightarrow \lambda^2 - \text{tr}(A_2)\lambda + \det(A_2) = 0, \quad (\text{A.6.2})$$

where $\text{tr}(A_2) \equiv -(s_w^D + s_w^R)k_t f'' - (s_r^D f'' + \Delta)s_a^D > 0$, which is the trace of A_2 , and $\det(A_2) \equiv \Delta(1 + s_w^R)s_a^D k_t f'' > 0$, which is the determinant of A_2 .

By solving the quadratic equation with respect to λ , (A.6.2), we obtain the two eigenvalues of A_2 , as follows:

$$\lambda_1 = \frac{\text{tr}(A_2) - \sqrt{[\text{tr}(A_2)]^2 - 4\det(A_2)}}{2}, \quad \lambda_2 = \frac{\text{tr}(A_2) + \sqrt{[\text{tr}(A_2)]^2 - 4\det(A_2)}}{2}.$$

(A.6.3)

The discriminant of the quadratic equation is $D \equiv [\text{tr}(A_2)]^2 - 4\det(A_2)$. If $D \geq 0$, that is, $[\text{tr}(A_2)]^2 \geq 4\det(A_2)$, the eigenvalues are real roots. On the

¹² For notational simplicity, we omit the subscript t from Δ and Γ throughout the Appendix.

other hand, if $D < 0$, that is, $[\text{tr}(A_2)]^2 < 4 \det(A_2)$, they are imaginary roots. The sufficient condition for a steady state to be monotonically convergent is as follows: Both eigenvalues of the 2×2 matrix A_2 , defined by (A.6.1), are between 0 and Δ if they are real roots, and if both eigenvalues are imaginary roots, the common real part of the complex numbers is between 0 and Δ .

First, we consider the case in which $D \geq 0$. The sufficient condition for dynamic stability is $0 < \lambda_1 < \lambda_2 < \Delta$. Here, $\lambda_1 > 0$ is evidently satisfied because $\det(A_2) > 0$. Then, $\lambda_2 < \Delta$ is arranged as follows:

$$\begin{aligned}
 \lambda_2 < \Delta &\Leftrightarrow \Delta^2 - \Delta \times \text{tr}(A_2) + \det(A_2) > 0 \\
 &\Leftrightarrow \Delta - \text{tr}(A_2) + (1 + s_w^R) s_a^D k_t f'' > 0 \\
 &\Leftrightarrow \Delta + (s_w^D + s_w^R) k_t f'' + s_a^D \{ \Delta + [s_r^D + (1 + s_w^R) k_t] f'' \} > 0 \\
 &\Leftrightarrow \Gamma + s_a^D \{ \Delta + [s_r^D + (1 + s_w^R) k_t] f'' \} > 0. \tag{A.6.4}
 \end{aligned}$$

Therefore, if (A.6.4) is satisfied, the dynamic stability condition holds. We show that (A.6.4) necessarily holds when $\Gamma > 0$, as follows: Under the utility assumption, $s_a^D \in (-1, 0)$ is satisfied, as shown in (6.15). When $\Delta + [s_r^D + (1 + s_w^R) k_t] f''$ is negative, (A.6.4) is satisfied immediately. On the other hand, suppose that $\Delta + [s_r^D + (1 + s_w^R) k_t] f''$ is positive. When $s_a^D = -1$, (A.6.4) is minimized. When substituting $s_a^D = -1$ into the final equation of (A.6.4), we obtain the following:

$$\Gamma - \{ \Delta + [s_r^D + (1 + s_w^R) k_t] f'' \} = -[s_r^D + (1 - s_w^D) k_t] f'' > 0, \tag{A.6.5}$$

because $s_r^D > 0$ and $s_w^D < 1$. Therefore, $\lambda_2 < \Delta$ is always satisfied.

Second, we consider the case in which $D < 0$. When both eigenvalues are imaginary roots, the sufficient condition for the stability is that the real part of the conjugate complex is between 0 and Δ . In this case, if the real part of λ_1 and λ_2 , that is, $\text{tr}(A_2)/2$, is between 0 and Δ , stability is satisfied. It is evident that $\text{tr}(A_2)/2 > 0$. On the other hand, $\text{tr}(A_2)/2 < \Delta$ if and only if

$$\begin{aligned}
 &2\Delta + (s_w^D + s_w^R) k_t f'' + (s_r^D f'' + \Delta) s_a^D > 0 \\
 &\Leftrightarrow \Gamma + s_r^D f'' s_a^D + (1 + s_a^D) \Delta > 0. \tag{A.6.6}
 \end{aligned}$$

The sign of (A.6.6) follows from $\Gamma > 0$ and $s_a^D \in (-1, 0)$. Thus, also when $D < 0$, the sufficient condition is satisfied.

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Part III

Intergenerational Allocation and Pension System

Chapter 7

Intergenerational Allocation and The Transfer Problem

We investigate the transfer problem between two countries in the steady state in a one-sector overlapping generations (OLG) model and explain how transfers should be shared between the young and old generations of the donor country and allocated across the generations of the recipient country. Except at the golden rule of capital accumulation, the ratios of the burden and distribution of transfers between the young and old generations affect welfare. We obtain the following results. First, the sharing of the transfer burden in the donor country depends on the relative size of two effects, namely, a negative direct effect and a positive indirect effect. If the former exceeds the latter, it is preferable for the donor country to allocate all of the transfer burden to the old generation and vice versa. Second, from the viewpoint of welfare maximization, it is preferable for the recipient country to distribute all of the transfers to the young generation. In contrast to the existing literature, these results suggest that the setting whereby the young generation of the donor country defrays all transfer costs may not be justifiable from the viewpoint of donor welfare maximization.

Keywords: transfer paradox, intergenerational allocation, overlapping generations model, recipient immiserization

JEL classifications: D91, E21, F35, F43

7.1 Introduction

How should different generations in a donor country share the burden of any international transfers to a recipient country? How should the recipient government distribute transfers to different generations? Although many studies focus on the dynamic aspect of this transfer problem in an overlapping generations (OLG) model, few have dealt with the intergenerational distribution of international transfers. In this chapter, we attempt to respond to the above questions by investigating how transfers between two countries affect the welfare of the young and old generations in both countries using a standard one-sector OLG model.

Generally speaking, the transfer must be financed by a tax on the donor country's people. In this sense, the allocation ratio of the transfer between the young and the old generations is a matter of concern for the donor country's people. This allocation problem relates to the individual's tax burden problem over the life cycle. For instance, if the public social security system, such as a pension plan for the retired, is fully equipped, but its funds are paid for by the working generation, the tax burden ratio of the working generation will be larger than that of the retired generation. Moreover, it is possible that even if the total amount of the tax burden remains unchanged in nominal terms, its present discounted value changes through the increase or decrease in the interest rate, which results from the change in the savings of the young. Thus, if the intergenerational allocation also affects welfare in the donor country, it will be preferable to scrutinize who should pay for the international charity from the viewpoint of the donor's welfare. At the same time, the donor also faces another problem on who should be given in a recipient country.

According to OECD (2014), development aid in 2013 attained a total of USD 134.8 billion, the highest level ever recorded, and is increasing by some 6.1% in real terms. OECD (2009) also mentions that donor countries belonging to the Development Assistance Committee (DAC) have distributed Official De-

velopment Assistance to a range of sectors in recipient countries, especially social infrastructure and services (30.5%), debt (13.2%), economic infrastructure (12.4%), and government and civil society (11.7%). However, although a considerable amount of data shows that the aid is used for a wide variety of purposes in recipient countries, there is little evidence concerning how it is distributed across generations in these recipient countries. This is also an important issue because if the distribution of aid across generations affects the recipient country's welfare, the donor can enhance the effectiveness of aid in improving recipient welfare by appropriately targeting the generation where welfare can be most improved. For example, if aid to the young generation contributes more to welfare improvement as a whole, the donor should prioritize aid tied to education, youth unemployment policy, and health care policy to reduce the infant mortality rate. In contrast, if aid to the elderly effectively contributes more to improving overall welfare, aid should be used to support the social security system and medical care for the elderly. If the distribution of the transfer between the young and the old itself, apart from the amount of transfer, does not have an impact on recipient welfare at all, intergenerationally neutral (untied) aid would be best. Thus, the donor country shares a concern about which generation should receive the aid transfer with the recipient country in order to fulfill its responsibility to its own people.

Apart from policy issues, the transfer between two countries has led to considerable controversy on the transfer paradox, which denotes the paradoxical situation in which a donor country is enriched and/or a recipient country is immiserized, seemingly despite the income transfer from the donor to the recipient. It has also been well known, at least since the seminal debate on the problem of German war reparations between Keynes (1929) and Ohlin (1929), that international transfers between countries affect the welfare of both and, in some cases, the transfer paradox arises.¹

In a dynamic framework, especially an OLG framework, the seminal works in this area clarify that the transfer paradox can arise without any distortions in a two-country model, even in a dynamically efficient economy. For example, Galor and Polemarchakis (1987) argued that a permanent lump-sum transfer can bring about the transfer paradox in a steady-state equilibrium. Haaparanta

¹ Polemarchakis (1983) provides an excellent survey of the transfer paradox.

(1989) showed that a transfer paradox can arise when the temporary transfer is financed by public debt in the donor country and/or is used for debt relief in the recipient country. Yanagihara (1998) clarified that how a transfer paradox occurs depends on the difference between the levels of public goods with a positive externality on production between the donor and the recipient. Cremers and Sen (2008) investigated the effect of the transfers on welfare, not only in the steady state, but also on the transitional path, and proved that the results obtained in Galor and Polemarchakis (1987) could also be applied to the transition. Overall, in a dynamic framework, these studies have shown that there is the possibility of the transfer paradox arising, even in the dynamically efficient region as well as at the golden rule.

In nearly all of the existing literature dealing with the transfer problem between two countries in an OLG model, it is assumed, mostly for analytical simplicity, that the transfer is made from the donor country's young to the recipient country's young. This assumption at first seems justified because when to pay is not at all important for an individual if the individual's lifetime income does not change. However, in practice, even if the recipient's young generation receives the same amount of transfer, the lifetime income of the donor changes depending on which generation, either the young or the old, defrays the transfer. Likewise, even if the donor's young generation pays the same amount of transfer, the impact on the welfare of the recipient country differs depending on who receives the transfer because the lifetime income of the recipient may change. Thus, the intergenerational allocation problem is an important policy issue. Hitherto, there has been few discussion on how to collect the amount of income to be transferred among those in the donor country. As one of the few exceptions, Dalgaard, Hansen, and Tarp (2004) pointed out that the allocation of received transfer between generations influences the steady state level of capital stock and production in the recipient economy in the OLG model. As the main focus of their study is empirical research on the effectiveness of aid, they do not argue the welfare consequence of generational allocation of transfer. Unlike their study, we focus on the effect that the intergenerational distribution of the transfer has on welfare, and examine which of the young and old generations in the donor country should be burdened with the transfer, and to which generations in the recipient country the transfer should be distributed.

Accordingly, we investigate how the transfer burden should be shared be-

tween the young and old generations using an OLG model. Similarly, we investigate to which generation, the young or the old, in the recipient country it is more suitable to distribute the transfer, from the viewpoint of welfare maximization. In the situation in which capital is accumulated at the golden rule, which generation pays or receives the transfer is unimportant because the ratios of the burden and the distribution of the transfer between the young and the old generations do not affect the welfare of either country. However, this indifference result does not hold when we extend the analysis to the more general situation in which the economy is dynamically efficient except at the golden rule. It is in this situation, where the ratios of transfer burden and distribution between the young and the old affect the welfare of both countries, that we investigate which of the generations in the donor and the recipient country should pay and receive the transfer, respectively.

Focusing on the steady state, we obtain the following results in the dynamically efficient economy. First, when the economy is dynamically efficient but not at the golden rule, from the viewpoint of welfare maximization, it is preferable for the recipient country to distribute all of the transfer to the young generation. Second, in contrast to the recipient's distribution, who should bear the transfer burden in the donor country to increase donor country welfare depends on the relative size of two effects, namely, a negative direct effect from the increase in the transfer burden and a positive indirect effect from the improvement in the intertemporal terms of trade, which results from the change in the interest rate. If the negative direct effect exceeds the positive indirect effect, it is preferable for the donor country to allocate all of the transfer burden to the old generation and vice versa. In contrast to the typical assumption in the existing literature, the setting in which the young generation of the donor country defrays all of the transfer cost is justifiable from the viewpoint of donor welfare maximization only in some situations.

The remainder of this chapter is organized as follows. In Section 7.2, we describe the transfer problem in a one-sector OLG model in which both the young and old generations share the transfer burden in the donor country and receive the transfer in a recipient country. In Section 7.3, we present our main results concerning the ratios of burden and distribution under which the transfer increases welfare. Section 7.4 concludes.

