Multi-period fringe projection interferometry using back-propagation method for shape measurement of glass plate

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ABSTRACT

A multi-period fringe-projection interferometry with back-propagation method is described to measure surface profiles of a glass plate. Phase distribution of the multi-period fringe formed by interfering the two calumniated laser beams is utilized to determine the position of the two surfaces of the glass plate. The multiple optical fields of the different fringe periods on the two surfaces, which are obtained from the sinusoidal phase-modulated interference signals, are back-propagated to a position where all of the phases of the multiple optical fields become zero. At the same time, the amplitude of the sum of the multiple back-propagated fields becomes maximum. The distances of the back-propagation provide the positions of the two surfaces of the glass plate. In the experiment a glass plates of 2mm- thickness is measured with a precision of 2.3μm.

Keywords: back-propagation method, multi-period fringe projection, shape of glass plate

1. INTRODUCTION

Moiré and fringe projection techniques are noncontact optical techniques that have been widely applied in industry for measuring 3-D profile of object surfaces. These two techniques use the fringe patterns projected onto the object surfaces which are generated with either a grating or two laser beams. The surface profile of the object is obtained from a phase distribution of the fringe patterns which is calculated by the Fourier transform method[1] or by the phase- shifting method[2]. Recently some methods using multi-period fringe projection have been proposed to measure object surfaces with the discontinuities.[3-6] But it is difficult for these methods to measure surface profile of a transparent glass plate. In this paper, a multi-period fringe projection interferometry using back-propagation method is applied for measuring the two surface profiles of a glass plate. In the back-propagation method which was proposed in Refs.7 and 8, the optical fields on each point of the object surface are back-propagated to the constant phase point. When the back-propagated fields reach to the constant phase point, the amplitude of the sum of the back-propagated fields with the different fringe periods becomes maximum and its phase becomes zero. The back-propagation distance from the each point of the object to the constant phase point provides the position of the object surface. In experiments, a glass plate is measured with a precision of 1μm.

2. PRINCIPLE

A schematic configuration for the multi-period fringe projection interferometry is shown in Fig.1. Two plane waves of wavelength λ propagating in different directions produce a parallel interference fringe in the space. The coordinate system (x_p, y_p, z_p) is involved with the fringe pattern as shown in Fig. 1. The intersecting angles between the two waves and the z_p axis are $\pm\theta_m$, respectively. The period of the fringe pattern is expressed by $P_m = \lambda / 2\theta_m$ since $\sin\theta_m \approx \theta_m$ when θ_m is small. Changing the intersecting angles θ_m as θ_0, $\theta_1 \ldots$, and θ_{M-1}, the periods of the fringe pattern P_m are scanned as P_0, $P_1 \ldots$, and P_{M-1} accordingly. The phase of the interference fringe at $x_p = 0$ is kept at $\pi/2$ during the period scanning by using a feedback control system. The phase distribution in the coordinate system (x_p, y_p, z_p) is written as

$$\alpha_m(z_p, x_p) = k_m x_p + \frac{\pi}{2}, \qquad (1)$$

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where $k_m = \frac{2\pi}{P_m}$, and it is called wave number of the fringe pattern.

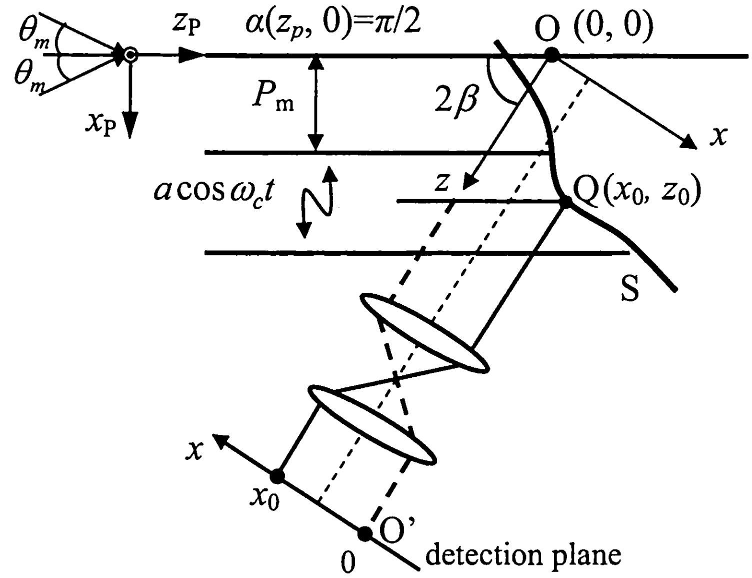

Fig.1 Schematic configuration for multiperiod fringe projection interferometer with sinusoidal phase-modulation.

