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Heavy quark free energies and screening at finite temperature and density

Matthias Döring∗
Fakultät für Physik, Universität Bielefeld, D-33615 Bielefeld, Germany
E-mail: doering@physik.uni-bielefeld.de

S. Ejiri[b], **O. Kaczmarek**[a], **F. Karsch**[a,c], **E. Laermann**[a]
[a]*Fakultät für Physik, Universität Bielefeld, D-33615 Bielefeld, Germany*
[b]*Department of Physics, The University of Tokyo, Tokyo 113-0033, Japan*
[c]*Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA*

We study the free energies of heavy quarks calculated from Polyakov loop correlation functions in full 2-flavour QCD using the p4-improved staggered fermion action. A small but finite baryon number density is included via Taylor expansion of the fermion determinant in the baryo-chemical potential μ. For temperatures above T_c we extract Debye screening masses from the large distance behaviour of the free energies and compare their μ-dependence to perturbative results.

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1. Introduction

Lattice QCD has widely been used to get information about the properties of matter at high temperature and vanishing net baryon density μ_b [1]. These studies have been extended to non-vanishing baryon density. I.e., the equation of state has been discussed using Taylor expansion [2], reweighting [3] or imaginary chemical potential techniques [4].

Furthermore the free energy of static quark anti-quark sources at $\mu_b = 0$ received much attention [5]. We will now apply here the Taylor expansion approach in order to extend the results on this free energy to non-zero quark chemical potential. A first attempt to do this was discussed in [6]. Furthermore we will analyze the large distance behaviour of the free energy and determine the corresponding expansion coefficients of the screening mass, which can be compared to perturbative predictions.

We will restrict the discussion of the expansion up to the 2nd order in μ_b here. Results up to the 6th order as well as furhter details on the simlulation and the Taylor expansion technique can be found in [7].

2. Taylor expansion method in QCD at finite density

The staggered fermion partition function

$$Z_\mu = \int DU \, \Delta(\mu) \, e^{-S} , \tag{2.1}$$

where in our 2-flavour case $\Delta(\mu)$ is the square root of the fermion determinant, can be Taylor expanded in powers of the quark chemical potential $\mu = \mu_b/3$ by inserting the expansion of $\Delta(\mu)$ in powers of μ.

$$\Delta(\mu) = \Delta(0) \left(1 + D_1\mu + D_2\mu^2 + \cdots\right) . \tag{2.2}$$

The odd orders in the expansion of Z_μ are vanishing because D_n is imaginary for odd and real for even n. The simulation can be done at $\mu = 0$, where the D_n are handled like observable quantities.

This procedure can be extended easily to the μ-dependent expectation value of an observable $\mathscr{A}$ that does not directly depend on μ,

$$\begin{aligned} \langle \mathscr{A} \rangle_\mu &= \frac{1}{Z_\mu} \int DU \, \mathscr{A} \Delta(\mu) \, e^{-S} = \frac{\langle \mathscr{A} \cdot 1 \rangle_0 + \langle \mathscr{A} D_1 \rangle_0 \, \mu + \cdots + \langle \mathscr{A} D_6 \rangle_0 \, \mu^6}{1 + \langle D_2 \rangle_0 \, \mu^2 + \langle D_4 \rangle_0 \, \mu^4 + \langle D_6 \rangle_0 \, \mu^6} + \mathscr{O}(\mu^7) , \\ &= \langle \mathscr{A} \rangle_0 \left(1 + a_1\mu + a_2\mu^2 + \cdots + a_6\mu^6\right) + \mathscr{O}(\mu^7) . \end{aligned} \tag{2.3}$$

We apply this scheme to the Polyakov loop correlation functions discussed in the next section.

