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A NEW METHOD TO OBTAIN THE OPTIMAL POLARIZATION STATES IN THE CO-POLARIZED CHANNEL

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1. Introduction

As regards the characteristic polarization states of a radar target for the completely polarized wave case, Boerner et al. [1],[2] have already derived eight characteristic polarization states based on the polarization transformation ratio, for which a radar receives optimum power. These states are two co-polarization maximums (CO-POL Maxs), two co-polarization nulls (CO-POL Nulls), two cross-polarization maximums (X-POL Maxs), two cross-polarization nulls (X-POL Mins), and two cross-polarization saddles (X-POL Saddles). Since the pair of X-POL Nulls and CO-POL Maxs is identical, there exists a total of eight physical characteristic polarization states. To obtain the CO-POL Maxs and CO-POL Nulls, some authors used the Lagrangian multiplier method and then solved nonlinear equations. So, it is not easy to obtain the characteristic polarization states in the co-polarized radar channel.

In this paper, a new method is provided to obtain the CO-POL Maxs and CO-POL Nulls very easily for the symmetric/asymmetric scattering matrix case. By use of this method, the formula of the CO-POL Maxs is presented. Finally, one numerical example is given, showing the results are identical with [2], [7].

2. The Formula for CO-POL Maxs

Let

$$T = \begin{bmatrix} t_1 & t_2 \\ t_2 & t_3 \end{bmatrix} \tag{1}$$

denote a scattering matrix of a radar target, $\vec{a}$ denote the polarization state of the transmitter as given by

$$\vec{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

and $\|\vec{a}\|=1$. Then the received power in the co-polarized radar channel can be expressed as

$$P = \left| T\vec{a} \cdot \vec{a} \right|^2 \tag{2}$$

Using Cauchy-Schwarz inequality, we know from (2) that P will be maximal if and only if

$$\vec{a} = \left(T\vec{a} \right)^* / \|T\vec{a}\| \tag{3}$$

where * denotes conjugation. Substituting (3) into (2), we have

$$P = \| T\vec{a} \|^2 = T^{\dagger} T\vec{a} \cdot \vec{a}^* \tag{4}$$

where † denotes conjugate transpose.

Let $T_p = T^{\dagger}T$, then the matrix is so-called power matrix [3]. From (1), we have

$$T_p = \begin{bmatrix} |t_1|^2 + |t_2|^2 & t_2 t_1^* + t_3 t_2^* \\ t_1 t_2^* + t_2 t_3^* & |t_2|^2 + |t_3|^2 \end{bmatrix} \tag{5}$$

It can be proved easily that

$$T_p = \frac{1}{2}\left(|t_1|^2 + 2|t_2|^2 + |t_3|^2\right) I - i\,\mathrm{Re}(t_1 t_2^* + t_2 t_3^*)\,K$$
$$+ \frac{1}{2} i\left(|t_1|^2 - |t_3|^2\right) L + i\,\mathrm{Im}(t_1 t_2^* + t_2 t_3^*)\,J \tag{6}$$

where I, J, K and L are Pauli matrices [4], and

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$
$$K = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix} \quad L = \begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix}$$

Let

$$\vec{g} = \left(1,\ g_1,\ g_2,\ g_3\right)^t \tag{7}$$

denotes the Stokes vector of $\vec{a}$, where t denotes transpose, then we know

$$g_1 = iL\vec{a}\cdot\vec{a}^*,\quad g_2 = -iK\vec{a}\cdot\vec{a}^*,\quad g_3 = iJ\vec{a}\cdot\vec{a}^* \tag{8}$$

Substituting (5)~(8) into (4), we can obtain

$$P = \frac{1}{2}\left(|t_1|^2 + 2|t_2|^2 + |t_3|^2\right) + \frac{1}{2}\left(|t_1|^2 - |t_3|^2\right) g_1$$
$$+ \mathrm{Re}(t_1 t_2^* + t_2 t_3^*)\,g_2 + \mathrm{Im}(t_1 t_2^* + t_2 t_3^*)\,g_3$$
$$= \frac{1}{2}\left(|t_1|^2 + 2|t_2|^2 + |t_3|^2\right) + v_1 g_1 + v_2 g_2 + v_3 g_3 \tag{9}$$

where

$$v_1 = \frac{1}{2}\left(|t_1|^2 - |t_3|^2\right) \qquad v_2 = \mathrm{Re}(t_1 t_2^* + t_2 t_3^*)$$
$$v_3 = \mathrm{Im}(t_1 t_2^* + t_2 t_3^*) \qquad v = \sqrt{v_1^2 + v_2^2 + v_1^2} \tag{10}$$

