

DIRECTION-OF-ARRIVAL ESTIMATION OF COHERENT WAVES WITH NONUNIFORM ARRAY BY USING SUPERRESOLUTION TECHNIQUE

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1. Introduction

In estimation of direction-of-arrival(DOA) of incident waves, an antenna array is often utilized. Superresolution techniques (such as the MUSIC algorithm[1]) have been attracting attention in this problem. However, when the waves are fully correlated(coherent), their correlation must be suppressed before applying the MUSIC algorithm. On the other hand, MODE(Method Of Direction Estimation) algorithm[2] which is one of the superresolution techniques can be applied directly to them without any signal preprocessings. Applying the MODE algorithm to the coherent signal estimation, as well as the MUSIC algorithm, array of sensors must be uniformly spaced. In this case, array length is limited by the number of sensors. Estimation errors of DOAs highly depend on the array length so that a nonuniform array is often preferable for small number of sensors. Here, the term “nonuniform” means that the sensors are located in the separation of multiple integer of “ Δx ” where “ Δx ” is the minimum separation. For example, when we denote a 4 element uniform array as [1111], one of the nonuniform array can be written by [110101]. In this example, the array length become 5/3 times longer than the uniform array. Hence, estimation performance will be improved. To apply the MODE algorithm to such a nonuniform array, we also adopt EM(Expectation Maximization) algorithm. The EM algorithm is essentially used for interpolation of received array data[3]. In this algorithm, initial DOA values(first guess) are required, then the values are renewed iteratively. Therefore, the initial value selection is important for convergence and true DOA estimation.

We investigate number of required iterations and successfully estimated ratio by computer simulations, and show that number of required iterations is less than 20 in this example. And almost 48.5 % of trials are successfully estimated even when initial values are selected randomly. This rate will be improved when a prior DOA information is available. Also, simulation results of RMSE(Root-Mean-Square-Error) of DOA are presented to show validity of the method.

2. Problem Formulation

We consider an array of L sensors impinging d narrowband waves. The received data at x_l , $r(x_l)$, are defined by

$$r(x_l) = \sum_{k=1}^d s_k e^{j2\pi \frac{x_l}{\lambda} \sin \theta_k} + n(x_l), \quad (1)$$

where s_k and θ_k denote the signal amplitude and DOA of the k -th signal respectively, and λ is wave-length of the waves. $n(x_l)$ is the noise term having zero mean and variance of σ^2 . Equation (1) can be expressed using vector notation as follows:

$$\mathbf{r} = \mathbf{A}\mathbf{s} + \mathbf{n}, \quad (2a)$$

$$\mathbf{r} = [r(x_1), r(x_2), \dots, r(x_L)]^T, \quad (2b)$$

where T denotes transpose, \mathbf{A} is the steering-vector matrix, and \mathbf{s} and \mathbf{n} are signal and noise vector, respectively. Hence, the data correlation matrix can be given by

$$\mathbf{R} = E[\mathbf{r}\mathbf{r}^H] = \mathbf{A}\mathbf{S}\mathbf{A}^H + \sigma^2\mathbf{I}, \quad (3)$$

where H denotes complex conjugate transpose, and $E[\cdot]$ is the ensemble average.

3. EM-MODE algorithm

EM algorithm is based on the concept that the complete data can be estimated from incomplete data set. This idea can be applied to the nonuniform array. This algorithm consists of two steps called E-step(Expectation-step) and M-step(Maximization-step). We apply the MODE algorithm to M-step in this paper.

E-step(1-th)

Set the expectation value of uniform(complete) data $\mathbf{r}^{(p)}$ from the realistic(incomplete) one \mathbf{r}_a with initial value(s), where p denotes the number of iterations in this algorithm.

M-step(MODE algorithm)

Calculate the correlation matrix \mathbf{R} from $\mathbf{r}^{(p)}$, and estimate DOA by the MODE algorithm. This algorithm is based on the ML(Maximum Likelihood) method that can be stated the minimization problem of following $f(\mathbf{b})$ with respect to $\mathbf{b} = [b_0, b_1, \dots, b_d]^T$.

$$f(\mathbf{b}) = \| (\mathbf{B}^H \mathbf{B})^{-\frac{1}{2}} (\mathbf{B}^H \mathbf{E}_S) (\Lambda_S - \sigma^2 \mathbf{I})^{\frac{1}{2}} \|^2, \quad (4)$$

where Λ_S is a diagonal matrix having d signal eigenvalues of \mathbf{R} , and \mathbf{E}_S is a matrix whose columns are the eigenvectors spanning signal subspace. In Eq.(4), \mathbf{B}^H is the matrix consisted of the elements of \mathbf{b} . $\|\cdot\|$ stands for the Euclidean norm. It is well known that \mathbf{b} which minimizes Eq.(4) have the property:

$$b_0 z^d + b_1 z^{d-1} + \dots + b_{d-1} z + b_d = b_0 \prod_{k=1}^d (z - e^{j\frac{2\pi\Delta x}{\lambda} \sin \theta_k}). \quad (5)$$

From Eq.(5), we can estimate angle θ_k of the k -th($k = 1, 2, \dots, d$) signal.