3. Heavy quark free energies

A heavy (static) quark Q at site x is represented by the Polyakov loop,

$$L(x) = \prod_{x_4=1}^{N_\tau} U_4(x, x_4) , \tag{3.1}$$

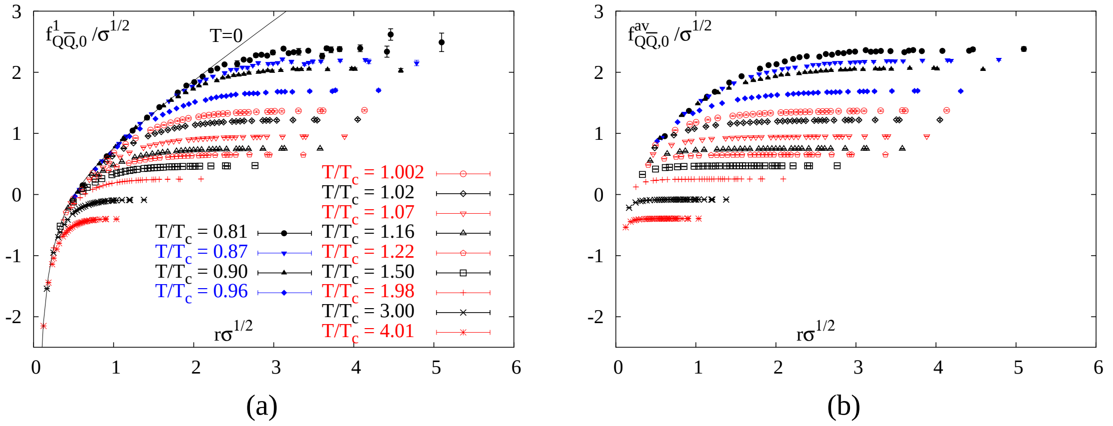


Figure 1: The 0^{th} order coefficients $f^1_{Q\bar{Q},0}$ and $f^{\text{av}}_{Q\bar{Q},0}$ for the singlet (a) and colour averaged (b) free energies. $f^1_{Q\bar{Q},0}$ is matched to the $T=0$ heavy quark potential at small distances (a).

which is an SU(3) matrix. A heavy anti-quark $\bar{Q}$ is described by the corresponding hermitian conjugate matrix. The colour averaged and singlet $Q\bar{Q}$-free energies $F^{\text{av}}_{Q\bar{Q}}(r,T,\mu)$ and $F^1_{Q\bar{Q}}(r,T,\mu)$ can be calculated from the corresponding correlation functions $\mathscr{O}^{\text{av},1}(r)$,

$$F^{\text{av},1}_{Q\bar{Q}}(r,T,\mu) = -T\ \ln\left\langle \mathscr{O}^{\text{av},1}(r)\right\rangle = f^{av,1}_{Q\bar{Q},0}(r,T) + f^{av,1}_{Q\bar{Q},2}(r,T)\left(\frac{\mu}{T}\right)^2 + \mathscr{O}\left(\left(\frac{\mu}{T}\right)^4\right) , \qquad (3.2)$$

where

$$\mathscr{O}^{\text{av}}(r) = \frac{1}{\mathscr{N}}\frac{1}{N_c^2}\sum_{x,y}\text{Tr}L(x)\,\text{Tr}L^\dagger(y)\,,$$
$$\mathscr{O}^{1}(r) = \frac{1}{\mathscr{N}}\frac{1}{N_c}\sum_{x,y}\text{Tr}L(x)L^\dagger(y)\,. \qquad (3.3)$$

In Fig. 1 and 2 we show the results for the 0th and 2nd order expansion coefficients in μ/T for temperatures above and below T_c. Because $f^{\text{av},1}_{Q\bar{Q},2}(r,T)$ is always negative, we find that for small μ the free energy of a static quark anti-quark pair decreases in a medium with a net excess of quarks or anti-quarks.

We determine the large distance value of these coefficients, shown in Fig. 3, by taking the weighted average of the values at the five largest distances. $f^{\text{av},1}_{Q\bar{Q},2}(\infty,T)$ signs the transition temperature by showing a pronounced peak at T_c. The coefficients of the colour averaged and singlet free energy are identical at infinite distance.

4. Screening masses

For temperatures above T_c and large distances r the heavy quark free energies are expected to be screened,

$$\Delta F^{\text{av},1}_{Q\bar{Q}}(r,T,\mu) = F^{\text{av},1}_{Q\bar{Q}}(\infty,T,\mu) - F^{\text{av},1}_{Q\bar{Q}}(r,T,\mu) \sim \frac{1}{r^n}e^{-m^{\text{av},1}(T,\mu)r} \qquad (4.1)$$

