

# Sinusoidal phase modulating laser diode interferometer with feedback control system to eliminate external disturbance

Osami Sasaki, Kazuhide Takahashi, and Takamasa Suzuki

Niigata University, Faculty of Engineering  
8050 Ikarashi 2, Niigata-shi, Japan

## 1. ABSTRACT

We propose a sinusoidal phase modulating laser diode interferometer which is insensitive to vibrations of optical devices and fluctuations in the optical wavelength of the laser diode. We analyze the sinusoidal phase modulation in a laser diode interferometer, and describe the principle of the feedback control of the injection current of the laser diode to eliminate the fluctuations in the phase of the interference signal caused by external disturbances. We construct two sinusoidal phase modulating interferometers for movement measurements and surface profile measurements, respectively. The experimental results make it clear that the interferometers can be used in mechanically noisy circumstances.

## 2. INTRODUCTION

Heterodyne interferometry and fringe scanning interferometry have been widely used to measure surface profiles with high accuracy. Recently laser diodes (LDs) have been incorporated into heterodyne interferometers<sup>1</sup> and fringe scanning interferometers<sup>2</sup> as light sources and phase modulators. As another interferometric technique, we proposed sinusoidal phase modulating (SPM) interferometry, in which surface profiles are obtained with the Fourier transform method<sup>3</sup> and the integrating-bucket method.<sup>4</sup> We also reported the method of movement measurements in SPM interferometry.<sup>5</sup>

In this paper we describe a SPM laser diode interferometer which is insensitive to vibrations of optical devices and fluctuations in the optical wavelength of the LD. Sinusoidal phase modulated interference signal is generated by modulating the injection current of the LD with a sinusoidal wave signal. The phase modulation in SPM interferometry is very simple compared with those in heterodyne interferometry and fringe scanning interferometry. The signal that is a trigonometric function of the phase difference between the object and reference waves can be easily obtained from the sinusoidal phase modulated interference signal. This signal is used as a feedback signal in controlling the injection current of the LD to reduce the fluctuations in the phase of the interference signal caused by external disturbances. These special characteristics of the sinusoidal phase modulated interference signal allow us to construct an interferometer with the feedback control system to eliminate the external disturbances.

The sinusoidal phase modulation in a LD interferometer is theoretically analyzed in Sec.2, and the principle of the feedback control of the injection current is described in Sec.3. In Sec.4, we describe a SPM interferometer for movement measurements in which we obtain a feedback signal from the interference signal generated with a stationary object. We measure movements of a piezoelectric transducer without suffering from external disturbances. In Sec.5, we also construct a SPM interferometer for surface profile measurements, and measure surface profiles of diamond-turned aluminum disks. The measurement repeatability is greatly improved by the feedback control of the injection current. The experimental results make it clear that the SPM interferometers presented here can be used in mechanically noisy circumstances.

## 3. SINUSOIDAL PHASE MODULATION IN LASER DIODE INTERFEROMETER

Let us consider a Twyman-Green type interferometer as shown in Fig.1. The injection current of a LD consists of a DC component  $i_0$  and a time-varying component  $\Delta i_c(t)$  as follows:

$$i(t) = i_0 + \Delta i_c(t) . \quad (1)$$

The DC component determines a central wavelength of the light  $\lambda_0$ , and the  $\Delta i_c$  produces a small change in the wavelength of the LD

$$\Delta \lambda(t) = \beta \Delta i_c(t) . \quad (2)$$

Then the wavelength of the LD is given by

$$\lambda(t) = \lambda_0 + \Delta \lambda(t) . \quad (3)$$

The optical wave emitted from the LD is represented by

$$\exp\{j2\pi c \int_0^t [1/\lambda(t)] dt\} = \exp\{j\phi(t)\} , \quad (4)$$

where  $c$  is the velocity of the light. The light reflected from an object is an objective wave, and the light reflected from a mirror (M) is a reference wave. The optical path length of these waves are denoted by  $l_0$  and  $l_r$ , respectively. The objective wave  $U_0$  and reference wave  $U_r$  on the photodiode (PD) are represented by

$$U_0 = \exp\{j\phi(t - \tau_0)\} , \quad U_r = \exp\{j\phi(t - \tau_r)\} , \quad (5)$$

where  $\tau_0 = l_0/c$  and  $\tau_r = l_r/c$ . The time-varying component of the interference signal produced with the two waves is given by

$$S(t) = \cos\{\phi(t - \tau_0) - \phi(t - \tau_r)\} = \cos\Phi(t) . \quad (6)$$

Using the approximation

$$1/\lambda(t) \simeq (1/\lambda_0)\{1 - [\Delta\lambda(t)/\lambda_0]\} , \quad (7)$$