7.2 The model

We consider a one-sector OLG model with two countries, namely, a donor country and a recipient country, with an international income transfer. The donor and the recipient are indexed by countries $i = D$ and R , respectively, such that country D provides a permanent transfer to country R . These two countries are identical except for the marginal propensity to save. In the model, time is discrete, starts at $t = 1$, and never ends.

L_t^i denotes the population of country $i = D, R$ in period t , which is equal to the labor force of the country. We assume that labor is inelastically supplied by individuals. For brevity, population in both countries is equal, that is, $L_t^D = L_t^R \equiv L_t$. Thus, the populations of both countries grow equally with the gross population growth rate of $1 + n \equiv L_{t+1}/L_t (\geq 1)$, which is exogenously given and constant over time.

Capital is fully mobile between the two countries, but labor is immobile. Because of the assumption of full capital mobility, the factor prices and also the levels of the capital stock are equal in both countries through factor price equalization.

7.2.1 Individuals

In each period, both countries are populated by two generations, the young, who inelastically supply one unit of their labor and earn wages either to consume or to save, and the old, who retire and consume savings accumulated during the young period plus accrued interest.² All individuals live for two periods. We refer to the individuals who are young in period t as generation t . Generation t in country $i = D, R$ chooses levels of consumption in their young period t and in their old period $t + 1$, (c_t^i, d_{t+1}^i) , in order to maximize their utility, subject to the budget constraints in their respective young and old periods.

² Similar to the existing literature, we assume that only the young earn wages but the old do not. However, even if we extend the model by allowing the old generation to earn wages, the qualitative results in our model do not change. This extension just adds an additional negative term in the indirect effect on welfare through a decrease in the discounted value of the wage in the old period. We thank an anonymous referee for pointing out that the result is unchanged even in more general settings.

The intertemporal utility of generation t in country i is given by

$$u^i(c_t^i, d_{t+1}^i). \quad (7.1)$$

We assume that u^i is twice differentiable and strictly quasi-concave. We assume $u_c^i, u_d^i, u_{cd}^i > 0$, $u_{cc}^i, u_{dd}^i < 0$, $\lim_{c \rightarrow 0} u_c^i(c, d) = +\infty$, $\lim_{d \rightarrow 0} u_d^i(c, d) = +\infty$, and $u_{cc}^i u_{dd}^i > (u_{cd}^i)^2$. These assumptions guarantee the interior solution and the second-order condition for utility maximization.

In existing studies dealing with the transfer paradox in an OLG model, an amount of transfer per young individual, denoted by T , is transferred from the donor's young to the recipient's young between the two countries of equal population size. When investigating the steady state, it is usually assumed that the transfer T is sufficiently small so as to focus on the infinitesimal change in transfer. In contrast, we examine how the permanent transfer should be shared and distributed between the young and old generations in a donor and a recipient country. $\theta^i \in [0, 1]$ and $1 - \theta^i$ denote the sharing and distribution ratio of the per capita transfer T between the young and the old generations in country i , respectively. T and $(1+n)T$ are then the per capita transfer shared or distributed by the young and the old generations, respectively. It should be noted that the total amount of transfer in each period $L_t T$ does not depend on the allocation ratio θ^i because $L_t \theta^i T + L_{t-1} (1 - \theta^i) (1+n)T = L_t T$ always holds. As by assumption the old generation has a smaller population than the young, the per capita sharing and distribution of the transfer for the old are larger than those for the young by the increase in the population growth rate.

The budget constraints in the respective young and old periods for generation t in country D and R are as follows:

$$c_t^D + s_t^D = w_t - \theta^D T \quad \text{and} \quad d_{t+1}^D = (1 + r_{t+1}) s_t^D - (1 - \theta^D) (1+n)T, \quad (7.2)$$

$$c_t^R + s_t^R = w_t + \theta^R T \quad \text{and} \quad d_{t+1}^R = (1 + r_{t+1}) s_t^R + (1 - \theta^R) (1+n)T, \quad (7.3)$$

where r , w , and s denote the net interest rate, wage rate, and savings, respectively. From (7.2) and (7.3), we obtain the intertemporal budget constraints for

generation t in both countries as follows:

$$c_t^D + \frac{1}{1+r_{t+1}}d_{t+1}^D = I_t^D \equiv w_t - \left[\theta^D + (1-\theta^D) \frac{1+n}{1+r_{t+1}} \right] T, \quad (7.4)$$

$$c_t^R + \frac{1}{1+r_{t+1}}d_{t+1}^R = I_t^R \equiv w_t + \left[\theta^R + (1-\theta^R) \frac{1+n}{1+r_{t+1}} \right] T. \quad (7.5)$$

Define $X^i \equiv \theta^i + (1-\theta^i)(1+n)/(1+r_{t+1}) = [1+n+\theta^i(r_{t+1}-n)]/(1+r_{t+1})$, which stands for the marginal effect of the transfer on lifetime income. The lifetime income of generation t in countries D and R is denoted by $I_t^D \equiv w_t - X^D T$ and $I_t^R \equiv w_t + X^R T$, respectively. We assume that $I_t^i > 0$ for all t .

The utility maximization problem for generation t in country i is formulated as follows:

$$\max_{\{c_t^i, d_{t+1}^i\}} u^i(c_t^i, d_{t+1}^i) \quad \text{s.t.} \quad c_t^i + \frac{1}{1+r_{t+1}}d_{t+1}^i = I_t^i. \quad (7.6)$$

The first-order condition is given by

$$u_c^i = (1+r_{t+1})u_d^i. \quad (7.7)$$

The second-order condition is satisfied by assumption.

The optimal consumption bundle depends on (I_t^i, r_{t+1}) so that the demand functions can be obtained as $c_t^i = c^i(I_t^i, r_{t+1})$ and $d_{t+1}^i = d^i(I_t^i, r_{t+1})$. Regarding the demand functions, we obtain the following properties:³

$$\begin{aligned} c_I^i &= -\frac{u_c^i(u_d^i u_{cd}^i - u_c^i u_{dd}^i)}{X}, \\ d_I^i &= \frac{u_c^i(u_d^i u_{cc}^i - u_c^i u_{cd}^i)}{X}, \end{aligned} \quad (7.8)$$

³ The subscripts denote the partial derivative with respect to that subscript. For example, $c_I^i \equiv \partial c^i / \partial I_t^i$ and so on.

$$c_r^i = \frac{(u_d^i)^2 u_c^i u_d^i - (u_d^i u_{cd}^i - u_c^i u_{dd}^i) d_{t+1}^i}{u_c^i X}, \quad (7.9)$$

$$d_r^i = \frac{u_d^i - (u_c^i)^2 u_d^i + u_d^i (u_d^i u_{cc}^i - u_c^i u_{cd}^i) d_{t+1}^i}{u_c^i X},$$

where $X \equiv (u_d^i)^2 u_{cc}^i - 2u_c^i u_d^i u_{cd}^i + (u_c^i)^2 u_{dd}^i < 0$ by the second-order condition. The signs, $c_I^i \in (0, 1)$, $d_I^i > 0$, and $d_r^i > 0$ follow from the assumptions on the utility function. When denoting the savings function by $s_t^i = s^i(I_t^i, r_{t+1})$, we obtain the properties of the savings function as follows:

$$s_I^i = \frac{u_d^i (u_d^i u_{cc}^i - u_c^i u_{cd}^i)}{X}, \quad (7.10)$$

$$s_r^i = -\left(\frac{u_d^i}{u_c^i}\right)^2 \frac{(u_c^i)^2 u_d^i - [u_c^i u_d^i u_{cd}^i - (u_c^i)^2 u_{dd}^i] d_{t+1}^i}{X}.$$

By assumption, $s_I \in (0, 1)$ is satisfied, which indicates that goods are normal and the marginal propensity to save is less than unity. As usual, we also assume that the savings function is increasing in the interest rate, that is, $s_r^i > 0$.

Throughout the analysis, we assume that the donor has a higher marginal propensity to save than the recipient, that is, $s_I^D > s_I^R$. Under this assumption, the donor saves more than the recipient in the steady state, that is, $s^D > s^R$. In many realistic situations, the donor and the recipient are developed and developing countries, respectively, and it is often observed that developed countries have a relatively higher marginal propensity to save. Moreover, in our setting for the OLG model, the country that has a higher (lower) marginal propensity to save necessarily has a larger (smaller) amount of savings in the steady state and this results in the country being a capital lender (borrower). To exclude from the analysis the unusual case in which the capital borrower becomes a donor, we make the assumption of $s_I^D > s_I^R$.

Substituting the above demand functions into the utility function, we obtain the indirect utility function as follows:

$$V^i(I_t^i, r_{t+1}) \equiv u^i\left(c_t^i(I_t^i, r_{t+1}), d_{t+1}^i(I_t^i, r_{t+1})\right). \quad (7.11)$$

As $u_c^i c_I^i + u_d^i d_I^i = u_c^i > 0$ and $u_c^i c_r^i + u_d^i d_r^i = u_d^i s_t^i > 0$ hold from the first-order condition, the indirect utility function has the following properties:

$$V_I^i = u_c^i > 0, \quad V_r^i = u_d^i s_t^i > 0. \quad (7.12)$$

7.2.2 Lifetime income

Throughout this chapter, we assume dynamic efficiency: $r_t \geq n$ for all t . Under this assumption, if the economy is at the golden rule, the sharing and distribution ratio of the transfer θ^i between the young and the old does not affect the size of the lifetime income because if $r_{t+1} = n$, $X^i = 1$. Therefore if we focus only on the golden rule, any intergenerational allocation of transfer brings about the same result as when the donor's young generation makes a transfer to the recipient's young, an issue that has already been thoroughly examined in existing work. Importantly, when we limit the argument to that only at the golden rule, it does not make sense that we consider transfer sharing and distribution between the young and the old. However, when we examine the more general case where the dynamic efficiency condition is satisfied except at the golden rule, θ^i affects the present discounted value of the lifetime income of both the young and the old generations because when $r_{t+1} > n$, $X^i = [1 + n + \theta^i(r_{t+1} - n)] / (1 + r_{t+1})$ is strictly increasing in θ^i . In particular, when all of the transfer is attributed to the young (the old), that is, $\theta^i = 1$ ($\theta^i = 0$), X^i is maximized (minimized). Therefore, when the donor's young (old) generation bears all of the transfer, their lifetime income is minimized (maximized) for a given level of T . On the other hand, when the recipient's young (old) generation receives all of the transfer, their lifetime income is maximized (minimized) for a given level of T .⁴

The lifetime income of generation t in country i is denoted in (7.4) and

⁴ Galor and Polemarchakis (1987) showed the possibility of the transfer paradox also in a dynamically inefficient economy. As is well known, in a dynamically inefficient economy, reducing the capital stock increases the welfare and the transfer paradox is more likely to occur. Moreover, the sign of the effect of θ^i on the lifetime income is also reversed when the economy is dynamically inefficient. These differences revise our results and for example, the results of Lemma 7.2 and Proposition 7.1 are the opposite of ours. However, we exclude the dynamically inefficient economy from the analysis since it is difficult to occur from both empirical and realistic point of view in this economy. See De la Croix and Michel (2002, p.84).

(7.5). When the dynamic efficiency condition is satisfied except at the golden rule, that is, $r_{t+1} > n$, the lifetime income I_t^i is a function with respect to $(w_t, T, r_{t+1}, \theta^i)$.⁵ The partial derivatives of $I_t^i \equiv I^i(w_t, T, r_{t+1}, \theta^i)$ are as follows:

$$I_w^D = 1, \quad I_w^R = 1, \quad (7.13)$$

$$I_T^D = -\left[\theta^D + (1 - \theta^D)\frac{1+n}{1+r_{t+1}}\right] < 0, \quad I_T^R = \theta^R + (1 - \theta^R)\frac{1+n}{1+r_{t+1}} > 0, \quad (7.14)$$

$$I_r^D = \frac{(1 - \theta^D)(1+n)}{(1+r_{t+1})^2} T > 0, \quad I_r^R = -\frac{(1 - \theta^R)(1+n)}{(1+r_{t+1})^2} T < 0, \quad (7.15)$$

$$I_\theta^D = -\frac{r_{t+1} - n}{1+r_{t+1}} T < 0, \quad I_\theta^R = \frac{r_{t+1} - n}{1+r_{t+1}} T > 0. \quad (7.16)$$

7.2.3 Firms

Firms in both countries produce their output using the inputs of labor and capital under perfect competition. The production function is constant returns to scale, independent of time, and identical in both countries. Capital does not depreciate. The per capita production function is $f(k_t^i)$, where k_t^i represents the per capita capital in country i in period t . We assume that (i) $f(k_t^i)$ is continuously differentiable; (ii) $f(k_t^i) > 0$, $f'(k_t^i) > 0$, and $f''(k_t^i) < 0$ for all $k_t^i > 0$; and (iii) $f(0) = 0$, $f'(0) = +\infty$, and $f'(\infty) = 0$. Firms maximize their profit in per capita terms denoted by $\pi(k_t^i) \equiv f(k_t^i) - r_t k_t^i - w_t$. The profit maximization requires the equivalence of the marginal productivity and the price of each input as:

$$f'(k_t^i) = r_t \quad \text{and} \quad f(k_t^i) - f'(k_t^i)k_t^i = w_t. \quad (7.17)$$

From (7.17), the interest rate along with the wage rate can be represented by a function of k_t^i . As capital is perfectly mobile, these become the same in both countries through factor price equalization, so that $k_t^D = k_t^R \equiv k_t$ holds. We

⁵ As already mentioned, if the economy is at the golden rule, $I_t^D = w_t - T$ and $I_t^R = w_t + T$ do not depend on r_{t+1} and θ^i .

obtain $k_t = k(r_t)$ and $w_t = w(k(r_t)) \equiv w(r_t)$, where $k'(r_t) = 1/f'' < 0$ and $w'(r_t) = -k_t < 0$.

