

# Rotation angle measurement using an imaging method

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## ABSTRACT

A system of rotation angle measurement based on the fringe projection is proposed and demonstrated. This system has potential for a broad range of uses and a robustness for the external disturbances, because it requires no coherent light. The setup is very simple and applicable to the automatic on-line measurement. Several measurements indicate a sensitivity of 5 arcsec.

**Keywords:** rotation angle measurement, image processing, fringe projection, on-line measurement

## 1. INTRODUCTION

Measurement of the rotation angle<sup>1,2)</sup> involves the use of such established measuring devices as interferometers and autocollimators. While the former are extremely accurate, their use is usually limited, due to financial constrains, and the fact that they require an operating environment totally isolated from external disturbances. The latter, while being simple to use, take much longer to obtain the target-measurement. Therefore, they not suitable for automatic measurement.

The system we proposed is not based on either of the above method, but on what is referred to as optical image processing or fringe projection method<sup>3)</sup>. Using Fourier transform method<sup>4,5)</sup>, we calculate the angle, from the relative phase shift of the viewed grating image. Although the method bears some resemblance to Moiré deflectometry<sup>6)</sup>, it is unique, in that it does away with the need for coherent light source, expensive optical equipment, or the Moiré fringe, relying instead on single grating. Its insensitivity to external disturbances ensures that is operationally robust. Moreover, measurements require little or no fine tuning, thus making it ideally suited to automatic on-line measurement. Detection sensitivity is also tunable, by varying the spatial frequency of the grating, or the distance between the test target and the CCD camera.

## 2. PRINCIPLE

Figure 1 illustrates how the system measures the relative angle between the reference mirror (M1) and the object mirror (M2) which rotates around the point of origin (O). A grating pattern projected simultaneously onto M1 and M2 is reflected to, and observed on the viewing plane. If M2 slightly rotates by  $\theta$ , the viewed grating pattern reflected by M2 shifts by  $s$  against the other grating reflected by M1. The

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shift  $s$  represented by

$$s = d \tan 2\theta + x_0 \tan \theta \tan 2\theta \quad (1)$$

is a function of viewing distance  $d$ , rotation angle  $\theta$ , and  $x_0$  the  $x$ -coordinate of the grating. Since the  $x_0$  is much smaller than  $d$ , and  $\theta$  is very small, the second term of Eq. (1) can be neglected. Then, the rotation angle  $\theta$  is given by

$$\theta = \frac{1}{2} \tan^{-1} \left( \frac{s}{d} \right). \quad (2)$$

If the relative phase shift  $\Delta\alpha$  is detected by means of the Fourier transform method, the shift  $s$  is calculated by

$$s = \frac{P}{2\pi} \Delta\alpha, \quad (3)$$

where  $P$  is the grating pitch. The rotation angle  $\theta$  is given by

$$\theta = \frac{1}{2} \tan^{-1} \left( \frac{P}{2\pi d} \Delta\alpha \right). \quad (4)$$

Since the amount of phase shift is the function of the grating pitch  $P$  and the viewing distance  $d$ , resolution can be adjusted by changing  $P$  and  $d$ .

### 3. EXPERIMENT

#### 3.1 EXPERIMENTAL SETUP

The experimental setup shown in Fig. 2 consists of a computer-generated grating image, a CCD camera with an imaging lens and a computer. The object rotates around the point of origin, by means of a micrometer head positioned 80 mm distance from the point of origin. The grating image reflected by the

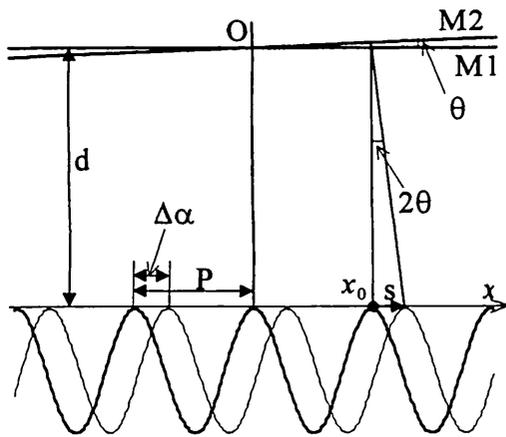


Fig. 1 Principle of the measurement, M1, Reference mirror; M2, Object mirror;  $s$ , Fringe shift;  $\Delta\alpha$ , Phase difference;  $P$ , Grating pitch;  $d$ , Viewing distance.