and the definition

$$\int \Delta\lambda(t) dt = \Delta\Lambda(t) , \quad (8)$$

the argument of Eq.(6) becomes

$$\Phi = (2\pi/\lambda_0)l - (2\pi c/\lambda_0^2)[\Delta\Lambda(t - \tau_0) - \Delta\Lambda(t - \tau_r)] , \quad (9)$$

where  $l = l_r - l_0$ . In the condition of  $\tau_r - \tau_0 \ll 1$ , we have the approximation

$$\Delta\Lambda(t - \tau_0) - \Delta\Lambda(t - \tau_r) \simeq (1/c)\Delta\Lambda(t) . \quad (10)$$

For sinusoidal phase modulation,

$$\Delta i_c(t) = a \cos(\omega_c t + \theta) , \quad (11)$$

we obtain the interference signal

$$S(t) = \cos[z \cos(\omega_c t + \theta) + \alpha] , \quad (12)$$

where

$$z = -(2\pi/\lambda_0^2)\beta a l , \quad \alpha = (2\pi/\lambda_0)l . \quad (13)$$

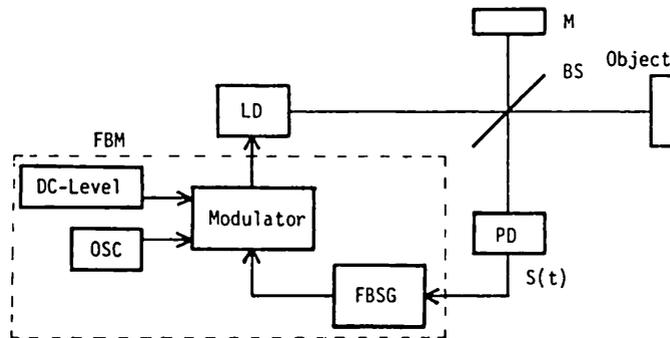


Fig. 1. Feedback control system in sinusoidal phase modulating laser diode interferometer.

#### 4. ELIMINATIO OF EXTERNAL DISTURBANCES WITH FEEDBACK CONTROL

The wavelength of the LD changes by  $\Delta\lambda_T$  with temperature. Optical devices in the interferometer vibrate in response to external mechanical vibrations. This caused the change  $\Delta l$  in the optical path difference between the object and reference waves. These  $\Delta\lambda_T$  and  $\Delta l$  cause the fluctuation in the phase of the interference signal. The fluctuation is compensated by controlling the injection current to produce the change  $\Delta\lambda_I$  in the wavelength of the LD. Considering the changes  $\Delta\lambda_T$ ,  $\Delta l$  and  $\Delta\lambda_I$  in Eqs.(9) and (10),  $l$  is replaced with  $l + \Delta l$  and  $\Delta\lambda$  is taken to be  $\Delta\lambda + \Delta\lambda_T + \Delta\lambda_I$ . By neglecting the term of  $(\Delta\lambda + \Delta\lambda_T + \Delta\lambda_I)\Delta l$ , the interference signal is written as

$$S(t) = \cos[z \cos(\omega_c t + \theta) + \alpha + \delta(t)] , \quad (14)$$

where

$$\delta(t) = (2\pi/\lambda_0)\Delta l - (2\pi l/\lambda_0^2)(\Delta\lambda_T + \Delta\lambda_I) . \quad (15)$$

We try to reduce the phase  $\delta(t)$  to zero by controlling the injection current or  $\Delta\lambda_I$ .

Let us explain how to generate the feedback signal for the injection current of the LD. The expansion of Eq.(14) is given by

$$S(t) = \cos[\alpha + \delta(t)][J_0(z) - 2J_2(z)\cos(2\omega_c t + 2\theta) + \dots] - \sin[\alpha + \delta(t)][2J_1(z)\cos(\omega_c t + \theta) - 2J_3(z)\cos(3\omega_c t + 3\theta) + \dots] . \quad (16)$$

Producing the signal  $S(t)\cos(\omega_c t + \theta)$  and passing this signal through a low-pass filter, we obtain the following feedback signal which is the output of the feedback signal generator (FBSG) shown in Fig.1:

$$J_1(z)\sin[\alpha + \delta(t)] . \quad (17)$$

This feedback signal is available in the region of  $z = 0.5 - 3.5$  where the value of the  $J_1(z)$  is not so small. When the phase  $\alpha$  is nearly multiplies of  $\pi$  rad, we can keep the phase  $\delta(t)$  to be zero stably with a proportional feedback control using the feedback signal given by Eq.(17). The phase  $\alpha$  is adjusted with the DC component of the injection current. The portion blocked with dot lines in Fig.1 is referred to feedback modulator (FBM). This modulator produces the injection current of the LD which is controlled so that the phase  $\delta(t)$  is reduced to zero.