E-step(2-th ~)

Reconstruct the complete data $\mathbf{r}^{(p+1)}$ from \mathbf{B}^H of M-step, $\mathbf{r}^{(p)}$, and \mathbf{r}_a .

Evaluation of convergence

Check convergence of the data vector \mathbf{r} . If \mathbf{r} is not converged, set $p = p + 1$ and go back to the M-step.

In the convergence check, we use following condition :

$$\beta = \frac{\| \hat{\mathbf{A}} (\hat{\mathbf{A}}^H \hat{\mathbf{A}})^{(-1)} \hat{\mathbf{A}}^H \mathbf{r}^{(p+1)} - \mathbf{r}^{(p)} \|}{\| \mathbf{r}^{(p)} \|} < \varepsilon_1, \quad (6a)$$

$$|\beta^{(p+1)} - \beta^{(p)}| < \varepsilon_2, \quad (6b)$$

where $\hat{\mathbf{A}}$ is the steering-vector matrix constructed by the estimated angles θ_k in each iteration of the EM algorithm. In this paper, we use $\varepsilon_1 = 0.09$ and $\varepsilon_2 = 1 \times 10^{-4}$.

4. Numerical Example

First, we show an estimated example by the EM-MODE method and its convergence property. Consider a uniform linear array of 5 sensors whose minimum element separation is $\lambda/2$. Now assume that #2 and #5 elements are skipped, then arrangement of array elements described by [1011011] in which 1 means that element exist, 0 means skipped. DOAs of the incident coherent waves are assumed to be 10° and 20° . And the waves have unit power. In this example, we use the initial values of 15° and 25° in the

EM-MODE algorithm. Estimated result is shown in Fig.1(a). Behavior of convergence value of β and estimated DOA in each iteration are shown in Fig.1(b). From these figures, we see that the estimated DOAs are converged to the inherent angles with adequate number of iterations by the EM-MODE algorithm.

Next, we show influence of the initial values to the estimated angles. Array parameters are the same as the first example. Figure 2(a) shows the convergence map of each initial value combinations (θ_1, θ_2) where \circ denotes successfully estimated and \bullet means fail. Figure 2(b) shows the convergence value of β for each initial values. In this simulation, probability of success is about 48.5%. Hence, this implies that even if initial values are selected randomly, the proposed method detect them successfully at rate of 48.5 %. From Fig.1(b), when this method cannot converge after 15 iterations, it is better to select different initial values and estimate again.

Finally, we show that resolution of DOA in nonuniform array is superior to that in uniform linear array. Here, we use the MODE algorithm for the DOA estimation in uniform array also. We examine three arrays as follows:

- array1:** nonuniform array of 5 sensors, where #2 and #5 elements are skipped from uniform linear array (element arrangement [1011011])
- array2:** uniform linear array of 7 sensors (element arrangement [1111111])
- array3:** uniform linear array of 5 sensors (element arrangement [11111])

Figure 3 and Figure 4 show the RMSE at $SNR = 20$, and 30 [dB], respectively. From these figures, we can see that the RMSE of array 1 is almost equal to that of array 2. This means that performance of the 7 elements linear array can be obtained with only the 5 element array.

5. Conclusion

In this paper, we showed that EM-MODE algorithm can analyze coherent signals with nonuniform array. This algorithm is available for high accuracy DOA estimation with small number of element array.

