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A Matrix Doubling Formulation for the Calculation of Mueller Matrices of Internal Emission from a Layer with Irregular Boundaries

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Abstract A formulation for internal emission from a thermal scattering layered medium with irregular boundaries is developed. The matrix doubling method is utilized to obtain the upward and downward thermal intensities inside the scattering layer. The scattering layer is assumed to be a slab imbedded with randomly-positioned thermal scattering particles, and the layer top and bottom boundaries are assumed to be random rough surfaces. The developed emission model may be useful for the understanding of the internal emission problem where a radiometer is located inside an emitting medium with irregular rough boundaries.

1 Introduction

In the study of passive microwave and optical remote sensing, it is often important to understand the radiometric emission behavior of area-extensive targets often encountered in natural environments. Recently, Twomey considered the problem of estimating internal emission [1], by extending the existing matrix-doubling emission model to planetary atmospheric passive sensing problems at optical frequencies. By internal emission, we mean the upwelling and downwelling emission process which takes place *within* the scattering layer which is embedded with a random collection of thermal particles. When one considers the measurements of sky or atmospheric temperatures with airborne passive sensors (radiometers, for instance), the effects of sea or terrain boundaries on the temperature measurements may be significant.

Hence, it is of interest to study how the boundaries may affect the radiometric emission behavior. The purpose of this paper is to develop an internal emission model for a thermal slab with rough interfaces, by generalizing Twomey's internal emission model [1]. In the next section, a theoretical formulation for internal emission for a slab without the boundaries is first presented. Then we develop an internal emission model which accounts for the boundary effects.

2 Emission from Plane-Parallel Medium

2.1 Emission from Unbounded Layer

We first consider an emission contribution from an infinitesimal layer of optical depth $\Delta\tau$ which resides in the thick scattering layer of τ_l. The up and downward intensities $\hat{u}$ and $\hat{v}$ at location τ_2 due to a small infinitesimal $\Delta\tau$ emitting layer located at τ_1 is given by

$$\begin{aligned}\hat{v}(\tau_2,\tau_1) &= (1-\omega)b\Delta\tau(I-S_{12}^{*}S_3)^{-1}(D_2+E_2)(I-S_1^{*}S_2)^{-1} \\ &\quad \cdot\ (I+S_1^{*})M^{-1}\hat{e} \qquad (2.1)\\ \hat{u}(\tau_2,\tau_1) &= S_3\hat{v}(\tau_2,\tau_1) \qquad (2.2)\\ S_{12}^{*} &= S_2^{*}+(D_2+E_2)(I-S_1^{*}S_2)^{-1}S_1^{*}(D_2^{*}+E_2)\end{aligned}$$

Note that the expressions given in Eq.(2.1) and (2.2) are in terms of the Green's function which is the response to the delta-source-like thin layer; hence, the total emitting intensity measured at location τ_2 due the Green's functions over the whole source points τ_1 (τ_1 can be anywhere within the layer of thickness τ_l) is:

$$\hat{u}(\tau_2) = \int_0^{\tau_l}\hat{u}(\tau_2,\tau_1)/\Delta\tau\, d\tau_1 \qquad (2.3)$$

$$\hat{v}(\tau_2) = \int_0^{\tau_l}\hat{v}(\tau_2,\tau_1)/\Delta\tau\, d\tau_1 \qquad (2.4)$$

2.2 Emission from Bounded Layer

Consider the emission problem shown in Fig.1. The total effective scattering matrix $\bar{S}_1$ and the total effective transmission matrix $\bar{D}_1$ are given by

$$\begin{aligned}\bar{S}_1 &= R^{01}+Q^{10}S_1Q^{01}+Q^{10}S_1R^{10}S_1Q^{01}+....\\ &= R^{01}+Q^{10}S_1(I-R^{10}S_1)^{-1}Q^{01} \qquad (2.5)\\ \bar{T}_1 &= T_1Q^{01}+T_1R^{10}S_1Q^{01}+T_1R^{10}S_1R^{10}S_1Q^{01}+...\\ &= T_1(I-R^{10}S_1)^{-1}Q^{01} \qquad (2.6)\end{aligned}$$

