

# ATTENUATION CONSTANTS OF NORMAL MODES IN HOLLOW CIRCULAR CYLINDER SURROUNDED BY DISSIPATIVE MEDIUM

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## Introduction

One of the important problems in vehicular communication is the attenuation of field intensity in tunnels which are surrounded by dissipative medium such as earth, rocks and concretes. It is too difficult to analyse the exact propagation characteristics of electromagnetic waves in tunnels considering all boundary conditions. The study reported here which models a tunnel as a hollow dielectric waveguide with circular cross section extends the earlier work by Glaser [1]. Attenuation constants of normal modes are investigated by numerical computation and compared with experimental data.

## Attenuation constants

Geometry of a modified tunnel is shown in Fig.1. At frequencies in the range of 200-4000 MHz, earth, rocks and concretes act as lossy dielectrics with relative dielectric constants in the range 5-10, conductivities  $10^{-2}$ - $10^{-1}$  U/m.

The normal modes in hollow circular dielectric waveguides are of four types; circularly symmetric  $TE_{0m}$ ,  $TM_{0m}$  modes and hybrid  $HE_{nm}$ ,  $EH_{nm}$  modes. These modes are assumed to have z variation of the form  $e^{jh_z z}$ , where

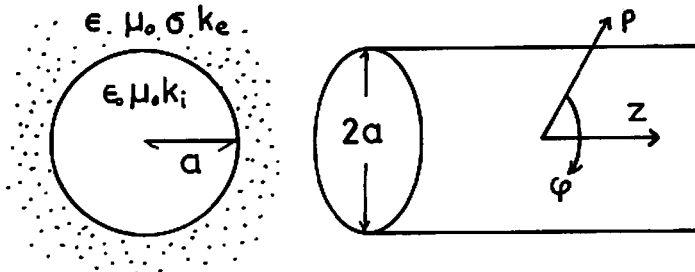


Fig.1

$h$  is the propagation constant. The propagation constant having  $n$  angular variations must satisfy the following deterministic equation as given by Stratton [2].

$$\left[ \frac{J_n'(u)}{uJ_n(u)} - \frac{H_n'(v)}{vH_n(v)} \right] \cdot \left[ \frac{k_i^2 J_n'(u)}{uJ_n(u)} - \frac{k_e^2 H_n'(v)}{vH_n(v)} \right] = n^2 h^2 \left( \frac{1}{u^2} - \frac{1}{v^2} \right)^2 \quad (1)$$

, where

$$u = \sqrt{k_i^2 - h^2} a$$

$$k_i^2 = \omega^2 \epsilon_0 \mu_0$$

$$h = \beta - j\alpha$$

$$v = \sqrt{k_e^2 - h^2} a$$

$$k_e^2 = \omega^2 \epsilon \mu_0 - j\omega \mu_0 \sigma$$

$\beta$ ; phase constant  
 $\alpha$ ; attenuation constant

$a$ ; radius  
 $\omega$ ; angular frequency  
 $\epsilon_0$ ; permittivity of free space  
 $\mu_0$ ; permeability of free space  
 $\sigma$ ; conductivity  
 $\epsilon_r = \epsilon / \epsilon_0$ ; relative dielectric constant

$J_n(u)$ ; Bessel Function of order  $n$  with complex argument  
 $H_n(v)$ ; Hankel Function of the second kind of order  $n$  with complex argument

' ; differentiation with respect to the indicated argument

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Equation (1) can be written simply in the form

$$F(u) = 0 \quad (2)$$

By numerical computation based on Newton's method, the exact propagation constants of normal modes can be obtained. Attenuation constants are shown in Fig.2-7. where parameters are

$$a=4 \text{ m} \quad \epsilon_r=2,5,10 \quad \sigma=10^{-1}, 10^{-2} \text{ v/m}$$

The attenuation constants of these modes decrease as frequency increases. This phenomena can be understood as the result of refraction loss rather than ohmic loss. The minimum-loss mode is TE<sub>01</sub> for large relative dielectric constant, EH<sub>11</sub> for small relative dielectric constant. However when  $k_1 a$  is small, HE<sub>11</sub> mode has the minimum loss, as shown later in Fig.9.