References

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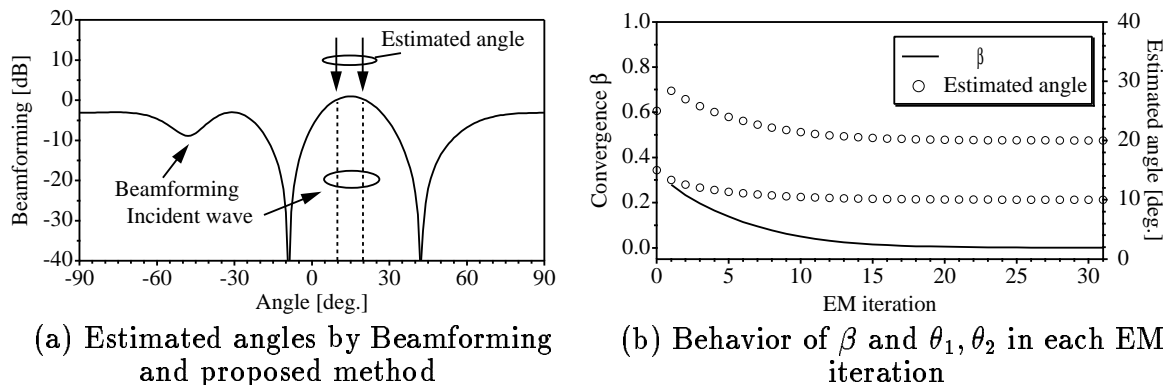
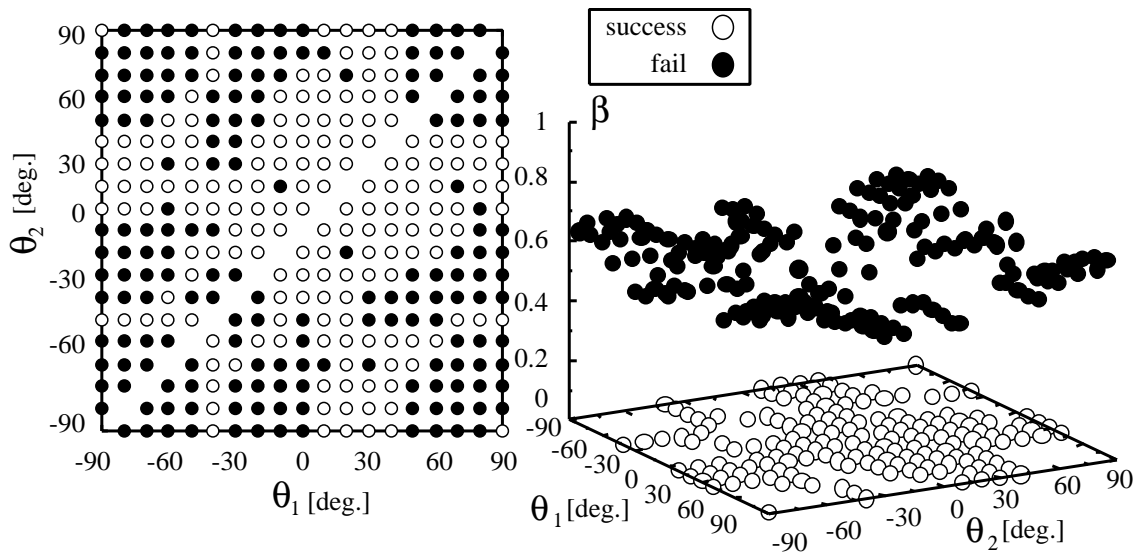
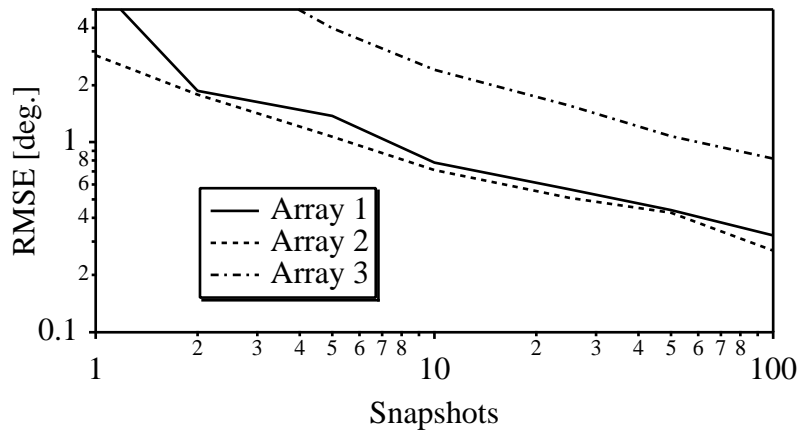


Fig.1 Conclusion of estimation DOA.

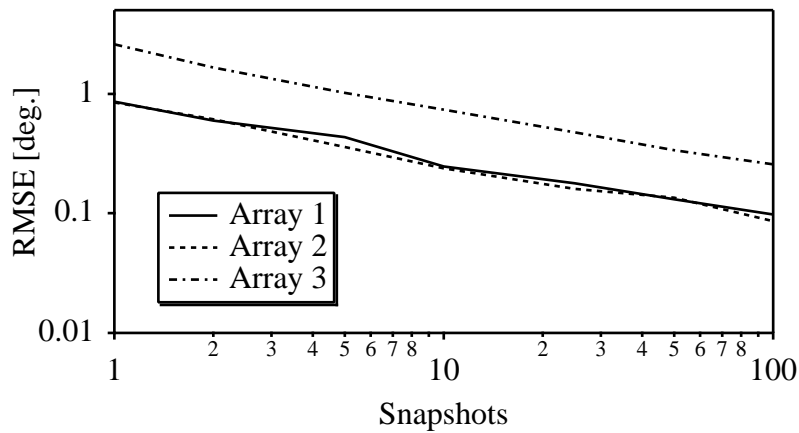


(a) Convergence map of each pair of initial values (b) Convergence value of β for each pair of initial values

Fig.2 Influence of initial value.



(a) RMSE at $SNR = 20$ [dB]



(b) RMSE at $SNR = 30$ [dB]

Fig. 3 Relation between RMSE and snapshots of #1 wave.