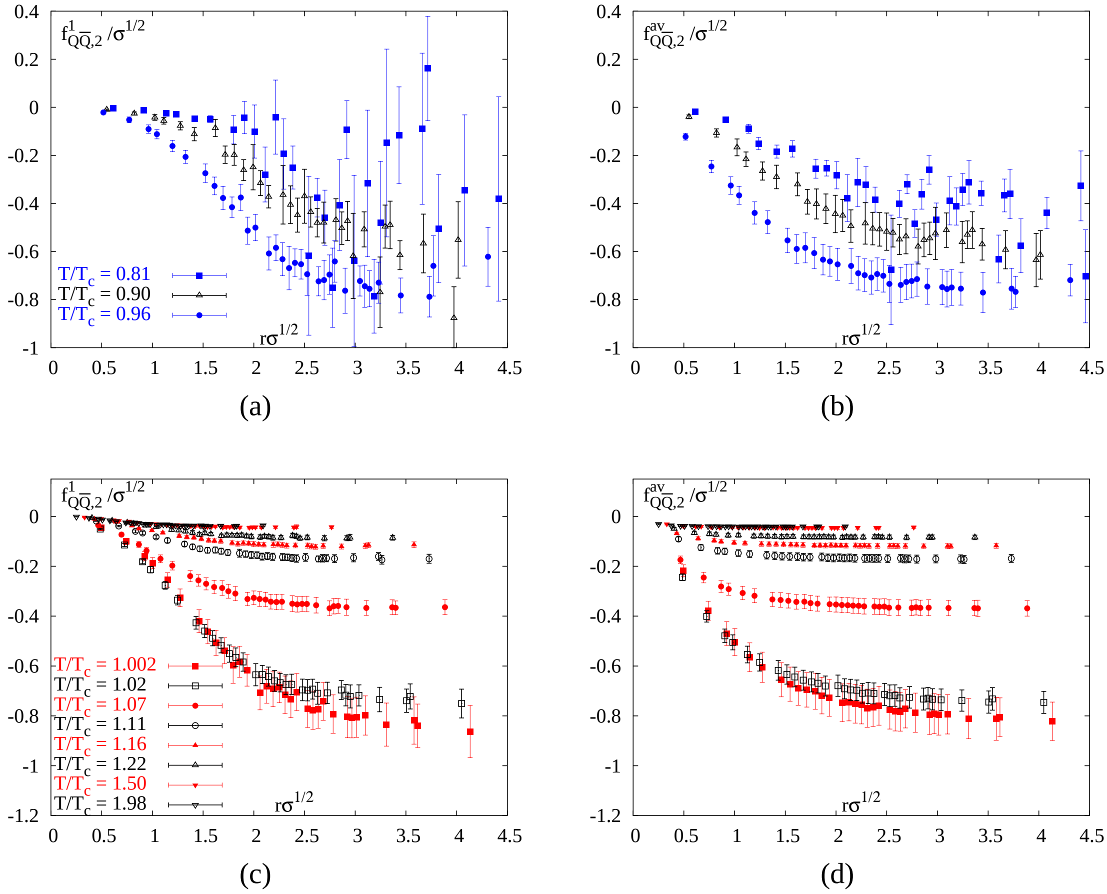


Figure 2: The 2^{nd} order coefficients of the singlet (a,c) and colour averaged (b,d) free energies below (a,b) and above (c,d) T_c.

with $n = 1,2$ for the singlet and colour averaged free energies respectively. In the infinite distance limit we thus get,

$$m^{\mathrm{av},1}(T,\mu) = -\lim_{r\to\infty} \frac{1}{r} \ln\left(\Delta F^{\mathrm{av},1}_{Q\bar{Q}}(r,T,\mu)\right) \ . \tag{4.2}$$

Expanding the logarithm in powers of μ we see that only the even orders of the expansion coefficients for the screening mass are non-zero, i.e. the second order coefficient can be written as

$$m_2^{av,1}(T) = -\lim_{r\to\infty} \frac{1}{r} \frac{\Delta f^{av,1}_{Q\bar{Q},2}(r,T)}{\Delta f^{av,1}_{Q\bar{Q},0}(r,T)} \ , \tag{4.3}$$

where we write $\Delta f^{\mathrm{av},1}_{Q\bar{Q},n}$ for the expansion coefficients of $\Delta F^{\mathrm{av},1}_{Q\bar{Q}}$. We use this expression to determine the expansion coefficient of order 2 from the infinite distance limit. For the 0th order coefficient we refer to fits of the form of eq. 4.1. In fact the rational expression under the limit in eq. 4.3 shows