7.2.4 Capital market equilibrium

The world capital market equilibrium in period t requires the aggregate (the sum of per capita) savings of generation t for both countries to equal the aggregate (the sum of per capita) capital demand in the subsequent period $t + 1$. Therefore, the capital market equilibrium in period t can be expressed as follows:

$$2(1+n)k_{t+1}(r_{t+1}) = s^D(I_t^D, r_{t+1}) + s^R(I_t^R, r_{t+1}). \quad (7.18)$$

Define the excess demand in the world capital market as $D(r_{t+1}) \equiv 2(1+n)k(r_{t+1}) - s^D(I_t^D, r_{t+1}) - s^R(I_t^R, r_{t+1})$. The Walrasian stability condition is given by

$$D'(r_{t+1}) = 2(1+n)k' - (s_I^D I_r^D + s_r^D + s_I^R I_r^R + s_r^R) < 0, \forall t. \quad (7.19)$$

(7.19) implies that the excess demand for capital is strictly decreasing in the interest rate r_{t+1} . We assume that (7.19) is satisfied.

7.2.5 Dynamic stability

As per capita capital k_t is the unique state variable, the dynamic stability condition for the steady-state equilibrium to converge monotonically is as follows:⁶

$$\frac{dr_{t+1}}{dr_t} \in (0, 1). \quad (7.20)$$

⁶ For brevity, we limit our argument to the monotonic convergence to the steady state and exclude the possibility of convergence with oscillation.

By totally differentiating the world capital market equilibrium (7.18) with respect to r_t and r_{t+1} , we obtain the following equation:

$$\begin{aligned}
 2(1+n)k'dr_{t+1} &= s_I^D(I_w^D w' dr_t + I_r^D dr_{t+1}) + s_r^D dr_{t+1} \\
 &\quad + s_I^R(I_w^R w' dr_t + I_r^R dr_{t+1}) + s_r^R dr_{t+1} \\
 &= s_I^D(-k_t dr_t + I_r^D dr_{t+1}) + s_r^D dr_{t+1} \\
 &\quad + s_I^R(-k_t dr_t + I_r^R dr_{t+1}) + s_r^R dr_{t+1}, \quad (7.21)
 \end{aligned}$$

$$\Leftrightarrow D'(r_{t+1})dr_{t+1} = -k_t(s_I^D + s_I^R)dr_t, \quad (7.22)$$

$$\Leftrightarrow \frac{dr_{t+1}}{dr_t} = -\frac{k_t(s_I^D + s_I^R)}{D'(r_{t+1})}. \quad (7.23)$$

By the Walrasian stability, $D'(r_{t+1}) < 0$, $dr_{t+1}/dr_t > 0$ necessarily holds. In order for the dynamic stability condition to be satisfied, it is sufficient that (7.23) is less than unity. We assume the following sufficient condition:

$$-\frac{k_t(s_I^D + s_I^R)}{D'(r_{t+1})} < 1 \Leftrightarrow \Gamma_t \equiv D'(r_{t+1}) + k_t(s_I^D + s_I^R) < 0. \quad (7.24)$$

7.3 Sharing and distribution of transfer

In this section, we present our main results relating to whether the transfer paradox arises in the steady state. The transfer paradox indicates the situation of either donor enrichment or recipient immiserization, in which in spite of the transfer, a donor is strictly enriched and a recipient is strictly immiserized, respectively.

7.3.1 The effect on the interest rate

First, we consider the effect of the transfer on the interest rate in the steady state, given the transfer sharing and distribution ratio between the young and the old θ^i . From (7.18), the world capital market equilibrium in the steady state

is given as follows:⁷

$$2(1+n)k(r) = s^D(I^D, r) + s^R(I^R, r). \quad (7.25)$$

Totally differentiating (7.25) with respect to (r, T) yields the following equation:

$$2(1+n)k' dr = s_I^D(-k dr + I_r^D dr) + s_r^D dr + s_I^R(-k dr + I_r^R dr) + s_r^R dr + (s_I^D I_T^D + s_I^R I_T^R) dT, \quad (7.26)$$

$$\Leftrightarrow \frac{dr}{dT} = \frac{s_I^D I_T^D + s_I^R I_T^R}{\Gamma}, \quad (7.27)$$

where $\Gamma \equiv D'(r) + k(s_I^D + s_I^R) < 0$ and $s_I^D I_T^D + s_I^R I_T^R = -s_I^D[\theta^D + (1 - \theta^D)(1+n)/(1+r)] + s_I^R[\theta^R + (1 - \theta^R)(1+n)/(1+r)]$. As $I_T^D < 0$ and $I_T^R > 0$ from (7.14), the sign of $(s_I^D I_T^D + s_I^R I_T^R)$ is indeterminate in general even when $s_I^D > s_I^R$. However, if $\theta^D \geq \theta^R$, the following lemma holds with regard to the effect of the transfer on the interest rate.

Lemma 7.1. *Suppose that $s_I^D > s_I^R$. If $\theta^D \geq \theta^R$, transfer T raises the interest rate r in the steady state.*

Proof. When $\theta^D \geq \theta^R$, $\theta^D + (1 - \theta^D)(1+n)/(1+r) \geq \theta^R + (1 - \theta^R)(1+n)/(1+r)$. Combining this inequality with $s_I^D > s_I^R$ by assumption, we immediately obtain $s_I^D I_T^D + s_I^R I_T^R < 0$. By (7.27), $dr/dT > 0$ is satisfied. \square

Lemma 7.1 implies that if the transfer sharing ratio for the donor's young generation is more than or equal to the transfer distribution ratio for the recipient's young generation, the transfer necessarily raises the interest rate. The increase in the interest rate can cause the transfer paradox because the higher interest rate brought about by the transfer potentially improves the terms of trade for the capital-exporting country (by definition, the donor) and at the same time, worsens those of the capital-importing country (by definition, the recipient).

⁷ The variables without time subscript denote steady-state values.

7.3.2 The effect of the transfer on welfare

Now we examine the effect of the transfer on welfare in the steady state, given the transfer sharing and distribution ratio between the young and the old θ^i . As already defined in (7.11), the indirect utility function in the steady state is given by $V^i(I^i, r) = u^i(c^i(I^i, r), d^i(I^i, r))$. By totally differentiating $V^i(I^i, r)$ with respect to (I^i, r) and normalizing V_I^i to unity without loss of generality, we obtain the following equation:

$$dV^i = V_I^i dI^i + V_r^i dr = dI^i + \frac{s^i}{1+r} dr. \quad (7.28)$$

Moreover, by differentiating the lifetime income I^i with respect to (r, T) , we obtain the following equations:

$$\begin{aligned} dV^D &= dw - \left[\theta^D + (1 - \theta^D) \frac{1+n}{1+r} \right] dT + \frac{(1 - \theta^D)(1+n)}{(1+r)^2} T dr + \frac{s^D}{1+r} dr \\ &= \left[-k + \frac{(1 - \theta^D)(1+n)}{(1+r)^2} T + \frac{s^D}{1+r} \right] dr - \left[\theta^D + (1 - \theta^D) \frac{1+n}{1+r} \right] dT, \end{aligned} \quad (7.29)$$

$$\begin{aligned} dV^R &= dw + \left[\theta^R + (1 - \theta^R) \frac{1+n}{1+r} \right] dT - \frac{(1 - \theta^R)(1+n)}{(1+r)^2} T dr + \frac{s^R}{1+r} dr \\ &= \left[-k - \frac{(1 - \theta^R)(1+n)}{(1+r)^2} T + \frac{s^R}{1+r} \right] dr + \left[\theta^R + (1 - \theta^R) \frac{1+n}{1+r} \right] dT. \end{aligned} \quad (7.30)$$

Substituting (7.27) into (7.29) and (7.30), we obtain the effect of the transfer on the welfare of both countries as follows:

$$\begin{aligned} \frac{dV^D}{dT} = & \left[-k + \frac{(1 - \theta^D)(1 + n)}{(1 + r)^2} T + \frac{s^D}{1 + r} \right] \frac{s_I^D I_T^D + s_I^R I_T^R}{\Gamma} \\ & - \left[\theta^D + (1 - \theta^D) \frac{1 + n}{1 + r} \right], \end{aligned} \quad (7.31)$$

$$\begin{aligned} \frac{dV^R}{dT} = & \left[-k - \frac{(1 - \theta^R)(1 + n)}{(1 + r)^2} T + \frac{s^R}{1 + r} \right] \frac{s_I^D I_T^D + s_I^R I_T^R}{\Gamma} \\ & + \left[\theta^R + (1 - \theta^R) \frac{1 + n}{1 + r} \right]. \end{aligned} \quad (7.32)$$

In (7.31) and (7.32), the first term is the indirect effect resulting from the change in the interest rate because of the transfer and the second term is the direct effect the income transfer has on welfare. As it is possible that the sign of the indirect effect differs from that of the indirect effect, there is a possibility that the transfer paradox arises if the indirect effect is greater than the direct effect. This possibility has already been discussed in seminal studies dealing with the transfer paradox in an OLG model, as represented by Galor and Polemarchakis (1987) and Cremers and Sen (2008).

Note that in the steady state, the donor's savings are larger than the recipient's savings, that is, $s^D > s^R$. Although the direct effect on the donor's welfare is negative, the indirect effect that is shown in the first term of (7.31) can be positive if the donor's savings are relatively high, that is, $s^D > (1 + r)k$ and the effect of the transfer on the interest rate is positive, that is, $dr/dT > 0$, the indirect effect is positive.⁸ In this case, the indirect effect countervails against the direct effect and if the former exceeds the latter, donor enrichment arises. As for the recipient's welfare, the direct effect is positive, while the indirect effect that is shown in the first term of (7.32) can be negative if the effect of the transfer on the interest rate is positive, because the recipient is the capital importer and $s^R < (1 + r)k$ necessarily holds under the dynamic efficiency condition. In this case, the indirect effect countervails against the direct effect

⁸ However, whether $s^D > (1 + r)k$ holds depends on the situation even under the dynamic efficiency condition. Although $s^D > (1 + n)k > s^R$ necessarily holds when the donor is the capital exporter, whether the donor's savings s^D are larger than $(1 + r)k$ is indeterminate in our model.

and if the former exceeds the latter, recipient immiserization arises. As shown in Lemma 7.1, if $\theta^D \geq \theta^R$, $dr/dT > 0$. Thus if $\theta^D \geq \theta^R$, the signs of the direct and indirect effects are opposite and it is possible for the paradox to occur. In contrast, if $dr/dT < 0$, the transfer paradox never occurs.

As θ^R changes from 1 to 0, it seems that the indirect effect increases in the negative direction while the positive direct effect decreases from (7.32). That is, it might be likely that transfers harm the donor's welfare. However, this intuition is not correct because a change in θ^R also affects the per capita capital stock k . We demonstrate a numerical simulation to see how the recipient's welfare changes as θ^R changes.⁹

We employ log-linear preferences and a constant elasticity of substitution production function:

$$U(c_t^i, d_{t+1}^i) = \ln c_t^i + \beta^i \ln d_{t+1}^i, \quad 0 < \beta < 1, \\ f(k) = A(ak^{-\gamma} + 1 - a)^{-1/\gamma}, \quad A > 0, 0 < a < 1, \gamma > -1, \gamma \neq 0, \quad (7.33)$$

for $i = D, R$. Under the log-linear preferences s_I^i becomes $\beta^i/(1 + \beta^i)$. To calibrate the model, we set $\gamma = 1$, $a = 0.7$, $n = 1.097$, $A = 20$, $s_I^D = 0.6$, and $s_I^R = 0.1$. Under these parameters, the steady state is $k = 5.4639$ with the interest rate, $r = 2.5586$. Hence, the dynamic efficiency condition is satisfied.

Figs. 7.1 and 7.2 plot out a change in the marginal effect of transfer on the donor's and the recipient's welfare, respectively, with respect to θ^R . The horizontal and vertical axis correspond to the change in θ^R and the marginal effect of transfer on welfare, respectively. It should be noted that when all transfers are given to the young (the old) generation, $\theta^R = 1$ ($\theta^R = 0$) holds. Both figures demonstrates that if the transfer is made from the young in the donor country to the young in the recipient country, the transfer paradox does not occur. When θ^R becomes smaller than a threshold value, that is, $\theta^R < 0.76$, the marginal effect of transfer on the recipient's welfare necessarily takes a negative sign and it implies that when the share of transfer by the old generation increases, the effect of the transfer on the recipient's welfare decreases.