Here, EH and HE modes are determined by the trace of roots of eq(2) as a function of conductivities. These modes can also be determined by examining the ratio  $E_z/\eta H_z$  for each mode.

$$Q = \left| \frac{E_z}{\eta H_z} \right| \quad \eta = \sqrt{\mu_0 / \epsilon_0} \quad (3)$$

That is, Q, by eq(3), greater than 1 for EH mode, while Q is less than 1 for HE mode. Fig.8 shows the value of Q for conductivities. It is hard to discriminate between EH and HE mode in dissipative medium as tunnels.

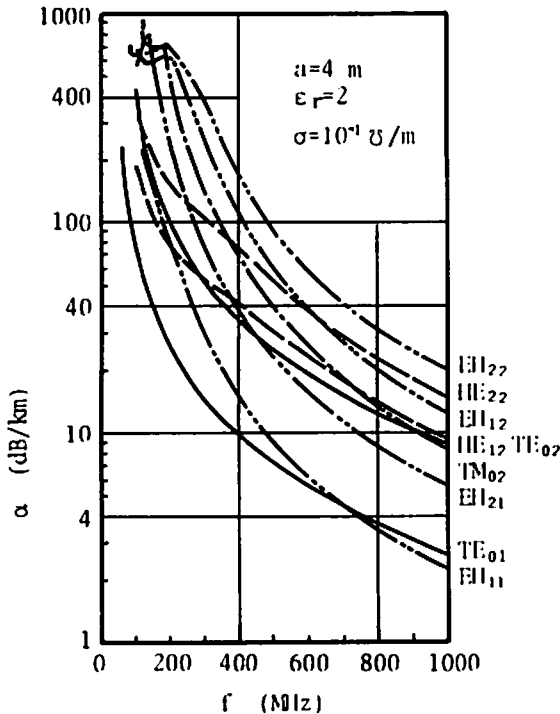


Fig.2

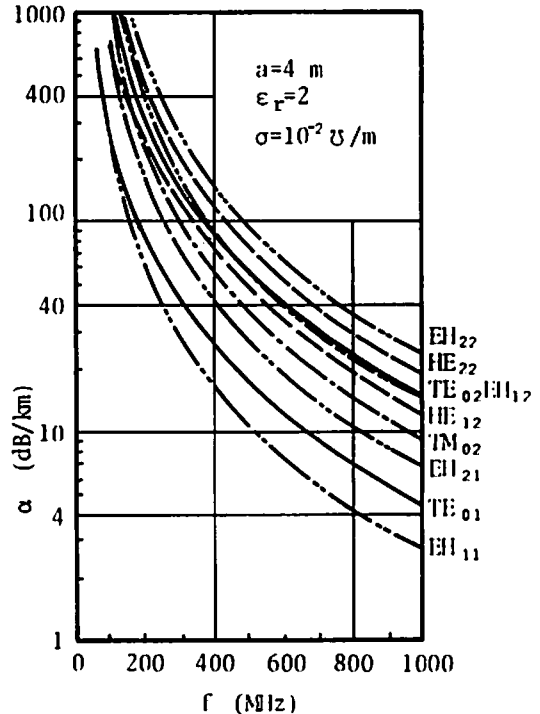


Fig.3

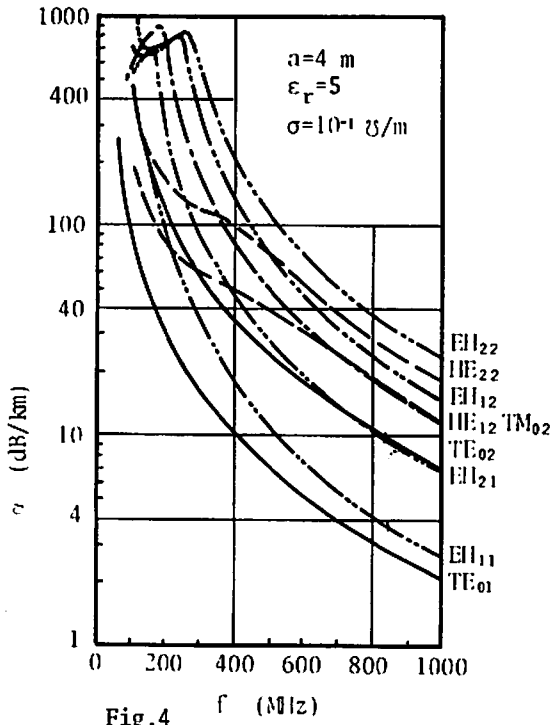


Fig. 4

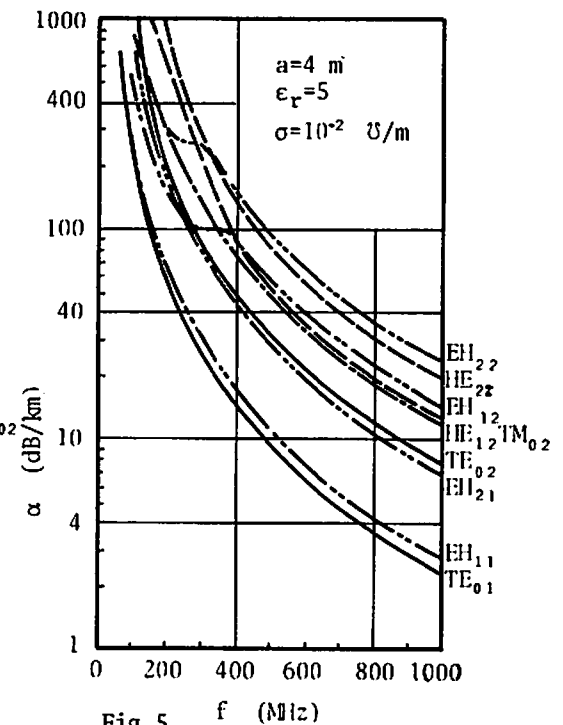


Fig. 5

