Syncretic Argumentation by means of Lattice Homomorphism and Fusion

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In his influential work on the abstract argumentation framework [5], Dung introduced the notion of "acceptability" of arguments that has played the most significant role in specifying the various kinds of semantics for argumentation: admissible, stable, preferred, grounded, complete. The abstract argumentation framework is specified as follows.

Definition 1 (**Argumentation Framework** [5]) *An argumentation framework is a pair* $AF =< AR, attacks >$ *where* AR *is a set of arguments, and attacks is a binary relation on* AR*, i. e.,* $attacks \subseteq AR \times AR$*.*

In Dung's theory of argumentation, we are not concerned with the internal structure of arguments and why and how arguments attack others. Everything is abstracted away in this way. This abstraction, however, was a good starting point for developing the formal argumentation semantics that is to capture what acceptable or admissible arguments are and the whole of justified arguments.

Definition 2 (**Acceptability and Admissibility** [5])

1. *An argument* $A \in AR$ *is acceptable w.r.t. a set* S *of arguments iff for each argument* $B \in AR$*: if* B *attacks* A *then* B *is attacked by* S*.*

2. *A conflict-free set of arguments* S *is admissible iff each argument in* S *is acceptable w.r.t.* S*.*

The notion of acceptability is a counterpart of the phenomenon observed in our daily argumentation and originates from an old saying, "The one who has the last word laughs best", as stated by Dung. It is an empirical social truth or wisdom that has been evolved in various cultural sphere over generations and considered useful by people. It is remarkable and suggestive that Dung's theory of argumentation had started from such a daily but philosophical observation. This might be because argumentation is humans' most normal but intelligent action for thought and communication by language.

There, however, can be a plurality of sets of justified arguments in argumentation as mentioned above, contrasting with the semantics of an ordinary logic that is to be uniquely given by the Tarskian semantics, for example. Naturally, this reflects a figure of argumentation, a decisive difference from a logic. The preferred semantics, for example, is defined as follows

Definition 3 (**Preferred Extension** [5]) *A preferred extension of an argumentation framework AF is a maximal (w.r.t. set inclusion) admissible set of AF.*

We developed the Logic of Multiple-valued Argumentation (LMA) [13] that is a variant of Dung's abstract argumentation framework concretized in such a way that the arguments are represented in terms of the knowledge representation language, Extended Annotated Logic Programming (EALP) and the attack relation consists of various sorts of attack such as rebuttal, undercut, defeat, etc. with three kinds of negation: ontological negation ($\sim$), default negation (**not**), and epistemological negation ($\neg$) that play a role of momentum in argumentation. EALP is an extension of ELP (Extended Logic Programming), and a very expressive knowledge representation language in which agents can express their knowledge and belief with annotations as truth-values that allow to represent various kinds of uncertainty of information. In a word, LMA is an argumentation framework that allows agents to participate in uncertain argumentation under uncertain knowledge bases if once the common annotation is shared among agents. Put it differently, agents are assumed to have a homogeneous recognition for propositions with the same annotation as truth-values.

In this paper, we make a clean break with this assumption, directing to a more natural but complex settings of argumentation named "Syncretic Argumentation". By the term "syncretic argumentation", it is meant to be such an argumentation that each agent can have its own knowledge base, based on its own epistemology, and participate in argumentation with it. More specifically, each agent can attend the argumentation in which arguments are represented in EALP and annotated with its own truth-values which are assumed to represent modes of truth or epistemic states of propositions [13]. The syncretic argumentation is a new framework that allows agents to argue about issues of mutual interest even when they have their own annotations, for example, agent A has two values $\mathcal{TWO} = \{f, t\}$ as annotation (this is typical in the Occident), and agent B has 4-values $\mathcal{FOUR} = \{\bot, \mathbf{t}, \mathbf{f}, \top\}$ as annotation (this is called tetralemma in the early philosophical literature and text of Buddhism [11][12]). This reflects an attitude against unilateralism, so that one agent world may not be forced to assimilate to another unilaterally. We realize the goal by means of the lattice homomorphism since the mathematical structure of annotations is a complete lattice and the homomorphism is a mathematical apparatus convenient to syncretize the difference of epistemic states of propositions.