The transfer sharing and distribution ratio between the young and the old θ^i affects the likelihood of the transfer paradox arising under a dynamically ef-

⁹ We draw upon Cremers and Sen (2008) for parameter values in the simulation.

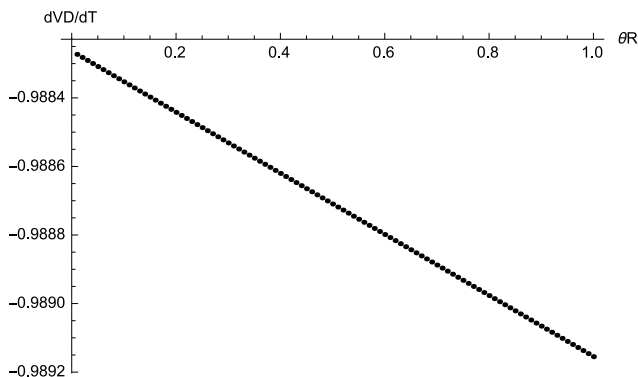


Fig. 7.1: The marginal effect of transfer on the donor's welfare

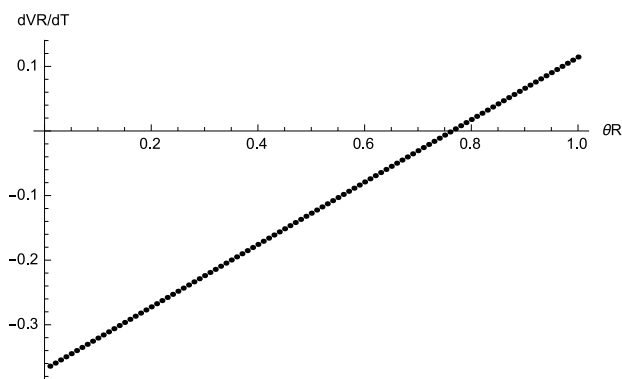


Fig. 7.2: The marginal effect of transfer on the recipient's welfare

ficient economy except at the golden rule. Therefore, the result that there is no need to think about the intergenerational allocation of the transfer is limited only to the situation at the golden rule, and one on which existing studies have mainly focused. However, to investigate both marginal effects on the welfare of the amount of transfer T itself and its intergenerational allocation θ^i complicates the analysis. In the following subsection, we proceed to examine the

effect that the change in the intergenerational allocation θ^i has on the interest rate and the welfare, providing that the total amount of transfer is constant.

7.3.3 The effect of the sharing or distribution ratio on the interest rate

In the previous subsection 7.3.1, we examined the effect of the transfer on the interest rate, given the transfer sharing and distribution ratio is constant. In this subsection, providing that the total amount of transfer is constant, that is, $dT = 0$, we examine how the change in the transfer sharing and distribution ratio between the young and the old θ^i affects the interest rate in the steady state.

By totally differentiating the world capital market equilibrium in the steady state (7.25) with respect to (r, θ^D) or (r, θ^R) , we obtain the following equations:

$$2(1+n)k'dr = s_I^D(-kdr + I_r^D dr) + s_r^D dr + s_I^R(-kdr + I_r^R dr) + s_r^R dr + s_I^D I_\theta^D d\theta^D, \quad (7.34)$$

$$\Leftrightarrow \frac{dr}{d\theta^D} = \frac{s_I^D I_\theta^D}{\Gamma} > 0, \quad (7.35)$$

$$2(1+n)k'dr = s_I^D(-kdr + I_r^D dr) + s_r^D dr + s_I^R(-kdr + I_r^R dr) + s_r^R dr + s_I^R I_\theta^R d\theta^R, \quad (7.36)$$

$$\Leftrightarrow \frac{dr}{d\theta^R} = \frac{s_I^R I_\theta^R}{\Gamma} < 0. \quad (7.37)$$

From (7.35) and (7.37), the following lemma is immediately obtained.

Lemma 7.2. *Suppose that the economy is dynamically efficient except at the golden rule. $dr/d\theta^D > 0$ and $dr/d\theta^R < 0$.*

Proof. From the dynamic stability condition, $\Gamma \equiv D'(r) + k(s_I^D + s_I^R) < 0$. Therefore, it is immediately obtained that $s_I^D I_\theta^D = -s_I^D(r-n)T/(1+r) < 0$ and $s_I^R I_\theta^R = s_I^R(r-n)T/(1+r) > 0$. \square

Lemma 7.2 indicates that the increase in θ^D raises the interest rate and the fall in θ^D lowers the interest rate. When the donor's young defray a larger

amount of transfer than the old, the transferred income decelerates capital accumulation in the donor country and raises the interest rate. In contrast, when the recipient's young receive a larger amount of transfer than the old, the transferred income increases capital accumulation in the recipient country and lowers the interest rate.

7.3.4 The effect of the sharing and distribution ratio on welfare

Now we examine how the intergenerational transfer sharing and distribution ratio affects welfare in both countries given that the total amount of transfer is constant. Note that the indirect utility function in the steady state is denoted by $V^i(I^i, r) = u^i(c^i(I^i, r), d^i(I^i, r))$, where $I^D = w - [\theta^D + (1 - \theta^D)(1 + n)/(1 + r)]T$ and $I^R \equiv w + [\theta^R + (1 - \theta^R)(1 + n)/(1 + r)]T$. Thus, total differentiation of $V^i(I^i, r)$ with respect to (r, θ^i) gives the following equations:

$$\begin{aligned} dV^D &= dw + I_\theta^D \theta^D + \frac{(1 - \theta^D)(1 + n)}{(1 + r)^2} T dr + \frac{s^D}{1 + r} dr \\ &= \left[-k + \frac{(1 - \theta^D)(1 + n)}{(1 + r)^2} T + \frac{s^D}{1 + r} \right] dr - \frac{r - n}{1 + r} T d\theta^D, \end{aligned} \quad (7.38)$$

$$\begin{aligned} dV^R &= dw + I_\theta^R \theta^R - \frac{(1 - \theta^R)(1 + n)}{(1 + r)^2} T dr + \frac{s^R}{1 + r} dr \\ &= \left[-k - \frac{(1 - \theta^R)(1 + n)}{(1 + r)^2} T + \frac{s^R}{1 + r} \right] dr + \frac{r - n}{1 + r} T d\theta^R. \end{aligned} \quad (7.39)$$

By substituting the effect of the transfer on the interest rate (7.35) and (7.37) into (7.38) and (7.39), respectively, we obtain the effect of the sharing and distribution ratio on welfare as follows:

$$\frac{dV^D}{d\theta^D} = \left[-k + \frac{(1 - \theta^D)(1 + n)}{(1 + r)^2} T + \frac{s^D}{1 + r} \right] \frac{s_I^D I_\theta^D}{\Gamma} + I_\theta^D, \quad (7.40)$$

$$\frac{dV^R}{d\theta^R} = \left[-k - \frac{(1 - \theta^R)(1 + n)}{(1 + r)^2} T + \frac{s^R}{1 + r} \right] \frac{s_I^R I_\theta^R}{\Gamma} + I_\theta^R, \quad (7.41)$$

where $I_\theta^D = -(r - n)T/(1 + r) < 0$ and $I_\theta^R = (r - n)T/(1 + r) > 0$. Both equations (7.40) and (7.41) indicate that the effect on welfare of the sharing

and distribution ratio of the young generation consists of two effects, namely, an intertemporal substitution effect and an income effect. As we have already shown, the former implies the indirect effect that is brought about by the change in the interest rate through the transfer, and the latter is the direct effect caused by income transfer itself. When the transfer share for the donor's young θ^D increases, the direct income effect is negative for the donor country. In contrast, when the receiving share for the recipient's young θ^R increases, the direct effect is positive for the recipient. As for the indirect effect, when θ^R increases, the indirect effect on the recipient's welfare is necessarily positive, while when θ^D increases, whether the indirect effect on the donor's welfare is positive depends on the situation. When we suppose that $s_I^D > s_I^R$ and the total amount of transfer is fixed in a dynamically efficient economy except at the golden rule, we can obtain the following main proposition from the above discussion with (7.40) and (7.41).

Proposition 7.1. *Whether the increase in the share of the transfer for the donor's young raises the donor's welfare is ambiguous. By contrast, raising the distribution ratio for the recipient's young necessarily improves the recipient's welfare.*

Proof. From (7.40), the direct income effect for the donor is negative because $I_\theta^D = -(r-n)T/(1+r) < 0$, while the sign of the indirect effect is indeterminate even when $s_I^D I_\theta^D/\Gamma > 0$ by Lemma 7.2, because $s^D > (1+r)k$ does not necessarily hold under the dynamic efficiency condition except at the golden rule even when the donor is the capital-exporting country. Thus, the sign of the bracket in (7.40) is ambiguous. If the sign of the bracket in (7.40) is negative, $dV^D/d\theta^D < 0$ and if it is positive, the sign of $dV^D/d\theta^D$ depends on the relative size of the negative direct effect and the positive indirect effect. From (7.41), the direct income effect for the recipient is positive because $I_\theta^R = (r-n)T/(1+r) > 0$, while the indirect effect is necessarily positive because $s_I^R I_\theta^R/\Gamma < 0$ by Lemma 7.2 and the recipient is the capital-importing country, that is, $s^R < (1+r)k$ under the dynamic efficiency condition except at the golden rule. As both the direct and indirect effects are positive, $dV^R/d\theta^R > 0$. \square

The reason why Proposition 7.1 holds is as follows. When the economy is dynamically efficient except at the golden rule, the increase in the distribution ratio of the young θ^i reduces (raises) the lifetime income of the donor's (recip-

ient's) individuals because the income transfer from (to) the young generation raises the present discounted value of lifetime income because the interest rate is higher than the population growth rate. Thus, the direct income effect of the transfer is always negative for the donor and positive for the recipient. On the other hand, there is also the indirect effect brought about by the change in the interest rate. As shown in Lemma 7.2, the interest rate increases with θ^D and decreases with θ^R because the transfer from the high-saving donor to the low-saving recipient decelerates world capital accumulation, which leads to an increase in the interest rate. The increase in θ^R then decreases the interest rate and this accounts for the positive indirect effect. Thus, as both the direct and the indirect effects are positive, the increase in θ^R necessarily brings about an improvement in the recipient's welfare. In contrast, whether the indirect effect for the donor is positive is indeterminate. When the indirect effect is also negative, the increase in θ^D worsens the donor's welfare. However, a case is possible in which the positive indirect effect countervails against the negative direct effect, which leads to an improvement in the donor's welfare.

Proposition 7.1 implies that whether the sharing ratio for the donor's young should increase in order to improve the donor's welfare depends on the specific situation, while the distribution ratio for the recipient's young necessarily improves the recipient's welfare. All of the extant studies assume without any reasonable explanation that the transfer is made from the donor's young to the recipient's young for the sake of analytical simplicity. In this study, we confirmed that when all of the transfer is distributed by the recipient's young, the increased recipient's welfare is maximized. This result justifies the reason why the recipient's young rather than the old should receive all of the transfer. On the other hand, the increase in the sharing ratio of the donor's young could either improve or worsen the donor's welfare depending on the relative size of the negative direct effect and the positive indirect effect. Therefore, in contrast to the case of a recipient country, the assumption that the donor's young should defray all of the transfer cannot be automatically justified. In fact, only when the negative direct effect is less than the positive indirect effect will a transfer payment by the donor's young be justified because the donor's welfare increases compared with when the donor's old pay all of the transfer, which is the situation that has been examined in the existing literature. However, as long as the direct effect is larger than the indirect effect, it is preferable that the

donor's old pay all of the transfer from the viewpoint of the donor's welfare.

In the existing research concerning the transfer problem, it is the typical situation for the donor that the direct income effect exceeds the indirect intertemporal terms-of-trade effect. In contrast, the transfer paradox occurs when the negative direct effect is less than the positive indirect effect that improves the terms of trade of the exporting capital through the increase in the interest rate. Therefore, Proposition 7.1 suggests that only if the transfer paradox occurs for the donor should the donor's young bear all of the transfer from the viewpoint of donor welfare maximization; otherwise, the donor's old should bear all of the transfer.¹⁰

7.4 Concluding remarks

In this chapter, we investigated how the burden of an international transfer should be shared between the young and old generations of a donor country and how it should be allocated between the young and old generations of a recipient country by considering the transfer problem between two countries in a one-sector OLG model. We clarified that in a dynamically efficient economy except at the golden rule level of capital, the ratios of transfer sharing and distribution between the young and old generations affect the level of welfare. In particular, we showed that from the viewpoint of welfare maximization, it is preferable for the recipient country to distribute all of the transfer to the young generation. On the other hand, who should share the transfer burden in the donor country depends on the relative size of two effects, namely, a negative direct effect and a positive indirect effect. If the former exceeds the latter, it is preferable for the donor country to allocate all of the transfer burden to the old

¹⁰ It should be noted that the indirect effect that we examined in Subsection 7.3.2 differs from that in Subsection 7.3.4, although at first glance they seem identical. The indirect effect that is brought about by the change in the amount of transfer dT depends on the term $(s_I^D I_T^D + s_I^R I_T^R)/\Gamma$ from (7.31) and (7.32). The sign and size of this indirect effect are determined by the relative size relationship of the marginal propensity to save between the donor and the recipient, that is, s_I^D and s_I^R . In contrast, the indirect effect brought about by the change of the intergenerational distribution ratio $d\theta^i$ depends on $s_I^i I_T^i/\Gamma$ from (7.40) and (7.41). This sign and size are determined by the absolute level of the marginal propensity to save of their own country. This difference comes from the fact that the change in θ^i only affects their own country's indirect utility.

generation and vice versa.