## 5. MOVEMENT MEASUREMENTS

### 5.1. Interferometer

Figure 2 shows a SPM interferometer with feedback control system for movement measurements. The light emitted from a LD is collimated with a lens 1 (L1). The light reflected from a mirror 1 (M1) is a reference wave. The light passed through a beam splitter (BS) is an object wave. A portion of the object wave is illuminated onto an object through a lens 2 (L2). The movement of the object is represented by  $r(t)$ . The reflected light from the object and the reference light are superimposed on a photodiode 1 (PD1). The interference signal detected with the PD1 is written as

$$S_1(t) = \cos[z_1 \cos(\omega_c t + \theta) + \alpha_{10} + \alpha_1(t) + \delta_1(t)] , \quad (18)$$

where  $\alpha_1(t) = (4\pi/\lambda_0)r(t)$ , and the  $\alpha_{10}$  is a constant. On the other hand, the rest of the object wave is illuminated onto a mirror 2 (M2). The reflected light and the reference light are deflected with a prism, and reach to a photodiode 2 (PD2). The interference signal detected with PD2 is written as

$$S_2(t) = \cos[z_2 \cos(\omega_c t + \theta) + \alpha_{20} + \delta_2(t)] . \quad (19)$$

The feedback signal is generated from this interference signal in the FBM. When the feedback control operates well, the phase  $\delta_2$  is reduced to a small value  $\Delta\delta$  as follows:

$$\delta_2(t) = (2\pi/\lambda_0)\Delta l_2 - (2\pi l_2/\lambda_0^2)(\Delta\lambda_T + \Delta\lambda_I) = \Delta\delta . \quad (20)$$

Then, the phase  $\delta_1(t)$  is written as

$$\delta_1(t) = (2\pi/\lambda_0)[\Delta l_1 - (l_1/l_2)\Delta l_2] + (l_1/l_2)\Delta\delta . \quad (21)$$

Since the optical path length  $l_1$  is longer than the optical path length  $l_2$  and the change  $\Delta l_1$  is not completely equal to the change  $\Delta l_2$  in this interferometer, the phase fluctuation  $\delta_1(t)$  cannot be always reduced to the amount  $\Delta\delta$ .

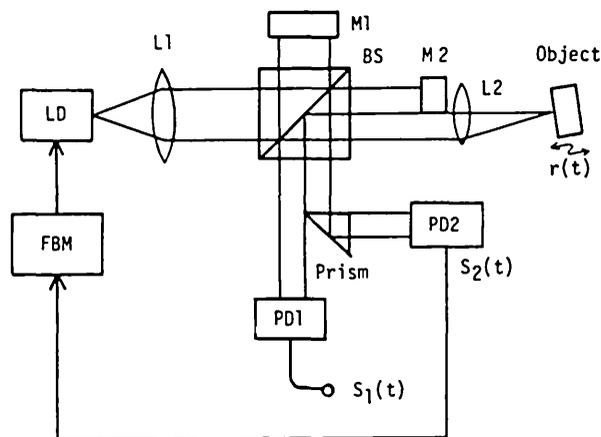


Fig. 2. SPM laser diode interferometer with feedback control system for movement measurements.

## 5.2. Experimental Results

We measured movements of the piezoelectric transducer which vibrated sinusoidally with a frequency of 100 Hz. The frequency of the sinusoidal phase modulation was 1KHz and the cutoff frequency of the low-pass filter employed in the FBM was 200 Hz. The movement  $r(t)$  and the phase  $\delta_2(t)$  were obtained by using the method described in Ref.5. Figures 3 and 4 show the movement  $r(t)$  and the phase  $\delta_2(t)$  measured when the feedback control did not operate. The measured movement contains the phase fluctuation  $\delta_1(t)$  which is almost equal to the measured phase  $\delta_2(t)$ . Figures 5 and 6 show the movement  $r(t)$  and the phase  $\delta_2(t)$  measured when the feedback control operated well. The measured  $\delta_2(t)$  corresponds to the  $\Delta\delta$  in Eq.(20). The optical path lengths  $l_1$  and  $l_2$  were 20 mm and 15 mm, respectively. The phase  $\delta_1(t)$  is reduced to be a small value, and the measured movement can be regarded to be a sinusoidal wave.