Definition 4 (Homomorphism [4]) *Let* $< L, \vee_L, \wedge_L, \leq_L >$ *and* $< K, \vee_K, \wedge_K, \leq_K >$ *be complete lattices. A map* $h : L \rightarrow K$ *is said to be a homomorphism if h satisfies the following conditions: for all* $a, b \in L$,

- $h(a \vee_L b) = h(a) \vee_K h(b)$
- $h(a \wedge_L b) = h(a) \wedge_K h(b)$
- $h(0_L) = 0_K$ *for the least element*
- $h(1_L) = 1_K$ *for the greatest element*

Example 1 *Let us consider two typical lattices: the two-valued complete lattice* $\mathcal{TWO}$ *and the four-valued one* $\mathcal{FOUR}$. *The former is typical in the West,*

and the latter in the early philosophical literature and text of Buddhism [11]. $\mathcal{TWO} =< \{f, t\}, \vee, \wedge, \leq >$, *where* $f \leq t$, *and* $\mathcal{FOUR} =< \{\bot, \boldsymbol{t}, \boldsymbol{f}, \top\}, \vee, \wedge, \leq >$, *where* $\forall x, y \in \{\bot, \boldsymbol{t}, \boldsymbol{f}, \top\}$ $x \leq y \Leftrightarrow x = y \vee x = \bot \vee y = \top$.

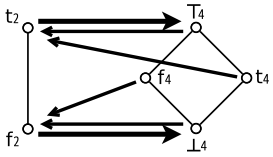

Fig. 1: Homomorphism: $h1 : \mathcal{TWO} \rightarrow \mathcal{FOUR}$ and $h2 : \mathcal{FOUR} \rightarrow \mathcal{TWO}$

With the lattice homomorphism above, we will illustrate how agents who have their own epistemology can reach an agreement and accept arguments through the grounded semantics or the dialectical proof theory of LMA [13].

Example 2 *Suppose two agents A and B have the following knowledge bases respectively.*

$K_A = \{\ a : t_2 \leftarrow,\ \sim b : t_2 \leftarrow,\ c : t_2 \leftarrow,\ \sim d : t_2 \leftarrow\ \}$

$K_B = \{\ \sim a : t_4 \leftarrow,\ b : t_4 \leftarrow,\ \sim c : \top_4 \leftarrow,\ d : \bot_4 \leftarrow,\ e : t_4 \leftarrow g : f_4,\ g : t_4 \leftarrow\ \}$

Then the agents A and B can make the following set of arguments $Args_{K_A}$ *and* $Args_{K_B}$ *from their knowledge bases respectively. (See [13] for the precise definition of arguments in LMA.)*

$Args_{K_A} = \{\ [a : t_2 \leftarrow],\ [\sim b : t_2 \leftarrow],\ [c : t_2 \leftarrow],\ [\sim d : t_2 \leftarrow]\ \}$

$Args_{K_B} = \{\ [\sim a : t_4 \leftarrow],\ [b : t_4 \leftarrow],\ [\sim c : \top_4 \leftarrow],\ [d : \bot_4 \leftarrow],\ [g : t_4 \leftarrow]\ \}$

The agents first assimilate their knowledge bases above to each other by the lattice homomorphism in Fig. 1, and compute justified arguments from them using the grounded semantics or the dialectical proof theory [13], in each direction of the homomorphism as follows.

[1] *Lattice homomorphism h1:* $\mathcal{TWO} \rightarrow \mathcal{FOUR}$ *(simply written as* $\mathcal{T} \rightarrow \mathcal{F}$*)*

$h1(K_A) = \{\ a : \top_4 \leftarrow,\ \sim b : \top_4 \leftarrow,\ c : \top_4 \leftarrow,\ \sim d : \top_4 \leftarrow\ \}$

$K_B = \{\ \sim a : t_4 \leftarrow,\ b : t_4 \leftarrow,\ \sim c : \top_4 \leftarrow,\ d : \bot_4 \leftarrow,\ e : t_4 \leftarrow g : f_4,\ g : t_4 \leftarrow\ \}$