Our results supporting the assumption in existing studies that the recipient's young receive all of the transfer can be justified from the viewpoint of the recipient's welfare maximization, while the assumption that the donor's young pay all of the transfer cannot be necessarily justified. If there emerges no paradoxical situation in which the indirect effect of the transfer exceeds the direct effect, it is preferable from the viewpoint of donor welfare maximization that the donor's old bear all of the transfer.

Throughout the analysis, we have only considered the situation in which the donor has a higher marginal propensity to save than the recipient. Although this assumption is quite natural for the actual transfer situation that we consider, our results are in fact not necessarily restricted by this assumption. For example, even if both the donor and the recipient have the same marginal propensity to save, that is, $s_I^D = s_I^R \equiv s_I$, a similar argument can be applied because dr/dT that determines the sign of the indirect effect depends on the sign of $s_I^D I_T^D + s_I^R I_T^R = s_I (I_T^D + I_T^R)$. Therefore, Proposition 7.1 suggests that even if the marginal propensity to save is the same for both the donor and the recipient, the difference in the income distribution policy between both governments could still cause the transfer paradox. As far as we know, few studies refer to the possibility that policy decisions by governments about intergenerational income distribution bring about the transfer paradox. A detailed investigation into the effects of income distribution policy is a future research challenge.

We conclude by presenting some possible research extensions. First, we do not incorporate the endogenous determination of the intergenerational sharing and distribution ratio into the model. One extension of the analysis would be then to consider the transfer problem in an endogenous model including the intergenerational allocation of transfers. Second, we do not consider decision making at the transfer level by the donor country. Further generalization of the model in our study is thus required in terms of the determination of the optimal transfer.

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Chapter 8

Pay-as-you-go Pension System and the Transfer Problem

This chapter examines how international transfers affect the welfare levels of a donor with a higher marginal propensity to save and a recipient with a lower marginal propensity to save when both countries adopt a pay-as-you-go (PAYG) pension system using a one-sector overlapping generations model. We demonstrate that in a dynamically efficient economy, except at the golden rule, when a per capita PAYG pension contribution of either a donor or a recipient increases marginally, the effect of the transfer on the donor's welfare can be reduced, whereas whether the effect of the transfer on the recipient's welfare is reduced is ambiguous. These results imply that the existence of a PAYG pension might hinder the effectiveness of the transfer on the donor's welfare, and the adoption of a PAYG pension system is likely to cause a weak transfer paradox in which both a donor and a recipient immiserize. Our results also suggest that the introduction of a PAYG pension system, which is used as a domestic policy instrument for intergenerational income redistribution, reduces the donor's incentive to make an international transfer to a recipient, which is a form of international income redistribution.

Keywords: pay-as-you-go pension, transfer paradox, overlapping generations model, capital accumulation

JEL classifications: D91, E21, F35, F43, H55

8.1 Introduction

This chapter examines how international transfers affect the welfare levels of both a donor with a higher marginal propensity to save and a recipient with a lower marginal propensity to save, each of which adopt a pay-as-you-go (PAYG) pension system in a one-sector overlapping generations (OLG) model.

The PAYG pension system is common in developed countries such as Japan, the UK, Germany, and Canada, and even in developing countries such as China.¹ In Japan, total social security benefits are around US\$ 1.1 trillion, which is equivalent to more than one-fifth of GDP in 2013. Pension benefits alone are around US\$ 0.5 trillion, or more than 10% of GDP. The influence of the pension system, and in particular the PAYG system, cannot be ignored. There has been a lively discussion about the ideal pension system (see World Bank 1994 and Orszag and Stiglitz 2001). This discussion made clear the various factors, including the influence on economic growth, the administrative cost, and the managerial and political risk of pension systems, which must be considered when comparing a PAYG pension system with a fully funded one.

The means by which international transfers have an effect on the welfare of a donor country and a recipient country have been examined since the late 1920s. Beginning with the controversy between Keynes (1929a, 1929b) and Ohlin (1929), the analysis has evolved to include the possibility of a transfer paradox: the donor becomes better off and/or the recipient becomes worse off. Following a considerable number of studies that used a static framework, Galor and Polemarchakis (1987) showed that the transfer paradox can arise in a dynamic framework, and specifically, in an OLG model.²

¹ In China, there are two types of pension system. The PAYG pension system is adopted in cities, and a fully funded scheme is adopted in rural areas.

² Following Galor and Polemarchakis (1987), Haaparanta (1989) demonstrated the occurrence of a transfer paradox even if only a temporary transfer occurs, involving the govern-

It is well-known that there are two causes of the transfer paradox in an OLG model. The first cause is the dynamic inefficiency of the market equilibrium. When the economy is dynamically inefficient, the transfer paradox can arise because the less capital that is accumulated, the larger welfare becomes. The second cause is the existence of the indirect effect of a transfer in the OLG model, brought about by a change in the interest rate. The indirect effect is usually comprised of two parts: the golden rule effect and the international capital movement effect. The golden rule effect implies that if the world (per capita) capital level approaches the golden rule level at which the amount of consumption is maximized in the steady state, the welfare levels of both countries increases. The international capital movement effect implies that if the interest rate rises (or equivalently, the capital level decreases), the welfare of a high-savings country as a creditor improves because the return from the capital lending to a low-savings country as a debtor increases, while the welfare of a low-savings country decreases. Even in a dynamically efficient economy in which a donor is a creditor and a recipient is a debtor, if the indirect effect exceeds the direct income effect of the transfer, it is possible that the transfer improves the donor's welfare and/or reduces the recipient's welfare.

A PAYG pension system is indispensable and fundamental for intergenerational income redistribution in modern capitalist economies, because it complements self-help private savings.³ PAYG pension systems have been examined in many studies using OLG models. For example, Fanti and Gori (2012) and Cipriani (2014) analyzed the relationship between the birth rate and pension benefits. Roberts (2003), Kaganovich and Zilcha (2012), and Bruce and Turnovsky (2013) investigated the effect of a PAYG pension on capital accumulation. However, there are few articles that have examined international transfers between countries with a PAYG pension system in a dynamic setting.

We investigate how PAYG social security alters the effect of international

ment issuing bonds. Yanagihara (1998) incorporated public goods that affected productivity in the private production sector and examined transfers in the form of lump-sum, debt-relief, and public goods. Yanagihara (2006) provided a visualization of the transfer paradox set out in Galor and Polemarchakis (1987). Cremers and Sen (2005) investigated the possibility of transfer paradoxes in the transitional path to the steady state as well as in the steady state.

³ In an OLG model, a fully funded pension cannot be distinguished from a system involving private savings only. Thus, introducing a fully funded pension into the model does not affect our results.

transfers on the welfare levels of countries in a one-sector OLG model. Taking a classical approach in which only physical capital is accumulated, we investigate whether the introduction of a PAYG pension system promotes or inhibits the effect of a transfer on the donor's and the recipient's welfare. We show that when a donor has a higher marginal propensity to save than a recipient, there is a possibility that a marginal increase in PAYG pensions reduces the effect of the transfer on the donor's welfare. This result implies that introducing a domestic policy on income redistribution between generations such as a PAYG pension system is likely to reduce the benefit of international transfers on the donor's welfare. This negative impact caused by the introduction of a PAYG pension system is brought about by the indirect effect of the transfer. The transfer from a donor with a high marginal propensity to save to a recipient with a low marginal propensity to save inhibits world capital accumulation and results in a rise in the interest rate. When this indirect effect on the interest rate of the transfer exceeds the direct income effect, the transfer paradox arises. When the government of a country introduces a PAYG pension, the per capita pension contribution reduces the lifetime income of each generation in a dynamically efficient economy except at the golden rule and indirectly affects the interest rate. This additional effect of the PAYG pension contribution sometimes has a negative effect because the negative indirect effect of the transfer on the interest rate might be reinforced by the increase in the pension contribution.

The remainder of this chapter is organized as follows: Section 8.2 sets out our one-sector, two-country OLG model. Section 8.3 shows the conditions for static and dynamic stability. Section 8.4 presents our main results and considers their implications. Section 8.5 provides concluding remarks.

8.2 The model

We consider a one-sector OLG model. In the world economy there exist two countries, a donor country and a recipient country that receives an international income transfer, which are denoted by country $i = D$ and R , respectively. These two countries are identical except for the time preferences of the individuals in a country. Both countries introduce a PAYG pension system. Capital is fully mobile between the two countries, but labor is immobile. The populations of both countries grow at the same rate, with a gross population growth rate of

$(1 + n) \geq 1$. n is exogenously given and constant over time.

8.2.1 Individuals

In each period, both countries are populated by two generations, the young who supply one unit of labor inelastically and earn wages, and the old who retire and consume savings accumulated in their youth. All individuals live for two periods. Individuals born in period t , which is denoted as generation t , and living in country $i = D, R$, choose consumption in their youth t and in their old age $t + 1$, (c_t^i, d_{t+1}^i) , so as to maximize their utility, subject to the budget constraints in their respective young and old periods.

The intertemporal utility of generation t in country i is given by:

$$u^i(c_t^i, d_{t+1}^i). \quad (8.1)$$

We assume that u^i is twice differentiable, $u_c^i > 0$, $u_d^i > 0$, $u_{cc}^i < 0$, and $u_{dd}^i < 0$.

The budget constraints in the respective young and old periods for generation t in country i are as follows:

$$c_t^i + s_t^i = w_t^i + T^i - P^i \quad \text{and} \quad d_{t+1}^i = (1 + r_{t+1}^i)s_t^i + (1 + n)P^i, \quad (8.2)$$

where r^i , w^i , and s^i , respectively, denote the net interest rate, wage rates, and savings in country i . T^i and P^i denote a per capita permanent transfer and the per capita pension contribution in country i , respectively. We assume that $w^i + T^i - P^i > 0$ for all periods. The intertemporal budget constraint can be rewritten as:

$$c_t^i + \frac{1}{1 + r_{t+1}^i} d_{t+1}^i = I_t^i, \quad (8.3)$$

where $I_t^i \equiv w_t^i + T^i - (r_{t+1}^i - n)P^i / (1 + r_{t+1}^i)$ defines the lifetime income of generation t in country i evaluated at period t . This expression has the properties that $dI^i/dw^i = 1$ and $dI^i/dr^i = -(1 + n)P^i / (1 + r^i)^2 < 0$. Throughout the chapter, we only consider the case in which the economy is dynamically efficient except at the golden rule, so that $r_t^i > n$ for all t . Therefore, the increase in the per capita PAYG pension contribution reduces the lifetime income. As we

consider a permanent transfer from country D to country R , in what follows, T denotes the amount of the permanent transfer from country D to country R , that is, $T \equiv T^R = -T^D > 0$.

By solving the utility maximization problem, we obtain the optimal consumption levels as well as the savings function, $s_t^i = s(I_t^i, r_{t+1}^i)$. We denote the indirect utility function of generation t in country i by $V_t^i \equiv V^i(I_t^i, r_{t+1}^i)$. As a usual assumption, we assume that the marginal propensity to save lies between 0 and 1, that is, $s_I^i \in (0, 1)$ and the savings increase with the interest rate r_{t+1}^i , that is, $s_r^i > 0$, where the subscripts of the savings function denote the partial derivatives. Furthermore, we assume that the marginal propensity to save does not decrease with income, that is, $s_{II}^i \geq 0$, which is required for the proofs of the following lemmas and propositions.

Throughout the analysis, we assume that the donor has a higher marginal propensity to save than the recipient, that is, $s_I^D > s_I^R$. Under this assumption, the donor saves more than the recipient in the steady state, that is, $s^D > s^R$. In many realistic situations, the donor and the recipient are developed and developing countries, respectively, and it is often observed that developed countries have a relatively higher marginal propensity to save. Moreover, in our setting for the OLG model, the country that has a higher (lower) marginal propensity to save necessarily has a larger (smaller) amount of savings in the steady state and this results in the country being a capital lender (borrower). To exclude from the analysis the unusual case in which the capital borrower becomes a donor, we make the assumption of $s_I^D > s_I^R$.