It is assumed that there is a point object Q on the surface of the object. An image of the point is formed on a detection plane with an afocal imaging system. Another coordinate system (x, y, z) involved with the imaging system is defined. The original O of the coordinate system (x, y, z) is the same as that of the coordinate system (x_p, y_p, z_p). The z axis is parallel to the optical axis of the imaging system. The intersection angle of the axis z_p and the axis z is 2β. The phase distribution in the coordinate system (x, y, z) is written as

$$\alpha_m(x,z) = k_m(x\cos 2\beta + z\sin 2\beta) + \frac{\pi}{2}. \quad (2)$$

The position of the point object Q is expressed by $x = x_0$ and $z = z_0$. The magnification of the imaging system is assumed to be unit for the sake of simplicity. The size of an object is limited to less than the aperture of the afocal imaging system. The position of $x = 0$ on the detection plane corresponds to the original O and the image of the point object Q is formed at $x = x_0$ on the detection plane. The interference fringe intensity oscillates sinusoidally in the direction of the x_p axis with the form of $a\cos\omega_c t$ in order to incorporate the sinusoidal phase-modulating interferometry. Then the following interference signal produced by the point object Q is detected on the detection plane:

$$I_m(x_0,t) = A_m + B_m \cos[a\cos\omega_c t + \alpha_m(x_0,z_0)], \quad m = 0,\cdots,M-1. \quad (3)$$

The amplitude B_m and the phase α_m of the interference signal are calculated with the sinusoidal phase modulation method. The calculated value of $\alpha_m(z_0)$ is wrapped in the range between $-\pi$ and π. Since the original point of the coordinate x on the detection plane is decided by finding a position where the phase α_m is a constant value of $\pi/2$ during the scanning of the fringe period, the coordinate x_0 of the object is provided by the position of the detection plane. The following phase α_{mz} is extracted from the phase $\alpha_m(x_0, z_0)$ by subtracting the value of $k_m x_0 \cos 2\beta + \pi/2$ from the calculated value of $\alpha_m(z_0)$:

$$\alpha_{mz}(z_0) = k_m z_0 \sin 2\beta. \quad (4)$$

The range of $\alpha_{mz}(z_0)$ is between $2n\pi-\pi$ and $2n\pi+\pi$, where n is an integer value and the component of $2n\pi$ is removed from the $\alpha_{mz}(z_0)$. Since the coordinate x_0 of the object is known, a method which is called the back-propagation method is used to get the coordinate z_0 of the point object.

In the back-propagation method, the detected field of the point object Q is made as

$$D_m(x_0, z_0) = B_m \exp[j\alpha_{mz}(z_0)]. \tag{5}$$

When the detected field D_m is back-propagated to a position z, the back-propagated field of $U_m(x_0, z)$ is given by

$$U_m(x_0, z) = D_m \exp[-j\alpha_{mz}(z)], \quad m = 0, \cdots, M-1. \tag{6}$$

The sum of the back-propagated fields over all of k_m produces the following reconstruction field as a function of back-propagation position z:

$$U_R(z) = \sum_{m=0}^{M-1} U_m(z) = \sum_{m=0}^{M-1} B_m \exp[-jSk_m(z - z_0)], \tag{7}$$

where S is the incline coefficient which is written as $S = \sin 2\beta$ and k_m is the wave number of the fringe pattern. k_m is expressed as $k_m = k_0 + m\Delta k$, where Δk is the scanning interval of k_m. If B_m and z_D are denoted by $B_m = 1$ and $z_D = z - z_0$, Eq. (7) is reduced to

$$\begin{aligned} U_R(z) &= \exp\{j[-k_0 + \frac{(M-1)}{2}\Delta k]Sz_D\} \frac{\sin(\frac{M}{2}\Delta k z_D S)}{\sin(\frac{1}{2}\Delta k z_D S)} \\ &= A_R \exp(j\phi_R) \end{aligned} \tag{8}$$

From Eq. (8), the amplitude of A_R becomes maximum and its phase ϕ_R becomes zero at $z_D = 0$ where z is equal to the coordinate z_0 of the point object Q. From this basic characteristic, the coordinate z_0 of the object point Q can be obtained.