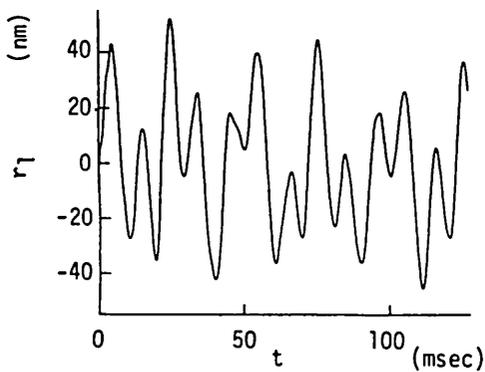


Fig. 3. Movement  $r(t)$  measured when the feedback control did not operate.

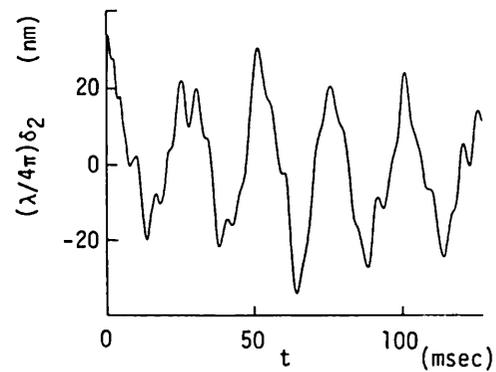


Fig. 4. Phase  $\delta_2(t)$  measured when the feedback control did not operate.

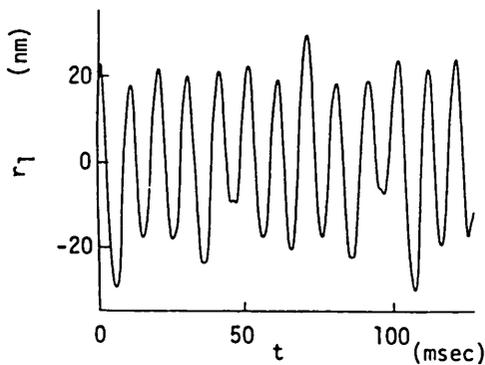


Fig. 5. Movement  $r(t)$  measured when the feedback control operated.

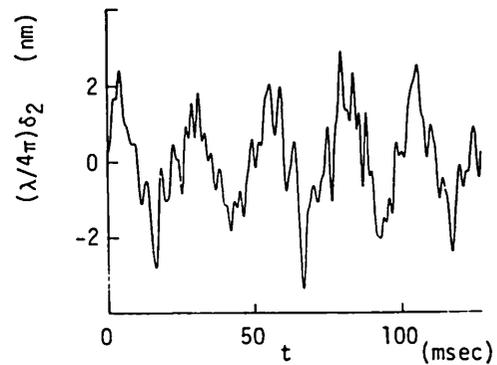


Fig. 6. Phase  $\delta_2(t)$  measured when the feedback control operated.

## 6. SURFACE PROFILE MEASUREMENTS

### 6.1. Interferometer

Figure 7 shows a SPM interferometer with feedback control system for surface profile measurements. The lens 2 (L2) makes an image of an object on a linear CCD image sensor. The surface profile of the object is represented by  $r(x)$ . The light reflected from the mirror 1 (M1) is a reference wave. The light near the CCD image sensor is reflected by the mirror 2 (M2) and reaches to the photodiode (PD). The interference signal detected with the CCD image sensor is written as

$$S_1(t,x) = \cos[z \cos(\omega_c t + \theta) + \alpha_{10} + \alpha_1(x) + \delta(t)] , \quad (22)$$

where  $\alpha_1(x) = (4\pi/\lambda_0)r(x)$ . The interference signal detected with the PD is written as

$$S_2(t) = \cos[z \cos(\omega_c t + \theta) + \alpha_{20} + \delta(t)] . \quad (23)$$

Since the distance between the measuring points for the CCD image sensor and the PD is short, the phase fluctuations in the signals  $S_1(t)$  and  $S_2(t)$  are considered to be identical. In other words, the conditions of  $l_1 = l_2$  and  $\Delta l_1 = \Delta l_2$  hold in this interferometer. The signal  $S_2(t)$  is fed to the FBM to generate the feedback signal. This feedback control system reduce the phase fluctuation  $\delta(t)$  in the signal  $S_1(t)$  to  $\Delta\delta$ .