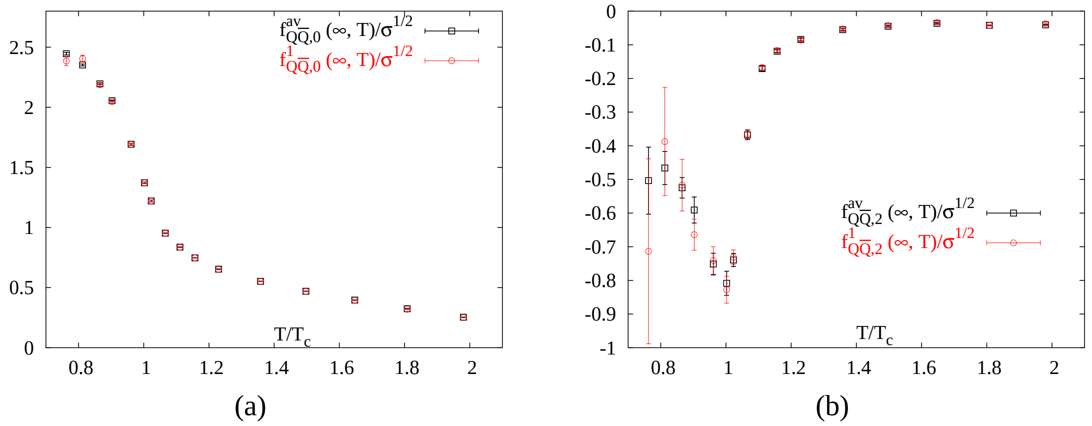


Figure 3: The coefficients for the singlet and colour averaged free energies at infinite distance rT versus temperature.

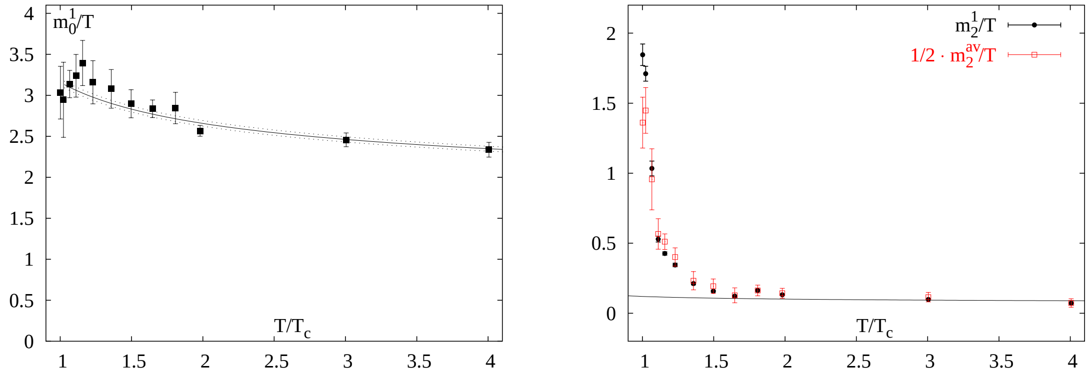
only little r-dependence and we see a wide plateau starting close to $rT = 0$. We therefore determine $m_2^{1,av}(T)$ by fitting the ratio in eq. 4.3 to a constant in the plateau range. The resulting coefficient, shown in Fig. 4 (b) is always positive and leads to an enhancment of the screening mass for small $\mu_b \neq 0$. At high temperatures $m_2^{1,av}(T)$ is in remarkable agreement with the LO high temperature perturbative prediction resulting from the μ-expansion of the Debye mass

$$m^D(T,\mu) = g(T)T\sqrt{\frac{N_c}{3} + \frac{N_f}{6} + \frac{N_f}{2\pi^2}\left(\frac{\mu}{T}\right)^2}\,, \tag{4.4}$$

where we introduce an additional scale factor A via $m^1(T) = A \cdot m^D(T)$, which is determined from fitting $m_0^1(T)$. We find $A = 1.397(18)$. Finally we see, that $m_2^1(T)$ and $m_2^{av}(T)$ differ by a factor $1/2$, which is expected at high temperature for all the coefficients but has not been observed for the 0th order so far.

5. Conclusions

We analyzed the dependence of heavy quark free energies and screening on the baryon-chemical potential. The free energies are decreasing to first order in μ^2, while the screening mass is increasing. This suggests that the screening length in a baryon or anti-baryon rich quark gluon plasma decreases with increasing value of the chemical potential, which is consistent with the expectation that a non-zero μ shifts the transition to lower temperatures. The screening behaviour is in good agreement with perturbation theory for $T/T_c \gtrsim 2$. We observed that the μ-dependent corrections of the colour averaged screening mass are twice as large as those of the colour singlet. This suggests that the contribution to the μ-dependent corrections of the colour averaged screening mass is due to two-gluon exchange.



(a) (b)

Figure 4: 0th and 2nd order expansion coefficients of the screening mass. The solid lines are connected to the LO perturbation theory.

6. Acknowledgments

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