Here, the notations R^{mn} and Q^{mn} (m,n = 0,1,3,4) signify the reflection and transmission phase matrices across the rough boundary between media m and n , when the incident intensity within medium m impinges on medium n. The reflection and transmission phase matrices account for the polarimetric surface scattering from the irregular randomly rough boundaries. Consider the case of incidence from below layer 1. The total effective scattering and transmission matrices are given by

$$\bar{S}_1^* = S_1^* + T_1 R^{10}(I - S_1 R^{10})^{-1} T_1^* \quad (2.7)$$
$$\bar{T}_1^* = Q^{10}(I - S_1 R^{10})^{-1} T_1^* \quad (2.8)$$

Next we consider the interaction which occurs at the lower boundary (interface between media 3 and 4). First, considering the multiple scattering and transmission which result from incidence from above the layer, we obtain the following effective scattering and transmission matrices:

$$\bar{S}_3 = S_3 + T_3^*(I - R^{34} S_3^*)^{-1} R^{34} T_3 \quad (2.9)$$
$$\bar{T}_3 = Q^{34}(I - S_3^* R^{34})^{-1} T_3 \quad (2.10)$$

Similarly, when the direction of the incident intensity is reversed, the effective scattering and transmission matrices at the lower boundary are given by

$$\bar{S}_3^* = R^{43} + Q^{34}(I - S_3^* R^{34})^{-1} S_3^* Q^{43} \quad (2.11)$$
$$\bar{T}_3^* = T_3^*(I - R^{34} S_3^*)^{-1} Q^{43} \quad (2.12)$$

Hence, the emission problem for the bounded thermal layer in Fig.1 may be thought of as a problem of emission from an equivalent thermal layer that has no boundaries. In view of Eq.(2.1) and (2.2), downward and upward intensities $\hat{v}_l$ and $\hat{u}_l$ at location τ_2 inside the bounded thermal layer are given by

$$\hat{v}_l(\tau_2) = \int_0^{\tau_l} (1-\omega) b (I - \bar{S}_{12}^* \bar{S}_3)^{-1} T_2 (I - \bar{S}_1^* S_2)^{-1} (I + \bar{S}_1^*) M^{-1} \hat{e}\, d\tau_1 \quad (2.13)$$
$$\hat{u}_l(\tau_2) = \int_0^{\tau_l} (1-\omega) b \bar{S}_3 (I - \bar{S}_{12}^* \bar{S}_3)^{-1} T_2 (I - \bar{S}_1^* S_2)^{-1} (I + \bar{S}_1^*) M^{-1} \hat{e}\, d\tau_1 \quad (2.14)$$
$$\bar{S}_{12}^* = S_2^* + T_2 (I - \bar{S}_1^* S_2)^{-1} \bar{S}_1^* T_2^*$$

It is possible to assume that the upward intensity from the bottom half space is a constant vector $\hat{u}_4$. The upward and downward intensities at τ_2 due to $\hat{u}_4$ are

$$\hat{u}_b(\tau_2) = (I - \bar{S}_3 \bar{S}_{12}^*)^{-1} \bar{T}_3^* \hat{u}_4 \quad (2.15)$$
$$\hat{v}_b(\tau_2) = \bar{S}_{12}^* (I - \bar{S}_3 \bar{S}_{12}^*)^{-1} \bar{T}_3^* \hat{u}_4 \quad (2.16)$$
$$\bar{S}_{12} = \bar{S}_1 + \bar{T}_1^* S_2 (I - \bar{S}_1^* S_2)^{-1} \bar{T}_1$$

When the nonscattering top half-space (medium 0) is at another constant temperature, its contribution measured at location τ_2 must be accounted for. The upward and downward intensities are shown to be

$$\hat{u}_t(\tau_2) = \bar{S}_3 (I - \bar{S}_{12}^* \bar{S}_3)^{-1} \bar{T}_{12} \hat{v}_0 \quad (2.17)$$
$$\hat{v}_t(\tau_2) = (I - \bar{S}_{12}^* \bar{S}_3)^{-1} \bar{T}_{12} \hat{v}_0 \quad (2.18)$$
$$\bar{T}_{12} = T_2 (I - \bar{S}_1^* S_2)^{-1} \bar{T}_1$$