$Args_{h1(K_A)} = \{\ [a : \top_4 \leftarrow],\ [\sim b : \top_4 \leftarrow],\ [c : \top_4 \leftarrow],\ [\sim d : \top_4 \leftarrow]\ \}$

$Args_{K_B} = \{\ [\sim a : t_4 \leftarrow],\ [b : t_4 \leftarrow],\ [\sim c : \top_4 \leftarrow],\ [d : \bot_4 \leftarrow],\ [g : t_4 \leftarrow]\}$

Note that $Args_{h1(K_A)} = h1(Args_{K_A})$ *since the homomorphism preserves the lattice ordering. From these argument sets, the agents can have the following set of justified arguments.*

$Justified\_Args_{\mathcal{T} \rightarrow \mathcal{F}} = \{\ [\sim b : \top_4 \leftarrow],\ [\sim d : \top_4 \leftarrow],\ [b : t_4 \leftarrow],\ [d : \bot_4 \leftarrow],\ [g : t_4 \leftarrow]\ \}$

[2] *Lattice homomorphism h2:* $\mathcal{FOUR} \rightarrow \mathcal{TWO}$ *(simply written as* $\mathcal{F} \rightarrow \mathcal{T}$*)*

$K_A = \{\ a : t_2 \leftarrow,\ \sim b : t_2 \leftarrow,\ c : t_2 \leftarrow,\ \sim d : t_2 \leftarrow\ \}$

$h2(K_B) = \{\ \sim a : t_2 \leftarrow,\ b : t_2 \leftarrow,\ \sim c : t_2 \leftarrow,\ d : f_2 \leftarrow,\ e : t_2 \leftarrow g : f_2,\ g : t_2 \leftarrow\ \}$

$$Args_{K_A} = \{\ [a : t_2 \leftarrow],\ \ [\sim b : t_2 \leftarrow],\ \ [c : t_2 \leftarrow],\ \ [\sim d : t_2 \leftarrow]\ \}$$

$$Args_{h2(K_B)} = \{\ [\sim a : t_2 \leftarrow],\ \ [b : t_2 \leftarrow],\ \ [\sim c : t_2 \leftarrow],\ \ [d : f_2 \leftarrow],\ \ [g : t_2 \leftarrow],\ [e : t_2 \leftarrow g : f_2,\ \ g : t_2 \leftarrow]\}$$

Note that $Args_{h2(K_B)} \neq h2(Args_{K_B})$ *in case of the homomorphism* $h2$ *since* $[e : t_2 \leftarrow g : f_2,\ \ g : t_2 \leftarrow]$ *has been qualified as an argument by* $h2$ *although its original form* $[e : t_4 \leftarrow g : f_4,\ \ g : t_4 \leftarrow]$ *in* K_B *is not an argument. From these argument sets, the agents can have the following set of justified arguments.*

$$Justified\_Args_{\mathcal{F}\rightarrow\mathcal{T}} = \{\ [\sim d : t_2 \leftarrow],\ \ [d : f_2 \leftarrow],\ \ [g : t_2 \leftarrow],\ \ [e : t_2 \leftarrow g : f_2,\ g : t_2 \leftarrow]\ \}$$

Through the two-way homomorphism, we had two different sets of justified arguments: $Justified\_Args_{\mathcal{T}\rightarrow\mathcal{F}}$ and $Justified\_Args_{\mathcal{F}\rightarrow\mathcal{T}}$. Next, we are interested in defining a set of justified arguments as a "common good" that is acceptable for both agents. Actually, we have three kinds of agent attitudes or criteria to chose it from among two different sets of justified arguments[7]. The following is the notion of skeptically justified arguments.

Definition 5 (Skeptically justified arguments)

- *An argument* a *in* $Args_{K_A}$ *is skeptically justified iff* $a \in Justified\_Args_{\mathcal{F}\rightarrow\mathcal{T}}$ *and* $h1(a) \in Justified\_Args_{\mathcal{T}\rightarrow\mathcal{F}}$.
- *An argument* a *in* $Args_{K_B}$ *is skeptically justified iff* $a \in Justified\_Args_{\mathcal{T}\rightarrow\mathcal{F}}$ *and* $h2(a) \in Justified\_Args_{\mathcal{F}\rightarrow\mathcal{T}}$.