The scanning interval of k_m is given by

$$\Delta k = \frac{2\pi}{M-1}\left(\frac{1}{P_0} - \frac{1}{P_{M-1}}\right). \tag{9}$$

Since U_m(z) is a discrete function with respect to k_m, $U_R(z)$ becomes a periodic function with respect to z. Considering Eqs. (4)-(7), the period of $U_R(z)$ or the measurement range of z is given by

$$z_{\max} = \frac{2\pi}{\Delta k S}. \tag{10}$$

The phase ϕ_R changes linearly with respect to z, and its period which is called central period is given by

$$P_C = \frac{2P_0 P_{M-1}}{(P_0 + P_{M-1})S}. \tag{11}$$

Now measurement of surface profiles of a glass plate is considered as shown in Fig.2. The arrowed lines in Fig.2 represent the propagation direction of an equiphase plane on which the phases of an interference fringe pattern are constant. The refractive index of the glass plate is n. The x axis is nearly parallel to the front surface of the glass plate and the intersection angle between the z axis and the equiphase plane of the interference fringe is β before the equiphase plane is incident into the glass plate. The propagation direction of the equiphase plane is determined by Snell's law. The interference signal detected at the coordinate of x_A on the detection plane contains two components: one is the interference signal produced from the phase $\alpha_m(Q_1)$ of the equiphase plane reflected at $Q_1(x_A, z_A)$, the other is the interference signal produced from the phase $\alpha_m(Q_3)$ of the equiphase plane at Q_3 which propagates to $Q_1(x_A, z_A)$ by reflect at $Q_2(x_B, z_B)$ on the back surface. It is assumed that the back-propagation method provides the point z_A corresponding to the equiphase plane at $Q_1(x_A, z_A)$ and a point z_Q corresponding to the equiphase plane at Q_3. The difference between z_A and z_Q is proportional to the phase difference of $\alpha_m(Q_1)$-$\alpha_m(Q_3)$. Therefore the coordinate of z_B is given by

$$z_B = \frac{(z_Q - z_A)\tan\beta}{2\tan\gamma} \tag{12}$$

where γ is the refraction angle of the equiphase plane which is incident to the glass plate at the angle β, and the relation of $\sin\beta = n\sin\gamma$ holds. The thickness d is given by z_A and z_B.

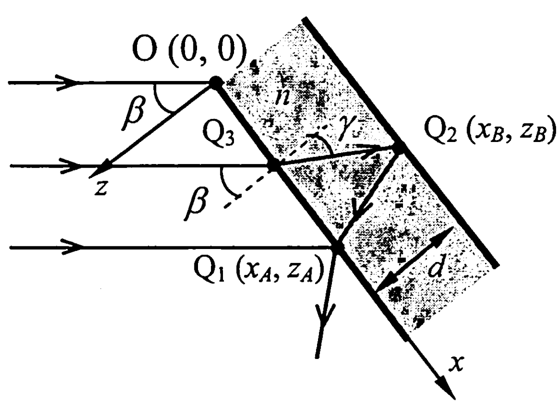

Fig. 2 Observed fringes reflected from the two surfaces of a glass plate.

3. EXPERIMENT

3.1 Experiment setup

The experimental setup is shown in Fig. 3. A 50mw laser diode is used as the light source. A laser beam collimated with a lens L_0 is divided into two beams by beam splitter BS1. The beams reflected from mirror M1 and mirror M2 produce the interference fringe. Piezoelectric transducer 1 (PZT1) changes the angle θ of mirror 1, which changes the periods P_m of the interference fringe. The piezoelectric transducer 2 (PZT2) vibrates with frequency f_c of $125Hz$ to produce the sinusoidal phase modulation in the interference fringe. The interference fringe is divided into two parts by beam splitter BS2. One part is incident on a photo diode (PD) which detects the interference signal at one point of x_f. Considering Eqs. (1) and (3), the detected interference signal is given by