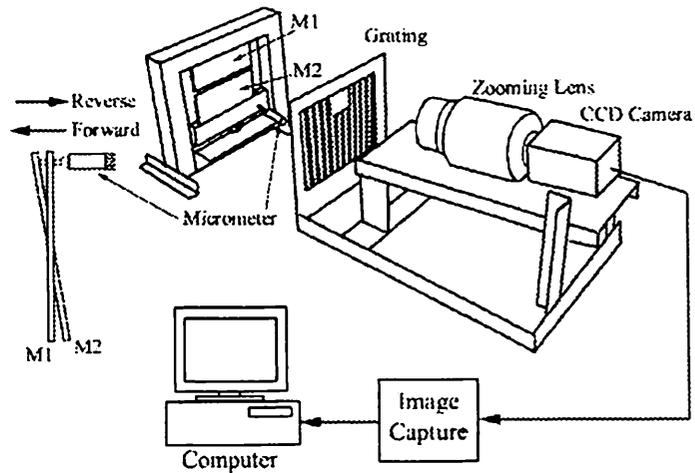


Fig. 2 Experimental setup.

mirror is captured by the CCD camera through a small aperture. The viewing distance  $d$  was 600 mm. No special lighting effects were employed, during the conduct of this experiment. We relied, instead, upon what was available naturally. The captured images were then processed by a computer.

### 3.2 SIGNAL PROCESSING

The image is analyzed by Fourier transformation. The flow chart shown in Fig. 3 details the major steps in our process, while Fig. 4 serves to explain how signals are processed. The original images reflected by M1 and M2 are displayed in the upper and lower areas of the Source Window, respectively. The images used in phase analysis are gotten of Source Window and displayed in the Object Window. The phase function obtained, is shown in the Process Window. Parameters such as grating pitch and viewing distance are displayed in the I/O field.

### 3.3 RESULTS

We used the micrometer head to rotate M2 around the point of origin, noting each  $10\ \mu\text{m}$  shift, whether forward or backward. Major results that are measured with a sinusoidal grating and a binary grating are shown in Figs. 5 and 6, respectively. The grating pitch in both instances was 3.6 mm. These experiments indicate a measurement sensitivity of 5 arcsec.

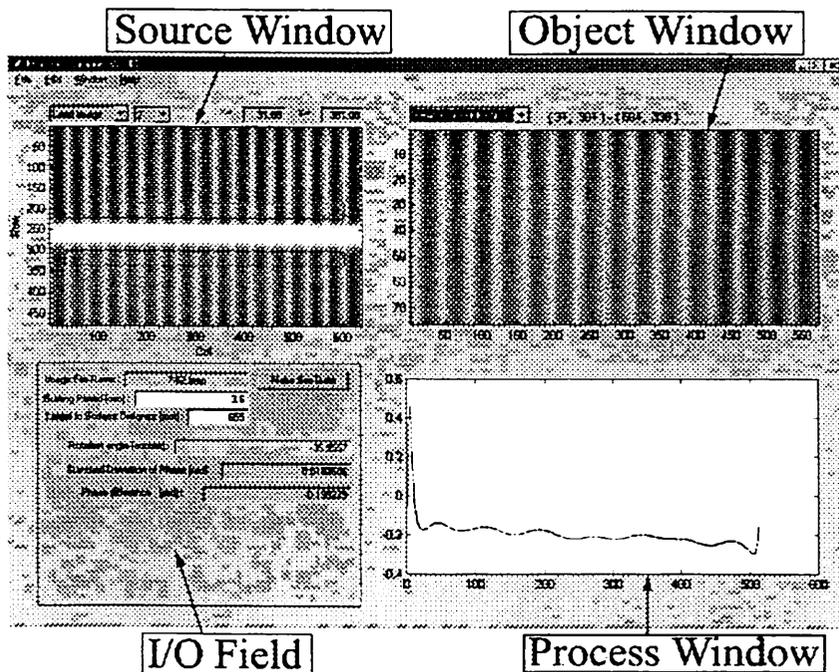
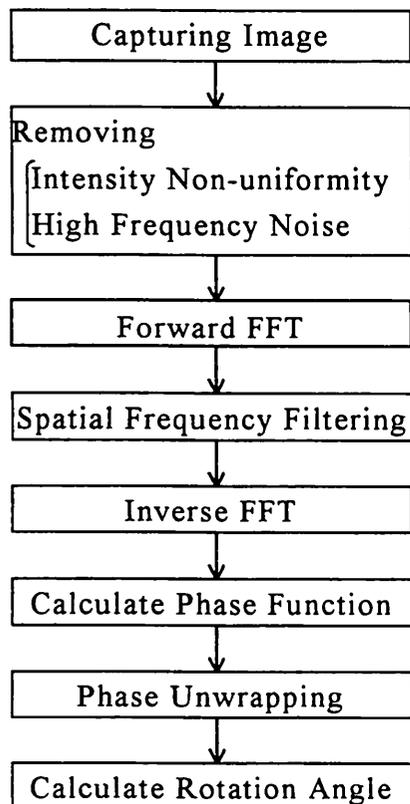


Fig. 3 Flow chart of the image processing.