8.2.2 Firms

Firms in both countries produce their output using labor and capital under perfect competition. The production function has constant returns to scale, is time independent, and is identical in both countries. Capital does not depreciate. From the properties of the production function given above, the per capita production function can be written as $f(k_t^i)$, where k_t^i represents the per capita capital in country i in period t . We assume that the per capita production function satisfies the following conditions: (i) $f(\cdot)$ is continuously differentiable; (ii) $f > 0$, $f' > 0$, and $f'' < 0$ for all $k > 0$; (iii) $f(0) = 0$, $\lim_{k \rightarrow 0} f' = \infty$, and $\lim_{k \rightarrow \infty} f' = 0$ (the Inada conditions).

Firms maximize their profit in per capita terms, denoted by $\pi(k_t^i) \equiv f(k_t^i) - r_t^i k_t^i - w_t^i$. Profit maximization requires the equivalence of the marginal productivity and the price of each input as follows:

$$f'(k_t^i) = r_t^i \quad \text{and} \quad f(k_t^i) - f'(k_t^i)k_t^i = w_t^i. \quad (8.4)$$

From (8.4), the level of capital stock as well as the wage rate can be represented as a function of r_t^i . As capital is fully mobile, the factor prices and the level of capital stock are equalized in each country because of factor price equalization, so that $r_t^D = r_t^R \equiv r_t$, $w_t^D = w_t^R \equiv w_t$, and $k_t^D = k_t^R \equiv k_t$. In sum, we obtain $k_t = k(r_t)$ and $w_t = w(r_t)$. The following relationships hold:

$$k' \equiv \frac{dk_t}{dr_t} = \frac{1}{f''(k_t)}, \quad w' \equiv \frac{dw_t}{dr_t} = -k_t. \quad (8.5)$$

8.2.3 Capital market equilibrium

The world capital market equilibrium condition in per capita terms in period t can be expressed as:

$$(1+n)k_{t+1}^D + (1+n)k_{t+1}^R = s_t^D + s_t^R. \quad (8.6)$$

The right-hand side of (8.6) represents the supply of capital from individuals in both countries in the present period t . The left-hand side indicates the demand for capital by firms in both countries, which is utilized in the next period $t+1$. As the savings depend on the lifetime income I_t^i , which is affected by not only the interest rate in period $t+1$ but also the wage rate in period t , that is, $(r_{t+1}, w(r_t))$, the world capital market equilibrium condition (8.6) involves the dynamics of the interest rates of r_t and r_{t+1} . Full international capital mobility causes the level of per capita capital in each country to be the same, that is, $k^D = k^R \equiv k$. By rewriting (8.6), we obtain the following equation:

$$2(1+n)k(r_{t+1}) = s^D(I_t^D(r_t, r_{t+1}), r_{t+1}) + s^R(I_t^R(r_t, r_{t+1}), r_{t+1}). \quad (8.7)$$

8.3 The stability conditions

In this section we check the static and dynamic stability conditions for this economy. To first check the static stability, namely, Walrasian stability, we define the excess demand in the world capital market that depends on (r_t, r_{t+1}) as follows:

$$D(r_t, r_{t+1}) \equiv 2(1+n)k(r_{t+1}) - s^D(I_t^D(r_t, r_{t+1}), r_{t+1}) - s^R(I_t^R(r_t, r_{t+1}), r_{t+1}). \quad (8.8)$$

Walrasian stability requires that excess demand for capital should decrease in r_{t+1} . As the lifetime income I_t^i depends on r_{t+1} , for given levels of transfer T and per capita pension contribution P^i , the condition for Walrasian stability to be satisfied is as follows:

$$\Omega_t \equiv \frac{\partial D(r_t, r_{t+1})}{\partial r_{t+1}} = 2(1+n)k' + \frac{(1+n)P^D}{(1+r_{t+1})^2} s_I^D + \frac{(1+n)P^R}{(1+r_{t+1})^2} s_I^R - s_r^D - s_r^R < 0, \forall t. \quad (8.9)$$

Under the assumptions, $k' < 0$, $s_I^i > 0$, and $s_r^i > 0$, the sufficient condition for (8.9) to be satisfied is that $(1+n)P^i/(1+r_{t+1})^2$ is not too large. This term implies a decrease in lifetime income caused by an increase in the per capita pension contribution. As we focus only on the marginal increase in the per capita pension contribution from $P^i = 0$ in the following analysis, Walrasian stability is always guaranteed under the assumption of $s_r^i > 0$.

Next, we consider the dynamic stability of this economy. For the steady state to be dynamically stable, $dr_{t+1}/dr_t \in (0, 1)$ is required. Totally differentiating (8.8) with respect to r_t and r_{t+1} , we obtain the following equations:

$$\Omega_t dr_{t+1} - (s_I^D w' + s_I^R w') dr_t = 0 \Leftrightarrow \frac{dr_{t+1}}{dr_t} = \frac{s_I^D w' + s_I^R w'}{\Omega_t}. \quad (8.10)$$

$dr_{t+1}/dr_t > 0$ is immediately derived from $s_I^i > 0$, $w' < 0$, and Walrasian stability condition $\Omega_t < 0$. Thus, in order to guarantee that the steady state is dynamically stable, it is sufficient to assume $dr_{t+1}/dr_t < 1$. Throughout the

chapter, we make the following assumption:

$$\frac{s_I^D w' + s_I^R w'}{\Omega_t} < 1 \Leftrightarrow \Gamma_t \equiv \Omega_t - s_I^D w' - s_I^R w' < 0, \forall t. \quad (8.11)$$

8.4 Main results

In this section, we investigate the effect of international transfer on social welfare in an economy in which a PAYG pension system is introduced in the donor and the recipient countries. We derive some main results about how the increase in per capita pension contributions affects the effect of the transfer on welfare. To avoid the complexity of analyzing the transitional path, we only focus on the steady state by omitting time subscripts.

8.4.1 The effect on the interest rate

As in the literature on the transfer problem in an OLG framework, the effect of the transfer on welfare can usually be decomposed into two components. The first component is the direct effect, which corresponds to the income effect brought about by the change in the income level directly caused by the transfer between a donor country and a recipient country. Obviously, the direct income effect is positive for the recipient country and negative for the donor country. The second component is the indirect effect: the transfer first affects the capital market equilibrium and thus affects the intertemporal terms of trade—the interest rate. Whether the transfer improves the welfare of a donor or a recipient depends on the relative sizes of the two abovementioned effects; if the indirect effect dominates the direct one, the transfer paradox arises. Therefore, before investigating the total effect that the transfer has on welfare, we examine the effect of the transfer on the interest rate and how the increase in the per capita pension contribution affects the latter effect. We arrive at the following lemmas:

Lemma 8.1.

- (i) *The transfer increases the interest rate, that is, $dr/dT > 0$.*
- (ii) *The increase in the per capita pension contribution in a country raises the interest rate, that is, $dr/dP^i > 0$.*

Lemma 8.2. *Suppose that the per capita pension contribution P^i marginally increases from zero and assume that $d\Gamma/dP^i \leq 0$, $s_{II}^D = s_{II}^R$, and $s_{Ir}^D = s_{Ir}^R$.*

(i) *The marginal increase in the donor's per capita pension contribution decreases the effect of the transfer on the interest rate, that is, $(d^2r/dTdP^D)|_{P^D=P^R=0} < 0$.*

(ii) *The marginal increase in the recipient's per capita pension contribution decreases the effect of the transfer on the interest rate only if $d\Gamma/dP^R < (r - n)s_{II}^R\Gamma/(1 + r)(s_I^D - s_I^R)(< 0)$.*

The proofs of all lemmas and propositions are in the Appendix.

Lemma 8.1(i) insists that when the donor has a higher marginal propensity to save than the recipient, the transfer raises the interest rate. The reason is that such an income transfer reduces the world's capital level and raises the interest rate. This result has already been mentioned in Galor and Polemar-chakis (1987). Lemma 8.1(ii) implies that irrespective of the relative sizes of the marginal propensities to save between countries, introducing a pension system in a country raises the interest rate. The reason why Lemma 8.1(ii) holds is that introducing a pension system in a country reduces a lifetime income at the steady state under the dynamic efficiency condition and leads to decrease the savings. Consequently, the pension contribution raises the interest rate by reducing capital accumulation in the world capital market.

Lemma 8.2(i) specifies the conditions under which the donor's per capita pension contribution decreases the effect of the transfer on the interest rate. The first condition, $d\Gamma/dP^i \leq 0$, guarantees dynamic stability even when we consider the marginal increase in pension contribution from zero because the sufficient condition for dynamic stability is $\Gamma < 0$, as shown in (8.11). $d\Gamma/dP^i < 0$ implies that Γ decreases as P^i increases and dynamic stability continues to hold. The second and the third conditions, $s_{II}^D = s_{II}^R$ and $s_{Ir}^D = s_{Ir}^R$, respectively, imply that the effects of lifetime income and the interest rate on the marginal propensity to save are the same between a donor and a recipient. Actually, Lemma 8.2 continues to hold even when s_{II}^i and s_{Ir}^i are approximately of the same size between a donor and a recipient. If the marginal propensity to save is not affected by income and the interest rate, these conditions are justified. More precisely, as the second-order derivatives of the savings depend on the second or higher derivatives of the utility function, their equivalence between a donor and a recipient is fundamentally attributed to the characteristics such that

the curvature of their utility functions is the same and not affected by variables. For example, the log-linear utility function always satisfies $s_{II}^D = s_{II}^R = 0$ and $s_{Ir}^D = s_{Ir}^R = 0$.

Lemma 8.2(i) claims that if the effect of lifetime income and the interest rate on the marginal propensity to save is similar between a donor and a recipient, the marginal increase in the donor's pension contribution decreases the effect of the transfer on the interest rate. On the other hand, Lemma 8.2(ii) suggests that whether the marginal increase in the recipient's per capita pension contribution decreases the effect of the transfer on the interest rate depends on the situation. Even when $d\Gamma/dP^i < 0$, $s_{II}^D = s_{II}^R$, and $s_{Ir}^D = s_{Ir}^R$, we need an additional condition to demonstrate that the marginal increase in the recipient's pension contribution decreases the effect of the transfer on the interest rate. The intuitive reason for the difference between Lemma 8.2(i) and (ii) is that the sign of dr/dT depends on the difference $(s_I^D - s_I^R)$, as shown in (A.8.2) and the sign of the effect of pension contribution is reversed between the donor and the recipient.

In the following analysis, we assume that $d\Gamma/dP^i \leq 0$, which implies that as the per capita pension contribution P^i in each country increases in the steady state, the sufficient condition for dynamic stability is more likely to hold. The details of this partial derivative are presented in Appendix A.8.3.

8.4.2 The effect on the donor's welfare

Now, we investigate the effect of the transfer on the donor's welfare in the steady state. By totally differentiating the indirect utility function of the donor country in the steady state, $V^D(I^D, r)$, with respect to (r, T) , we obtain the effect of a permanent transfer on the donor's welfare. By normalizing the marginal utility of income V_I^i , at the level of unity without loss of generality and noting $V_r^i = s^i/(1 + r)$ from Roy's identity, we obtain the following equa-

tion:

$$dV^D = V_I^D \left[w' dr - dT - \frac{(1+n)P^D}{(1+r)^2} dr \right] + V_r^D dr \quad (8.12)$$

$$= \underbrace{-dT}_{\text{(the direct income effect)}} - \underbrace{\left[k + \frac{(1+n)P^D}{(1+r)^2} - \frac{s^D}{1+r} \right] dr}_{\text{(the indirect effect on the interest rate)}}. \quad (8.13)$$

The first term in (8.12) indicates the effect of the transfer on welfare through the change in lifetime income. It consists of three parts. The first effect is brought about by the change in the wage rate caused by the change in the interest rate. The second effect is brought about by the decrease in income directly caused by the transfer. The third effect is the effect of the change in the interest rate on lifetime income via the pension contribution. On the other hand, the second term in (8.12) indicates the effect of the interest rate on savings, except for the already-mentioned effect that the interest rate has on the change in lifetime income. However, it is obvious that the first term in (8.12) is affected by the change in the interest rate. In (8.13), the equation is rearranged to distinguish the direct income effect of the transfer from the indirect effect that the transfer has on the interest rate. The first term in (8.13) is the direct income effect of the transfer; its sign is always negative. The second term is the overall indirect effect that the change in the interest rate has on welfare, irrespective of whether the effect is brought about via the change in lifetime income.