From (9), we know that P will be maximal if

$$g_i = v_i/v \qquad i = 1, 2, 3 \tag{11}$$

Notice that, if $\vec{g}$ denotes one state of the CO-POL Maxs, the other one is

$$\left(1,\ -g_1,\ -g_2,\ -g_3\right)^t$$

So, the CO-POL Maxs are

$$\vec{g} = \left(1,\ \pm v_1/v,\ \pm v_2/v,\ \pm v_3/v\right)^t \tag{12}$$

where v_i and v are given by (10).

Note that for the asymmetric scattering matrix (denoted by S) case, the power expression in the co-polarized radar channel remains the same if the scattering matrix S is replaced by a symmetric scattering matrix $T = (S + S^{\tau})/2$. In this way, we can also obtain the CO-POL Maxs by the above method.

3. The method for obtaining the CO-POL Nulls

Let S denote a scattering matrix of a radar target, where S may be a asymmetric matrix. From (2), we know that $P = 0$ if and only if

$$S\vec{a} = \lambda \begin{bmatrix} a_2 \\ -a_1 \end{bmatrix} = \lambda \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \vec{a}$$

or

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} S\vec{a} = \lambda \vec{a} \tag{13}$$

Obviously, this eigenvalue problem can be solved easily. The two eigenvectors are so-called formulations of the CO-POL Nulls.

4. Numerical Example

If a scattering matrix is

$$T = \begin{bmatrix} 2i & \frac{1}{2} \\ \frac{1}{2} & -i \end{bmatrix}$$

Using the formula (12), we obtain the two CO-POL Maxs (expressed by Stokes vectors) are

$$\vec{g} = (1, \pm\frac{\sqrt{2}}{2}, 0, \pm\frac{\sqrt{2}}{2})^t$$

From (13), we can obtain the two CO-POL Nulls are

$$\vec{a} = (i, -\frac{1}{2} \pm \frac{\sqrt{7}}{2} i)^t$$

or (expressed by Stokes vectors)

$$\vec{g} = (1, -\frac{1}{3}, \pm\frac{\sqrt{7}}{3}, -\frac{1}{3})^t$$

These results are identical with [6][7]. Fig.1 shows the CO-POL Maxs and CO-POL Nulls on the Poincaré sphere.

5. Conclusion

In this paper, a new method is provided for obtaining the characteristic polarization states in the co-polarized channel. It is pointed out that, the CO-POL Maxs can be obtained very easily by our formula. Finally, one numerical example is given, and the results are identical with [2],[7]. This shows the validity of our formula.

References

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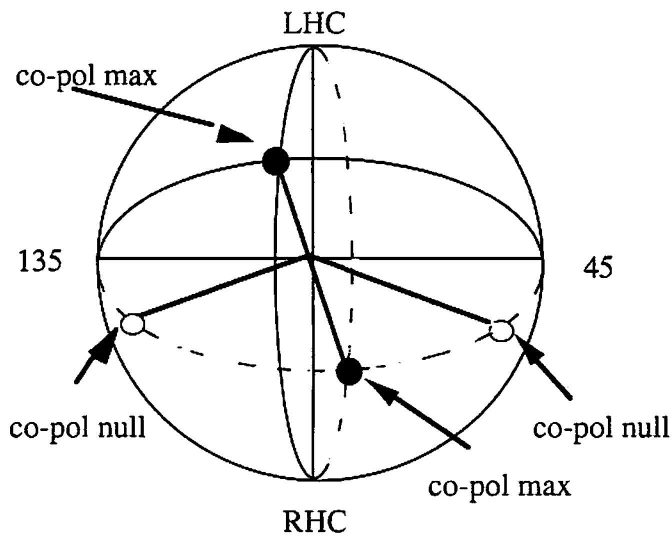

Fig.1 The CO-POL Maxs and CO-POL Nulls on the Poincaré sphere