This is a fair and unbiased notion of justified arguments in the sense that the both sides can attain a perfect consensus by the two-way homomorphism. Morally, it reflects such a compassionate attitude that agents look from the other agents' viewpoint, or place themselves in the other agents' position.

The syncretic argumentation is obviously a radical departure from the past argumentation frameworks [1][9] [10] in the sense that they are basically frameworks using two-valued knowledge base, or simply a fixed multi-valued one [2]. Here we should emphasize that our approach to the syncretic argumentation is not only technically new but also has a profound philosophy that underlies our syncretic argumentation. They are,

- Golden Rule in the ethics of reciprocity(of positive form): "Treat others (only) as you consent to being treated in the same situation." [6]
- Confucius' Golden Rule(of negative form): "Never impose on others what you would not choose for yourself". " [3]

and may be said to be ethical in contrast with Dung's background idea on the acceptability.

Next we turn to another construction of syncretic argumentation since there are cases where lattice homomorphism does not exist. We devise the new notions: the lattice fusion operator and fusion lattice that are induced through the lattice product, and can be considered as providing a natural way to syncretize the difference of epistemic states of propositions. Figure 3 shows an example of the fusion lattice constructed from two lattices: $\mathcal{TWO}$ and $\mathcal{FOUR}$, via. their

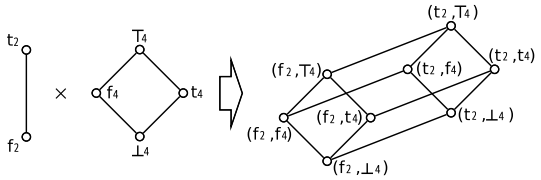

Fig. 2: Product of $\mathcal{TWO}$ and $\mathcal{FOUR}$

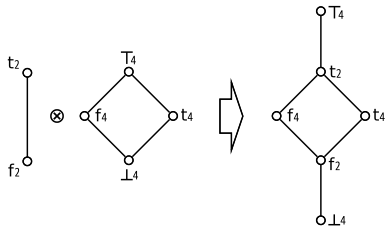

Fig. 3: Fusion of $\mathcal{TWO}$ and $\mathcal{FOUR}$

product depicted in Figure 2. The fusion lattice provides for agents a common argumentation field where agents can start syncretic argumentation using their knowledge bases with annotation specified in the fusion lattice. Our approach to fusing lattices has such advantages as majority principle, order preserving and commutativity (for the details, see [8]).

Agents have to live in the multi-cultural computer-networked virtual society as well as humans living in the multi-cultural society. This implies that agents also get involved in arguing about issues of mutual interest on the basis of their own belief and knowledge. But, if they insisted only on their epistemology, we would lose chances to interact or communicate with each other. The enterprise in this paper is an attempt to avoid such a cul-de-sac appearing even in argument-based problem solving.

There has been no work on argumentation frameworks in which each agent has its own knowledge representation language, its own epistemology, and its own argumentation framework. They have been all common to agents who participate in argumentaion. Our work goes to the polar opposite direction from the perspective of the past works.

The general golden rule has its roots in a wide range of world cultures: ancient Greece, ancient Egypt, ancient China, etc. and almost all religion and philosophy such as Buddhism, Christianity, Islam, Judaism, Confucianism, etc. The human history accepts it as a universal standard with which we resolve conflicts among different civilization and culture. Although the Golden Rule has had its critics on the one hand, the key element of it is that a person attempting to live by this rule should treat all people, not just members of his or her in-group, with consideration and compassion. Therefore it is reasonable for us to employ it and formalize the syncretic argumentation under the general golden rule as the rationale of our attempt. Our bi-directional homomorphism (operation) between different annotations and the fusion lattice approach could

realize the key and may be said to the general golden rule itself in the syncretic argumentation. We hope that the syncretic argumentation could lead to overcome and bridge the gulf of incommensurability among different cultural agents, and result in fair and equal argumentation without unilateral imposition.

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