Using (8.13), we investigate how the marginal increase in the donor's per capita pension contribution P^D affects the donor's welfare. In the dynamically efficient steady state, with the exception of the golden rule when the donor has a higher marginal propensity to save than the recipient, we summarize the main result in the following proposition:

Proposition 8.1. *Suppose that the per capita pension contribution marginally increases from zero and assume that $d\Gamma/dP^i \leq 0$, $s_{II}^D = s_{II}^R$, and $s_{Ir}^D = s_{Ir}^R$. If the donor's saving level and marginal propensity to save are relatively high, the marginal increase in the donor's or the recipient's per capita pension contribution reduces the effect of the transfer on the donor's welfare. More precisely, if $s^D \geq (1+r)k$ and $s_I^D \geq [s_r^D - (1+r)k']/k$, then $(d^2V^D/dTdP^D)|_{P^D=P^R=0} < 0$ and $(d^2V^D/dTdP^R)|_{P^D=P^R=0} < 0$.*

Proposition 8.1 asserts that if both the donor's saving level and marginal propensity to save are relatively high, the marginal increase in the donor's pension contribution decreases the effect of the transfer on the donor's welfare. Such a decrease means the effect has not only a smaller absolute value but also a smaller value even if this value is already negative. The intuitive reason why the pension dilutes the impact of the transfer on welfare is that the marginal increase in the pension contribution reduces the impact of the transfer on the interest rate, as Lemma 8.2 shows; consequently, it reduces the indirect effect of the transfer on welfare through the change in the interest rate.

The more precise reason why Proposition 8.1 holds can be explained as follows: Because dV^D/dr is replaced with the bracketed term of (A.8.12), (A.8.12) implies that the pension only affects the indirect effect of the transfer, which is represented by $-(dV^D/dr)(dr/dT)$. The pension affects both dV^D/dr and dr/dT . The first-term and the second-term of the right hand in (A.8.15) and (A.8.16), respectively, correspond to the effect that the pension has on the effect of the interest rate on welfare, that is, $-(d^2V^D/dr dP^i)(dr/dT)$ and the effect that it has on the effect of the transfer on the interest rate, that is, $-(dV^D/dr)(d^2r/dT dP^i)$. As Lemma 8.1(i) shows, because $dr/dT > 0$, the first term becomes negative if $d^2V^D/dr dP^i > 0$. When the donor's marginal propensity to save is so high that $s_I^D \geq [s_r^D - (1+r)k']/k$ holds, $d^2V^D/dr dP^i > 0$ is satisfied and the first term becomes negative. On the other hand, because $d^2r/dT dP^i < 0$ when $P^D = P^R = 0$, the second term also becomes negative if $dV^D/dr < 0$. When the donor's savings are so high that $s^D \geq (1+r)k$ holds, the negative sign of the second term is guaranteed. Therefore, under the assumptions of Proposition 8.1, which implies that the donor's savings level and marginal propensity to save are relatively high, the effect of the transfer on the interest rate becomes negative as the pension marginally increases because the pension attenuates the effect of the transfer on the interest rate.

Introducing a pension system as an instrument to implement income redistribution in a donor country reduces the possibility that the donor improves its welfare by making the international transfer. This result suggests that the implementation of a domestic policy significantly affects the effectiveness of an international transfer. When the transfer enriches the donor, the donor has an incentive to make the transfer to a recipient. When the assumption of Proposition 8.1 holds, the transfer is less likely to cause donor enrichment. It suggests

that the pension system reduces the incentive of the donor country to make income transfers to other countries.

8.4.3 The effect on the recipient's welfare

We proceed to investigate the effect of the transfer on the recipient's welfare in the steady state. Following a procedure similar to that in Subsection 8.4.2, we derive the effect of the transfer on the recipient's welfare as follows:

$$dV^R = V_I^R \left[w' dr + dT - \frac{(1+n)P^R}{(1+r)^2} dr \right] + V_r^R dr \quad (8.14)$$

$$= \underbrace{dT}_{\text{(the direct income effect)}} - \underbrace{\left[k + \frac{(1+n)P^R}{(1+r)^2} - \frac{s^R}{1+r} \right] dr}_{\text{(the indirect effect on the interest rate)}}. \quad (8.15)$$

In the same manner as that previously described in (8.12), the first term in (8.14) indicates the effect of the change in lifetime income on welfare and is decomposed into three parts. The first effect is attributed to the change in the wage rate caused by the interest rate. The second effect is attributed to the decrease in income directly caused by the transfer. The third effect is the effect through which the change in the interest rate affects lifetime income via pension contribution. The second term in (8.14) indicates the effect of the interest rate on savings, except for the already-mentioned effect that the interest rate has on the change in lifetime income. In (8.15), the equation is rearranged to distinguish the direct income effect of the transfer from the indirect effect of the transfer on the interest rate. The first term in (8.15) is the direct income effect of the transfer; its sign is always positive. The second term is the overall indirect effect that the change in the interest rate has on welfare, irrespective of whether the effect is brought about via the change in lifetime income.

Using (8.15), we investigate how the marginal increase in the recipient's per capita pension contribution P^R affects the recipient's welfare. For the dynamically efficient steady state, with the exception of the golden rule, when the donor has a higher marginal propensity to save than the recipient, we summarize the main result in the following proposition:

Proposition 8.2. *Suppose that the per capita pension contribution marginally increases from zero and assume that $d\Gamma/dP^i \leq 0$, $s_{11}^D = s_{11}^R$, and $s_{1r}^D = s_{1r}^R$. Even if the donor's marginal propensity to save is sufficiently high, whether the marginal increase in the donor's or the recipient's per capita pension contribution reduces the effect of the transfer on the recipient's welfare depends on the situation. That is, the signs of $(d^2V^R/dTdP^R)|_{PD=PR=0}$ and $(d^2V^R/dTdPD)|_{PD=PR=0}$ are not determined.*

Unlike Proposition 8.1 on the effect on the donor's welfare, Proposition 8.2 asserts that even if the recipient's marginal propensity to save is relatively high, whether the marginal increase in the per capita pension contribution of a country decreases the effect of the transfer on the recipient's welfare is indeterminate. The difference in the results between Propositions 8.1 and 8.2 lies in the size of the recipient's saving level. The recipient with a lower marginal propensity to save becomes a capital debtor and the recipient's level of savings is relatively small and cannot satisfy the condition for the donor's savings level to be satisfied in Proposition 8.1. Proposition 8.2 suggests that in some cases, introducing a PAYG system might amplify the positive effect of the transfer on the recipient's welfare, while Proposition 8.1 insists that a PAYG pension hinders the positive effect that the transfer has on the donor's welfare.

Proposition 8.2 states that it is possible that introducing a PAYG pension system in a recipient country raises the possibility that the recipient improves its welfare by the international transfer. It suggests that in some cases a recipient country has an incentive to introduce a pension system because the introduction of a PAYG pension is likely to raise the effectiveness of the transfer. However, whether the marginal increase in the per capita PAYG pension contribution amplifies the effect of the transfer on the recipient's welfare depends on the situation. In the opposite case, if introducing the pension reduces the effectiveness of the transfer, the recipient will never introduce a pension system. A substantial international transfer might promote or hinder the development of an income redistribution policy between generations by the government of a recipient country.

8.5 Concluding remarks

In this chapter, we introduced a PAYG pension system in a two-country OLG model in order to investigate how PAYG pension system influences the effect of an international transfer on welfare in both a donor country and a recipient country. We demonstrated that a marginal increase in the per capita PAYG pension contribution might reduce the effect of the transfer on the donor's welfare, while in contrast, introducing a PAYG pension does not necessarily reduce the effect of the transfer on the recipient's welfare. Therefore, adoption of a PAYG pension system is likely to cause only a weak transfer paradox at best and reduce the welfare of both the donor and the recipient. When the marginal propensity to save in the donor country is higher than that in the recipient country in the dynamically efficient economy, except at the golden rule, the decrease in lifetime income as a result of the per capita PAYG pension contribution inhibits the rise in the interest rate associated with the transfer and as a result, the adoption of a PAYG pension becomes less beneficial for the donor, but it might be more beneficial for the recipient. We showed a situation in which a PAYG pension crowds out savings and capital accumulation. Our study has for the first time demonstrated that a PAYG pension reduces the effect of an international transfer and has clarified how the indirect effect of the transfer on the interest rate is affected by the marginal increase in the PAYG pension contribution in an OLG model.

Our model can be extended in several directions. The first extension is the endogeneity of fertility. Throughout this chapter, we have assumed that the population growth rate is constant. However, if the fertility rate were endogenized, the per capita transfer level as well as the per capita PAYG pension contribution could affect the fertility of each family. As the per capita return on a PAYG pension is affected by the population growth rate which is determined endogenously, the effect of an international transfer on welfare would be changed via a change in the population growth rate. The second extension is incorporation of human capital accumulation into the model. It might also be worthwhile to introduce human capital formation in children into the utility function of their parents. When human capital accumulates, which is a fundamental source of economic growth, human capital formation associated with an international transfer will change the growth rate of the two countries dras-

tically. By scrutinizing what the transfer is used for, we can further investigate the effect of the transfer on not only the interest rate but also the growth rate because the low level of human capital in developing countries is a major concern for economic development.

Appendix

A.8.1 The proof of Lemma 8.1

From (8.7) and $r \equiv r_t = r_{t+1}$, the world capital market equilibrium in the steady state is rewritten as follows:

$$2(1+n)k(r) = s^D(I^D, r) + s^R(I^R, r). \quad (\text{A.8.1})$$

(i) By the definition of a transfer from a donor to a recipient, $dI^D/dT = -1$ and $dI^R/dT = 1$. By differentiating (A.8.1) with respect to r and T , we obtain the following equation:

$$\Gamma dr = -s_I^D dT + s_I^R dT \Leftrightarrow \frac{dr}{dT} = -\frac{s_I^D - s_I^R}{\Gamma}. \quad (\text{A.8.2})$$

$\Gamma < 0$ holds in the steady state under the dynamically stable condition, (8.11). Thus, if $s_I^D > s_I^R$, $dr/dT > 0$.

(ii) By the definition of lifetime income I^i , $dI^i/dP^i = -(r-n)/(1+r) < 0$. By differentiating (A.8.1) with respect to r and P^i , we obtain the following equation:

$$\Gamma dr = -s_I^i \left(\frac{r-n}{1+r} \right) dP^i \Leftrightarrow \frac{dr}{dP^i} = -\frac{(r-n)s_I^i}{(1+r)\Gamma} > 0. \quad (\text{A.8.3})$$

□

A.8.2 The proof of Lemma 8.2

Differentiating dr/dT , which is given by (A.8.2), with respect to P^i , we obtain the following equation:

$$\begin{aligned} \frac{d^2 r}{dT dP^D} = & \frac{s_{II}^D \left[w' \frac{dr}{dP^D} - \frac{r-n}{1+r} - \frac{(1+n)P^D}{(1+r)^2} \frac{dr}{dP^D} \right] + s_{Ir}^D \frac{dr}{dP^D} - s_{II}^R \left[w' \frac{dr}{dP^D} - \frac{(1+n)P^R}{(1+r)^2} \frac{dr}{dP^D} \right] - s_{Ir}^R \frac{dr}{dP^D}}{\Gamma} \\ & + \frac{(s_{II}^D - s_{II}^R) \frac{d\Gamma}{dP^D}}{\Gamma^2}, \end{aligned} \quad (\text{A.8.4})$$

$$\begin{aligned} \frac{d^2 r}{dT dP^R} = & \frac{s_{II}^D \left[w' \frac{dr}{dP^R} - \frac{(1+n)P^D}{(1+r)^2} \frac{dr}{dP^R} \right] + s_{Ir}^D \frac{dr}{dP^R} - s_{II}^R \left[w' \frac{dr}{dP^R} - \frac{r-n}{1+r} - \frac{(1+n)P^R}{(1+r)^2} \frac{dr}{dP^R} \right] - s_{Ir}^R \frac{dr}{dP^R}}{\Gamma} \\ & + \frac{(s_{II}^D - s_{II}^R) \frac{d\Gamma}{dP^R}}{\Gamma^2}. \end{aligned} \quad (\text{A.8.5})$$

When we evaluate (A.8.4) and (A.8.5) at $P^D = P^R = 0$, and noting that $(r-n)/(1+r) = -(\Gamma/s_I^i)(dr/dP^i)$ and $w' = -k$, the following equations are obtained:

$$\begin{aligned} \frac{d^2 r}{dT dP^D} \Big|_{P^D=P^R=0} &= \frac{1}{\Gamma^2} \left\{ \underbrace{\left[\frac{s_{II}^D \Gamma}{s_I^D} + s_{Ir}^D - s_{Ir}^R - (s_{II}^D - s_{II}^R)k \right]}_{(-) \text{ or } 0} \underbrace{\frac{(r-n)s_I^D}{1+r}}_{(+)} + \underbrace{(s_{II}^D - s_{II}^R)}_{(+)} \underbrace{\frac{d\Gamma}{dP^D}}_{(-)} \right\}, \end{aligned} \quad (\text{A.8.6})$$

$$\begin{aligned} \frac{d^2 r}{dT dP^R} \Big|_{P^D=P^R=0} &= \frac{1}{\Gamma^2} \left\{ \underbrace{\left[-\frac{s_{II}^R \Gamma}{s_I^R} + s_{Ir}^D - s_{Ir}^R - (s_{II}^D - s_{II}^R)k \right]}_{(+ \text{ or } 0)} \underbrace{\frac{(r-n)s_I^R}{1+r}}_{(+)} + \underbrace{(s_{II}^D - s_{II}^R)}_{(+)} \underbrace{\frac{d\Gamma}{dP^R}}_{(-)} \right\}. \end{aligned} \quad (\text{A.8.7})$$