Fig. 4 Display of the image processing.

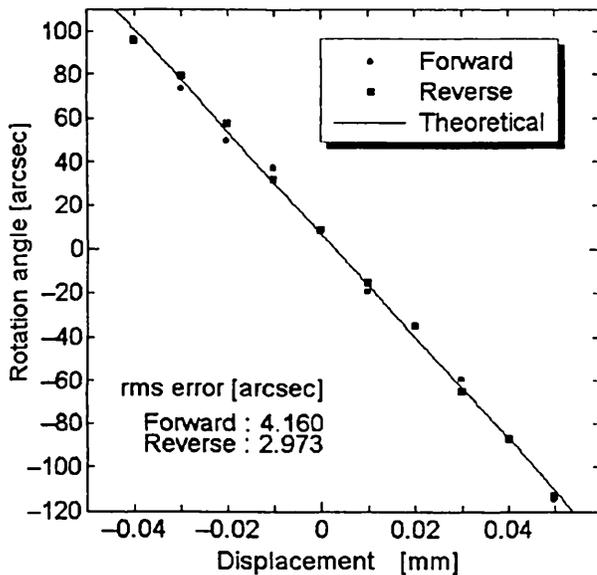


Fig. 5 Rotation angle measured with a sinusoidal grating.

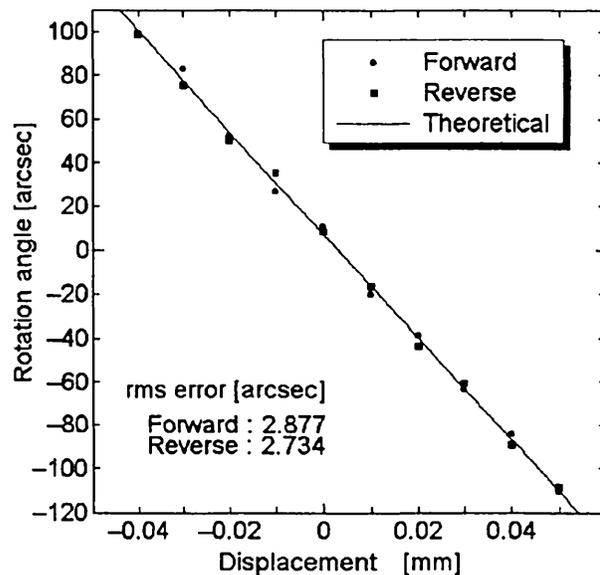


Fig. 6 Rotation angle measured with a binary grating.

#### 4. CONCLUSION

We proposed and demonstrated the rotation angle measurement system. It is simple, wide range, and has high accuracy. It enables us easily to make a automatic on-line measurement system for rotation angle, because of its robustness for the external disturbances and simplicity of construction.

#### REFERENCES

- 1) J. Z. Malacara, "Angle, Distance Curvature, and Focal Length Measurements," Chap. 18, in *Optical Shop Testing*, D. Malacara, Ed., (John Wiley and Sons, New York, 1992), pp. 718-720.
- 2) P. Shi and E. Stijns, "New optical method for measuring small-angle rotations," *Appl. Opt.* 27,4342-4344 (1988).
- 3) K. Leonhardt, U. Droste, and H. J. Tiziani, "Microshape and rough-surface analysis by fringe projection," *Appl. Opt.* 33, 7477-7488 (1994).
- 4) T. Mitsuo, H. Ina, and S. Kobayashi, "Fourier-transform method of fringe pattern analysis for computer-based topography and interferometry," *J. Opt. Soc. Am.*, 72, 156-160 (1982).
- 5) T. Mitsuo, K. Mutoh, "Fourier transform profilometry for the automatic measurement of 3-D object shapes," *Appl. Opt.* 22, 3977-3982 (1983).
- 6) O. Kafri and J. Glatt, "Moiré deflectometry: a ray deflection approach to optical testing," *Opt. Eng.*, 24, 944-960 (1985).