

Experiment of DOA Estimation with RD-CUBA-MUSIC Using 7-element ESPAR Antennas

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1. Introduction

The ESPAR (Electronically Steerable Parasitic Array Radiator) antenna [1] is an attractive antenna to steer the beam a full 360 degrees by means of varactor-loaded parasitic elements surrounding the main monopole radiator. Because of its low-cost, small size, and low power consumption, the application of the ESPAR antenna to user terminals and DOA (Direction Of Arrival) detection are being vigorously researched. However, because the RF port is only one system, the DBF (Digital Beam Forming) approach is not applicable. Thus, "Reactance Domain Signal Processing (RD)", which is peculiar to the ESPAR antenna, has been proposed [2].

In recent years, many DOA estimation methods that use the RD technique in a circular ESPAR antenna have been reported. Among these papers, SSP (Spatial Smoothing Preprocessing) by diamond sub-array [3] and by CUBA (Circular Uniform Beam Arrays) [4]-[6] has been proposed for the DOA estimation of coherent waves. Here, the former is limited to the estimation of up to three coherent waves due to its 7-element hexagonal structure, while the latter has the advantage that the number of DOA estimations increases when the number of elements increases. However, there was a problem in that the DOA estimation was difficult unless number of after-converted frequency data was less than half of number of before-converted sampling data. So it was considered to be impossible to apply it to a 7-element ESPAR antenna.

This paper shows, first of all, that the cause of this problem is frequency data with remarkably deteriorated accuracy. As a solution, the deteriorated data is shifted to the end, and then removed. Finally, this paper shows that RD-CUBA-MUSIC with a 7-element ESPAR antenna can estimate the DOAs of coherent waves in an experiment.

2. RD-CUBA-MUSIC

2.1. Beamforming of the ESPAR antenna

In general, a directional beam of the array antenna is given by the mode vector and the weight vector. The mode vector is shown as a function of the azimuth angle  $\theta$  as follows:

$$\mathbf{a}(\theta) = \left[ 1, \exp(j \frac{2\pi d}{\lambda} \cos(\theta)), \exp(j \frac{2\pi d}{\lambda} \cos(\theta - \frac{1}{3}\pi)), \dots, \exp(j \frac{2\pi d}{\lambda} \cos(\theta - \frac{5}{3}\pi)) \right]^T \quad (1)$$

Here,  $\lambda$  is a wave length,  $d(= \lambda / 4)$  is an element interval, and superscript  $T$  denotes the transpose.

Because the RF port of the ESPAR antenna is only one system, the weight vector which is used with a conventional DBF array antenna is not applicable. So, the equivalent weight vector is derived as a function of the reactance of the varactor diode [7].

$$\mathbf{w}_m = 2z_s (\mathbf{Z} + \text{diag}[z_s, jx_{m1}, jx_{m2}, \dots, jx_{m6}])^{-1} \mathbf{u}_0 \quad (m=1, \dots, 6) \quad (2)$$

where  $\mathbf{w}_m$  is an equivalent weight vector forming the  $m$ -th directional pattern, and  $\mathbf{Z}$  is an impedance matrix including mutual coupling between elements.  $z_s$  is the internal impedance of the ESPAR antenna, and  $\mathbf{u}_0$  is a unit vector shown by  $[1, 0, 0, \dots, 0, 0]^T$ .

However, each element of impedance matrix  $\mathbf{Z}$  is actually unknown due to a fabrication loss. So, in this paper,  $\tilde{\mathbf{w}}_m$  is derived by employing the calibration scheme proposed in [2], where  $\tilde{\mathbf{w}}_m$  is the calibrated weight vector. The beam pattern is shown by

$$b(n\theta_0) = \tilde{\mathbf{w}}_m^T \mathbf{a}(n\theta_0) \quad (m=1, n=0, \dots, N-1) \quad \text{with} \quad (\theta_0 = \frac{2\pi}{N}). \quad (3)$$

## 2.2. RD-CUBA-MUSIC

In RD-CUBA by an ESPAR antenna with  $(N+1)$ -elements, the main beam directions are rotated by  $\theta_0$  as shown in Fig. 1. Here, when the beam pattern is assumed to be  $b(\theta)$ , an arrival signal with the complex amplitude  $s_i$  and the direction  $\theta_i$  is shown by the following equation.