If $s_{II}^D = s_{II}^R$ and $s_{Ir}^D = s_{Ir}^R$, $(d^2r/dT dP^D)|_{P^D=P^R=0}$ is negative from (A.8.6), while only when $d\Gamma/dP^R < (r-n)s_{II}^R\Gamma/(1+r)(s_I^D - s_I^R)(< 0)$, $(d^2r/dT dP^R)|_{P^D=P^R=0}$ is negative from (A.8.7). \square

A.8.3 The derivation of $d\Gamma/dP^D$

From (8.11), the dynamic stability condition in the steady state can be rewritten as follows:

$$\Gamma = 2(1+n)k' + \frac{(1+n)P^D}{(1+r)^2} s_I^D + \frac{(1+n)P^R}{(1+r)^2} s_I^R - s_r^D - s_r^R - s_I^D w' - s_I^R w'. \quad (\text{A.8.8})$$

Differentiating (A.8.8) with respect to P^D , we obtain the following equation:

$$\begin{aligned} \frac{d\Gamma}{dP^D} = & 2(1+n)k'' \frac{dr}{dP^D} + \frac{1+n}{(1+r)^3} \left(1+r-2P^D \frac{dr}{dP^D} \right) s_I^D \\ & - \frac{(1+n)P^D}{(1+r)^2} \left\{ s_{II}^D \left[k \frac{dr}{dP^D} + \frac{r-n}{1+r} + \frac{(1+n)P^D}{(1+r)^2} \frac{dr}{dP^D} \right] - s_{Ir}^D \frac{dr}{dP^D} \right\} \\ & - 2 \frac{(1+n)P^R}{(1+r)^3} s_I^R \frac{dr}{dP^D} \\ & - \frac{(1+n)P^R}{(1+r)^2} \left\{ s_{II}^R \left[k \frac{dr}{dP^D} + \frac{(1+n)P^R}{(1+r)^2} \frac{dr}{dP^D} \right] - s_{Ir}^R \frac{dr}{dP^D} \right\} \\ & + \left\{ s_{rI}^D \left[k \frac{dr}{dP^D} + \frac{r-n}{1+r} + \frac{(1+n)P^D}{(1+r)^2} \frac{dr}{dP^D} \right] - s_{rr}^D \frac{dr}{dP^D} \right\} \\ & + \left\{ s_{rI}^R \left[k \frac{dr}{dP^D} + \frac{(1+n)P^R}{(1+r)^2} \frac{dr}{dP^D} \right] - s_{rr}^R \frac{dr}{dP^D} \right\} \\ & + s_I^D k' \frac{dr}{dP^D} - k \left\{ s_{II}^D \left[k \frac{dr}{dP^D} + \frac{r-n}{1+r} + \frac{(1+n)P^D}{(1+r)^2} \frac{dr}{dP^D} \right] - s_{Ir}^D \frac{dr}{dP^D} \right\} \\ & + s_I^R k' \frac{dr}{dP^D} - k \left\{ s_{II}^R \left[k \frac{dr}{dP^D} + \frac{(1+n)P^R}{(1+r)^2} \frac{dr}{dP^D} \right] - s_{Ir}^R \frac{dr}{dP^D} \right\}. \end{aligned} \quad (\text{A.8.9})$$

When marginally evaluating at $P^D = 0$ and $P^R = 0$, (A.8.9) is arranged as follows:

$$\begin{aligned} \frac{d\Gamma}{dP^D} \Big|_{P^D=P^R=0} &= 2(1+n)k'' \frac{dr}{dP^D} + \frac{1+n}{(1+r)^2} s_I^D + s_{rI}^D \left(k \frac{dr}{dP^D} + \frac{r-n}{1+r} \right) \\ &\quad - s_{rr}^D \frac{dr}{dP^D} + s_{rI}^R k \frac{dr}{dP^D} - s_{rr}^R \frac{dr}{dP^D} \\ &\quad + s_I^D k' \frac{dr}{dP^D} - k \left[s_{II}^D \left(k \frac{dr}{dP^D} + \frac{r-n}{1+r} \right) - s_{Ir}^D \frac{dr}{dP^D} \right] \\ &\quad + s_I^R k' \frac{dr}{dP^D} - k \left(s_{II}^R k \frac{dr}{dP^D} - s_{Ir}^R \frac{dr}{dP^D} \right). \end{aligned} \quad (\text{A.8.10})$$

By arranging (A.8.10) with $dr/dP^D = -(r-n)s_I^D/(1+r)\Gamma > 0$, we obtain the following equation:

$$\begin{aligned} \frac{d\Gamma}{dP^D} \Big|_{P^D=P^R=0} &= \\ &\left\{ 2(1+n)k'' + \underbrace{(s_I^D + s_I^R)k'}_{(-)} - \left[(s_{II}^D + s_{II}^R)k^2 - 2(s_{Ir}^D + s_{Ir}^R)k + s_{rr}^D + s_{rr}^R \right] \right. \\ &\quad \left. - \left[\frac{s_{rI}^D - s_{II}^D k}{s_I^D} + \frac{1+n}{(1+r)(r-n)} \right] \underbrace{\Gamma}_{(-)} \right\} \times \underbrace{\frac{dr}{dP^D}}_{(+)}. \end{aligned} \quad (\text{A.8.11})$$

The signs follow from $k' = 1/f'' < 0$, $\Gamma < 0$, and $dr/dP^D > 0$, from Lemma 8.1(ii). The sufficient condition under which $(d\Gamma/dP^D)|_{P^D=P^R=0}$ is negative is that the expression inside the braces of the left-hand side of (A.8.11) is negative. A similar procedure can be used to derive $d\Gamma/dP^R$.

A.8.4 The proof of Proposition 8.1

By rearranging (8.13), we obtain the following equation:

$$\frac{dV^D}{dT} = -1 - \left[k + \frac{(1+n)P^D}{(1+r)^2} - \frac{s^D}{1+r} \right] \frac{dr}{dT}. \quad (\text{A.8.12})$$

By differentiating (A.8.12) with respect to P^D and P^R , respectively, we obtain the second-order cross-partial derivatives:

$$\begin{aligned} \frac{d^2 V^D}{dT dP^D} = & - \left\{ \left[k' - \frac{2(1+n)P^D}{(1+r)^3} + \frac{s^D}{(1+r)^2} \right] \frac{dr}{dP^D} + \frac{1+n}{(1+r)^2} \right. \\ & \left. - \frac{s_I^D \left[w' \frac{dr}{dP^D} - \frac{r-n}{1+r} - \frac{(1+n)P^D}{(1+r)^2} \frac{dr}{dP^D} \right] + s_r^D \frac{dr}{dP^D}}{1+r} \right\} \frac{dr}{dT} \\ & - \left[k + \frac{(1+n)P^D}{(1+r)^2} - \frac{s^D}{1+r} \right] \frac{d^2 r}{dT dP^D}, \end{aligned} \quad (\text{A.8.13})$$

$$\begin{aligned} \frac{d^2 V^D}{dT dP^R} = & - \left\{ \left[k' - \frac{2(1+n)P^D}{(1+r)^3} + \frac{s^D}{(1+r)^2} \right] \frac{dr}{dP^R} \right. \\ & \left. - \frac{s_I^D \left[w' \frac{dr}{dP^R} - \frac{(1+n)P^D}{(1+r)^2} \frac{dr}{dP^R} \right] + s_r^D \frac{dr}{dP^R}}{1+r} \right\} \frac{dr}{dT} \\ & - \left[k + \frac{(1+n)P^D}{(1+r)^2} - \frac{s^D}{1+r} \right] \frac{d^2 r}{dT dP^R}. \end{aligned} \quad (\text{A.8.14})$$

Evaluating (A.8.13) and (A.8.14) at $P^D = P^R = 0$, we obtain the following equations:

$$\begin{aligned} \frac{d^2 V^D}{dT dP^D} \Big|_{P^D=P^R=0} = & - \frac{\overbrace{1+n+(r-n)s_I^D}^{(+)} + \left\{ s^D + (1+r) \left[k s_I^D - s_r^D + (1+r)k' \right] \right\} \overbrace{\frac{dr}{dP^D}}^{(+)} \Big|_{P^D=P^R=0}}{(1+r)^2} \underbrace{\frac{dr}{dT}}_{(+)} \Big|_{P^D=P^R=0} \\ & - \underbrace{\frac{(1+r)k - s^D}{1+r} \frac{d^2 r}{dT dP^D}}_{(-)} \Big|_{P^D=P^R=0}, \end{aligned} \quad (\text{A.8.15})$$

$$\begin{aligned}
 & \frac{d^2 V^D}{dT dP^R} \Big|_{P^D=P^R=0} = \\
 & - \frac{s^D + (1+r)[ks_I^D - s_r^D + (1+r)k']}{(1+r)^2} \underbrace{\frac{dr}{dP^R}}_{(+)} \Big|_{P^D=P^R=0} \times \underbrace{\frac{dr}{dT}}_{(+)} \Big|_{P^D=P^R=0} \\
 & - \underbrace{\frac{(1+r)k - s^D}{1+r}}_{(-)} \underbrace{\frac{d^2 r}{dT dP^R}}_{(-)} \Big|_{P^D=P^R=0}. \tag{A.8.16}
 \end{aligned}$$

Note that $dr/dT > 0$, $dr/dP^D > 0$, and $(d^2 r/dT dP^i)|_{P^D=P^R=0} < 0$ from Lemmas 8.1 and 8.2, respectively. From (A.8.15) and (A.8.16), we confirm that if $s^D \geq (1+r)k$ and $s_I^D \geq [s_r^D - (1+r)k']/k$, both $(d^2 V^D/dT dP^D)|_{P^D=P^R=0}$ and $(d^2 V^D/dT dP^R)|_{P^D=P^R=0}$ are negative. \square

A.8.5 The proof of Proposition 8.2

The proof procedure is similar to that of Proposition 8.1. By rearranging (8.15), we obtain the following equation:

$$\frac{dV^R}{dT} = 1 - \left[k + \frac{(1+n)P^R}{(1+r)^2} - \frac{s^R}{1+r} \right] \frac{dr}{dT}. \tag{A.8.17}$$

By differentiating (A.8.17) with respect to P^R and P^D , respectively, we obtain the second-order cross-partial derivatives. Evaluating them at $P^D = P^R = 0$,

we obtain the following equations:

$$\begin{aligned}
 & \frac{d^2 V^R}{dT dP^R} \Big|_{P^D=P^R=0} = \\
 & - \frac{\overbrace{1+n+(r-n)s_I^R}^{(+)} + \{s^R + (1+r)[ks_I^R - s_r^R + (1+r)k']\}}{(1+r)^2} \\
 & \times \underbrace{\frac{dr}{dP^R}}_{(+)} \Big|_{P^D=P^R=0} \underbrace{\frac{dr}{dT}}_{(+)} \Big|_{P^D=P^R=0} - \underbrace{\frac{(1+r)k - s^R}{1+r}}_{(+)} \underbrace{\frac{d^2 r}{dT dP^R}}_{(-)} \Big|_{P^D=P^R=0},
 \end{aligned} \tag{A.8.18}$$

$$\begin{aligned}
 & \frac{d^2 V^R}{dT dP^D} \Big|_{P^D=P^R=0} = \\
 & - \frac{s^R + (1+r)[ks_I^R - s_r^R + (1+r)k']}{(1+r)^2} \times \underbrace{\frac{dr}{dP^D}}_{(+)} \Big|_{P^D=P^R=0} \underbrace{\frac{dr}{dT}}_{(+)} \Big|_{P^D=P^R=0} \\
 & - \underbrace{\frac{(1+r)k - s^R}{1+r}}_{(+)} \underbrace{\frac{d^2 r}{dT dP^D}}_{(-)} \Big|_{P^D=P^R=0}.
 \end{aligned} \tag{A.8.19}$$

Note that $dr/dT > 0$, $dr/dP^R > 0$, and $(d^2 r/dT dP^i)|_{P^D=P^R=0} < 0$ from Lemmas 8.1 and 8.2, respectively. By the capital market equilibrium and the dynamically efficient condition, $s^R < (1+n)k \leq (1+r)k$. Thus, $(1+r)k - s^R > 0$ always holds irrespective of the size of s^R . Therefore, the second terms of (A.8.18) and (A.8.19) are necessarily positive, while the sign of the first terms is not determined. For example, when $s_I^R \geq [s_r^R - (1+r)k']/k$, which is a similar condition to that used in Proposition 8.1, the first terms of (A.8.18) and (A.8.19) are negative and therefore the signs of both (A.8.18) and (A.8.19) depend on the relative sizes of the first term and the second term. \square

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