$$I_m(t, x_f) = A_m + B_m\cos(a\cos\omega_c t + k_m x_f) \tag{13}$$

In the feedback control circuit, a feedback signal of $S_f = B_m\cos(k_m x_f)$ is produced by eliminating the DC component of A_m and sampling $I_m(t, x_f)$ when $\cos(\omega_c t) = 0$. A feedback control signal made from the signal S_f is applied to the PZT2 so that S_f becomes zero by moving mirror M2. Thus the feedback control system makes the phase of the point x_f fixed at $\pi/2$ while the period P_m of the interference fringe is changed. The point x_f is defined as the origin point of the x_p axis or x axis, and the expression of Eq. (1) is realized. The other part of the interference fringe is projected on the surface of the object. The image of the object's surface is formed with the afocal imaging system whose magnification is $M_{12} = f_2 / f_1$, where f_1 and f_2 are the focal length of L_1 and L_2, respectively. A high-speed CCD camera with the frame period of 1ms is used to detect the sinusoidally phase-modulated interference fringe.

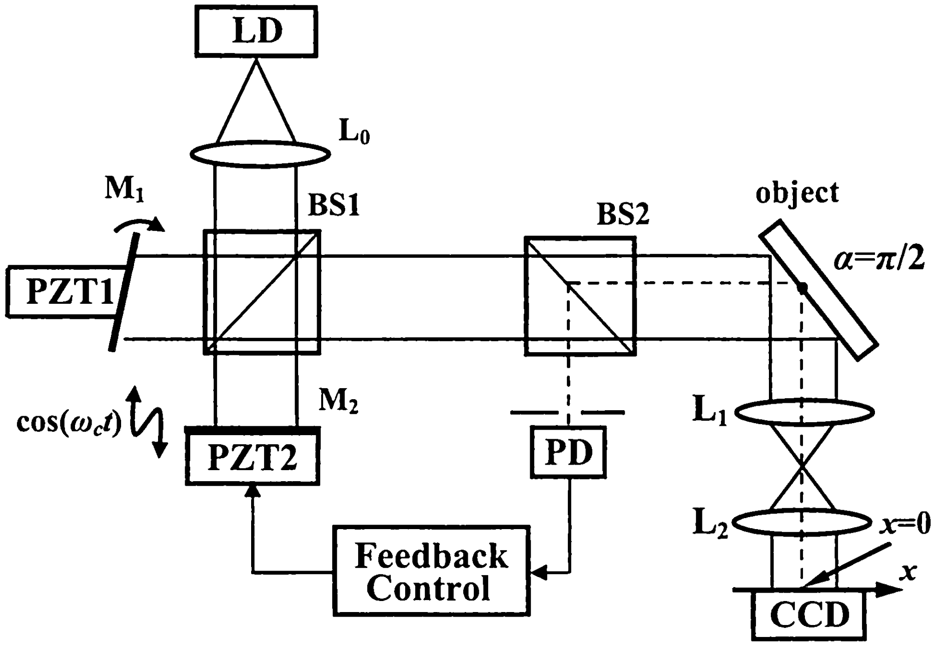

Fig. 3 Experiment setup of the multiperiod fringe projection interferometer.

3.2 Experiment results

A glass plate was used as an object. The thickness d of the glass plate is 2mm. The parameters used in the experiment were as follows: $P_1 = 143.0\mu m$, $P_{16} = 1002.4\mu m$, $M = 16$, $\Delta k = 0.0023 rad/\mu m$, β=45° $f_1 = 100mm$, and $f_2 = 25mm$. The image size on the x-y plane of the CCD image sensor were 256×240 pixels with the pixel size of 7.4μm, and the measuring region on the object was about $7.4 \times 7.2mm^2$ with $M_{12} = 1/4$. Figure 4 shows the amplitude A_R and phase ϕ_R for one measuring point of $x = 2.90mm$ and $y = 3.58mm$ on the front surface of the glass plate. Figure 4 indicates that the amplitude distribution A_R of U_R has two peaks at $z = z_{AA}$ and $z = z_{AQ}$. The phase distribution ϕ_R becomes zero at $z = z_{\phi A}$ and $z = z_{\phi Q}$ which is corresponding to the two peaks in the amplitude distribution. From these results, the measured values of $z_{\phi A}$ and $z_{\phi Q}$ were -2124μm and 14μm. The measured value of d was 2001μm at the point $x = 2.90mm$, $y = 3.58mm$. The measured values of z_{AA} and z_{AQ} were -2128μm and 16μm. The measured value of d was 2005μm. It is obvious that the measured values obtained from the phase distribution have higher precision than that obtained from the amplitude distribution. Figures 5 shows the measured position of the two surfaces of the glass plate where the axis of z is shown separated. The average value of the thickness d was 2001μm. The measurements were repeated 3 times at intervals of a few minutes. The repeatability in the thickness measurement was about 2.3μm which was the rms value of the differences between the average values of the thickness.