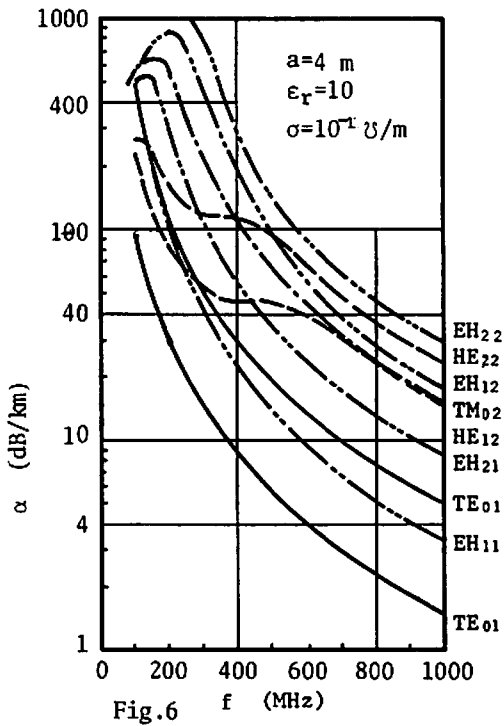


Fig. 6

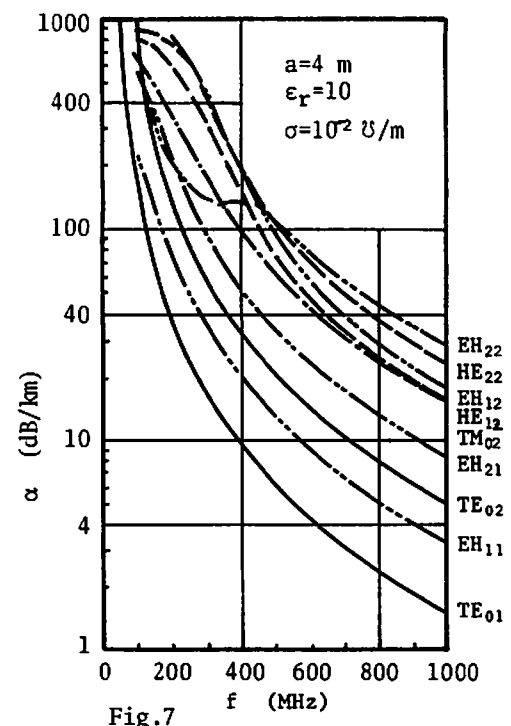


Fig. 7

Finally, we measured attenuation constants of some modes to confirm these theoretical results, as shown in Fig.9. Experimental results are in good agreements with theoretical ones. A method to decrease the attenuation of  $TE_{01}$  mode has been proposed using circular metallic rings around the wall [3].

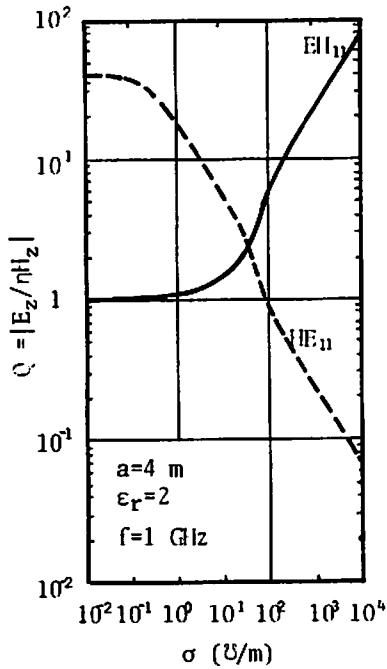


Fig.8

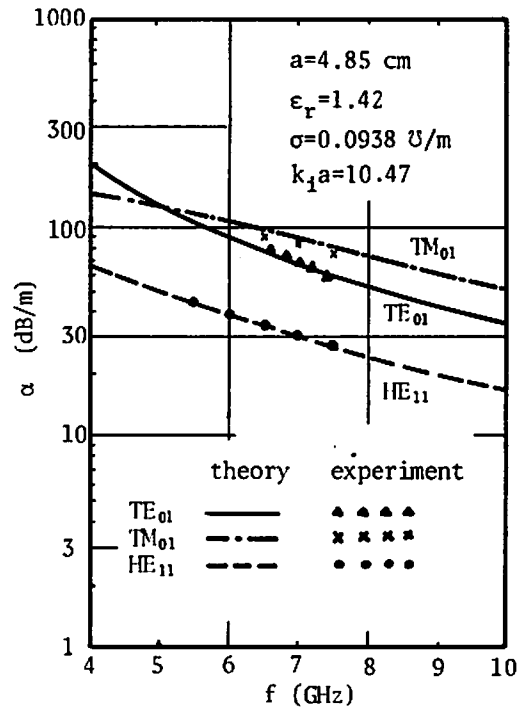


Fig.9

### Conclusion

Attenuation of normal modes obtained through numerical computation shows that the minimum-loss mode in ideal hollow dielectric waveguides is  $TE_{01}$  mode, and that  $EH_{11}$  mode has the second minimum loss. These theoretical results show fine agreements with experimental ones. But  $TE_{01}$  mode can exist only in the waveguides with circular cross section. Therefore in actual tunnels  $EH_{11}$  mode becomes the dominant. Additionally, it is proved experimentally that  $EH_{11}$  mode is excited quite easily in these structures.

### Reference

- [1] J. I. Glaser, "Attenuation and Guidance of modes on hollow dielectric waveguides" IEEE Trans. MTT, vol MTT, pp,173-174, Mar.1969
- [2] J. A. Stratton, "Electromagnetic Theory" New York: McGraw-Hill, 1941
- [3] Y. Yamaguchi, T. Sekiguchi, "Improvement of attenuation characteristics on hollow dielectric waveguide" IECE AP 77-62, Oct.1977