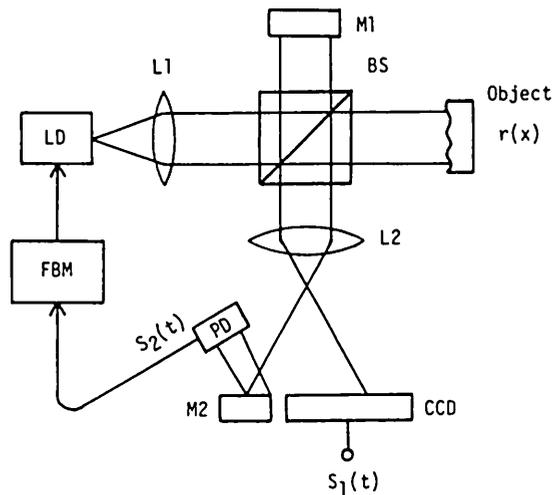


Fig. 7. SPM laser diode interferometer with feedback control system for surface profile measurements.

### 6.2 Experimental Results

We measured surface profiles of diamond-turned aluminum disks. The surface profile was obtained from the CCD output using the Fourier transform method described in Ref. 3. The same surface profile was measured at an interval of a few minutes. Figure 8 shows two surface profiles measured at the interval when the feedback control did not operate. There are slight differences between the two surface profiles. The measurement repeatability was between about 3.5 nm and 7.0 nm. Figure 9 shows two surface profiles measured at the interval when the feedback control operated well. The two surface profiles are almost identical. The measurement repeatability was greatly improved by the feedback control and was between about 0.5 nm and 1.0 nm.

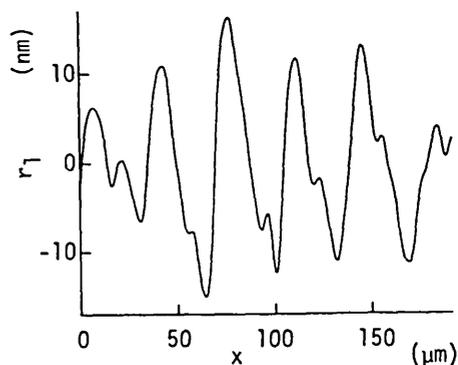
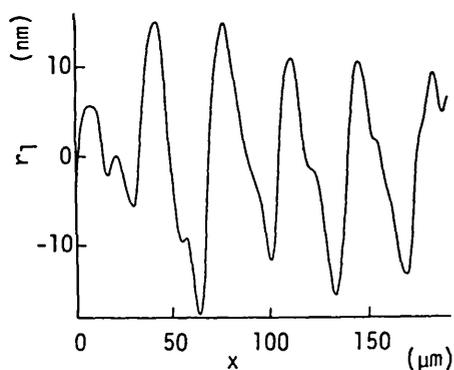
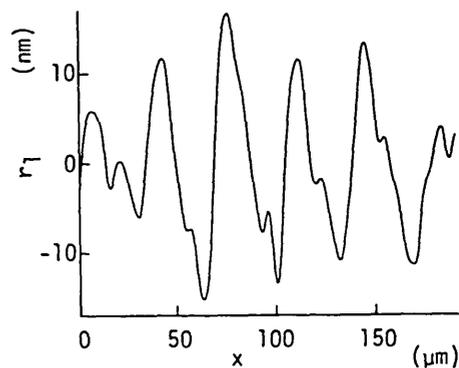
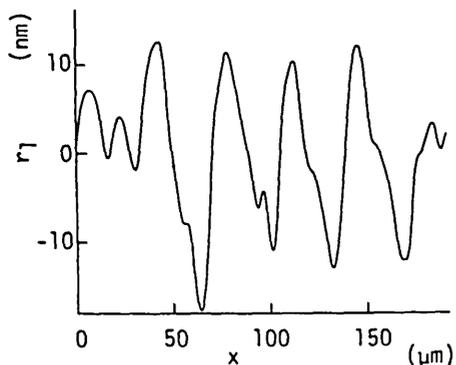


Fig. 8. Two surface profiles measured at an interval of a few minutes when the feedback control did not operate.

Fig. 9. Two surface profiles measured at an interval of a few minutes when the feedback control operated.

## 7. CONCLUSIONS

We constructed the SPM LD interferometers which were very insensitive to external disturbances such as mechanical vibrations and fluctuations in temperature. Since the optical path lengths and its fluctuations in the two different interference signals are not equal in the interferometer for the movement measurements, the measured movements contain phase fluctuations which can not be eliminated by the feedback control. However, the measured movements approach to the real movements. On the other hand, since they are equal in the interferometer for the surface profile measurement, the phase fluctuations are almost eliminated to improve greatly the measurement repeatability. The experimental results show clearly that the SPM LD interferometers presented in this paper can be used in mechanically noisy circumstances.

## 8. REFERENCES

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