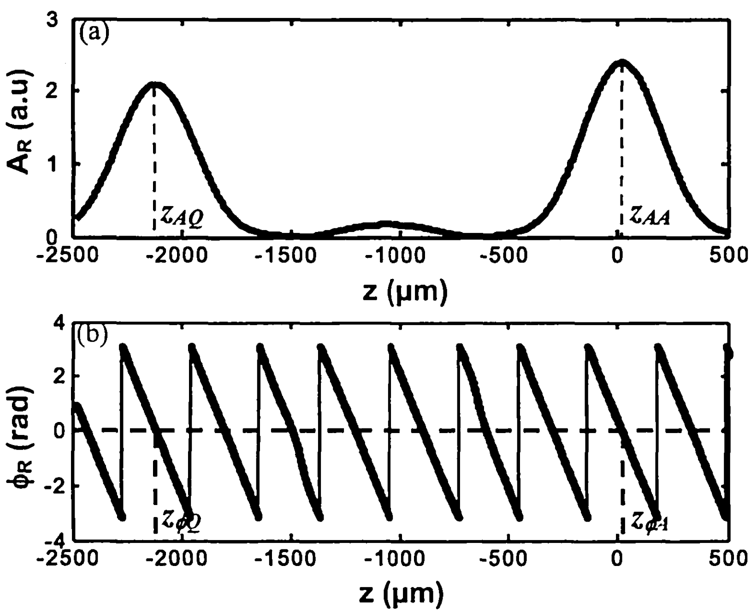

Fig. 4 Results of the back-propagation for one point of x_A =2.90mm on the front surface and x_B =3.30mm on the back surface of glass plate: (a) amplitude distribution A_R, (b) phase distribution ϕ_R.

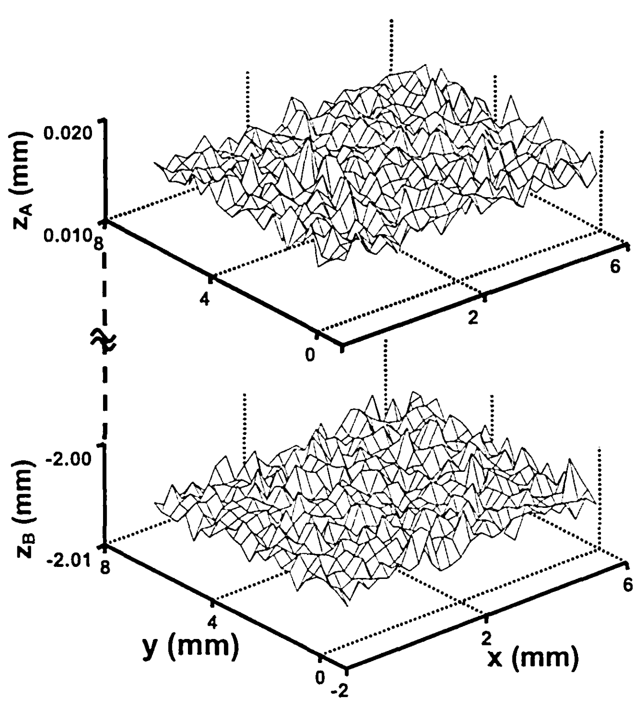

Fig. 5 Measured positions of the two surfaces

4. CONCLUSION

A multi-period fringe projection interferometry by using the back-propagation method was presented. In the back-propagation method the optical fields with different fringe periods were back-propagated to the stationary point of the phase. The position of the object surface was obtained from the distance of the back-propagation on which the amplitude of the sum of the back-propagated optical fields became maximum and its phase became zero. A glass plate of 2mm-thickness was measured with a precision of 2.3μm and the measurement repeatability is about 2.3μm.

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