It is possible to evaluate the total emission inside the layer at location τ_2 by adding up three different contributions, namely the scattering layer contributors $\hat{u}_l(\tau_2), \hat{v}_l(\tau_2)$, the bottom half-space contributors $\hat{u}_b(\tau_2), \hat{v}_b(\tau_2)$, and the top half-space contributors $\hat{u}_t(\tau_2), \hat{v}_t(\tau_2)$:

$$\hat{u}(\tau_2) = \hat{u}_l(\tau_2) + \hat{u}_b(\tau_2) + \hat{u}_t(\tau_2) \quad (2.19)$$

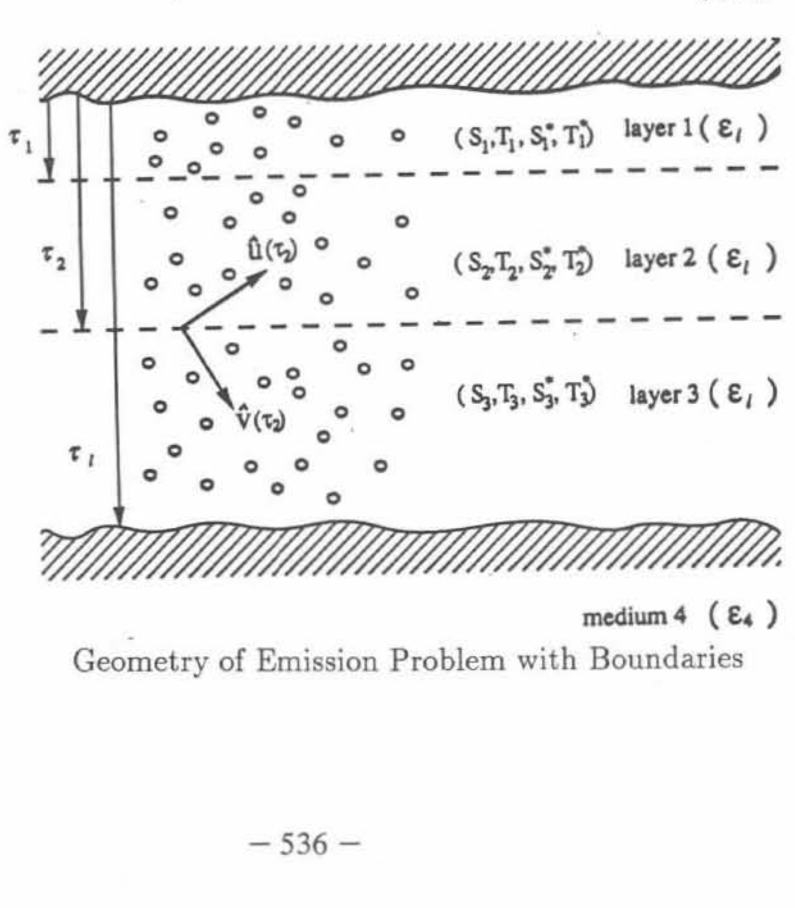
$$\hat{v}(\tau_2) = \hat{v}_l(\tau_2) + \hat{v}_b(\tau_2) + \hat{v}_t(\tau_2) \quad (2.20)$$

3 Conclusions

The internal emission problem for a scattering layer bounded by irregular boundaries is considered. The formulation, which is based on the matrix doubling method, gives the closed form solution for emission inside the thermal layer with interfaces. This theoretical formulation is useful in the study of atmospheric and terrestrial temperature sensing, in particular, when boundary effects are significant.

References

[1] S.Twomey,"Green's function formulae for the internal intensity in radiative transfer computations by matrix-vector methods" JQSRT vol.33, no.6, pp.575-579, 1985



Geometry of Emission Problem with Boundaries