$$\mathbf{y}_i = [y_i(0), y_i(\theta_0), \dots, y_i((N-1)\theta_0)]$$

$$y_i(n\theta_0) = s_i b(n\theta_0 - \theta_i) \quad (n=0, 1, \dots, N-1). \quad (4)$$

Moreover, because the antenna beam pattern is periodic in  $2\pi$ , the equivalent virtual aperture function is defined by the DFT (Discrete Fourier Transform):

$$\mathbf{b} = [b(0), b(\theta_0), \dots, b((N-1)\theta_0)] \quad (5)$$

$$\mathbf{F}_{NL} = \begin{bmatrix} 1 & \dots & 1 & \dots & 1 \\ 1 & \dots & e^{-j2\pi/N} & \dots & e^{-j2\pi(L-1)/N} \\ \vdots & \dots & \vdots & \dots & \vdots \\ 1 & \dots & e^{-j2\pi(N-1)/N} & \dots & e^{-j2\pi(N-1)(L-1)/N} \end{bmatrix} \quad (6)$$

and

$$\mathbf{B} = \mathbf{b}\mathbf{F}_{NL} \text{ with } \mathbf{B} = [B(0), B(s_0), \dots, B((L-1)s_0)], (s_0 = \frac{1}{2\pi}). \quad (7)$$

$\mathbf{F}_{NL}$  is an  $N \times L$  DFT matrix. Using (6), the DFT of (4) is shown by the following expressions:

$$\mathbf{Y}_i = \mathbf{y}_i \mathbf{F}_{NL}$$

$$= [s_i B(0), s_i B(s_0) \exp(-j\theta_i), \dots, s_i B((L-1)s_0) \exp(-j(L-1)\theta_i)]. \quad (8)$$

Equation (8) is standardized by  $\mathbf{B}$ , and with  $D$  waves impinging on the array, the virtual aperture function of the array output can be described with the element of  $L$  pieces of  $l=0$  to  $L-1$  by

$$\mathbf{Y} = \sum_{i=1}^D s_i \mathbf{v}(\theta_i) + \mathbf{n} \quad (9)$$

$$\mathbf{v}(\theta) = [1, \exp(-j\theta), \dots, \exp(-j\theta), \dots, \exp(-j(L-1)\theta)]. \quad (10)$$

$\mathbf{n}$  is a term concerning a noise component. The RD-CUBA-MUSIC is a DOA estimation scheme applying MUSIC to (9). Since each element of the mode-vector (10) increases by  $\theta$ , SSP can suppress the correlation between signals, and makes it possible to estimate the DOAs.

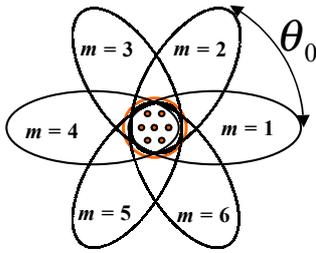


Fig. 1 Rotation of beam pattern ( $N=6$ ).

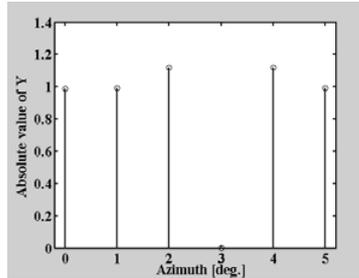


Fig. 2 Absolute value of  $\mathbf{Y}$ . (positive spatial frequency)

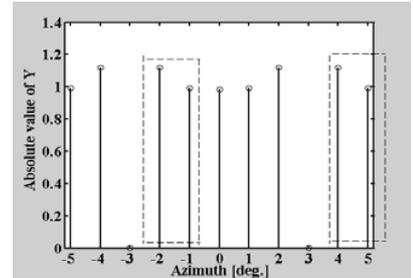


Fig. 3 Absolute value of  $\mathbf{Y}$ . (spatial frequency including negative)

## 2.3. Negative spatial frequency

It is impossible to estimate the DOAs unless the number  $L$  of after-converted frequency data is less than half of the number  $N$  of before-converted spatial sampling data. This is due to the remarkably deteriorated data that is in the vicinity of the center of the frequency data. Fig. 2 shows the absolute of  $\mathbf{Y}$  (Eq.9) by computer simulation. Assuming that the number of arriving waves is one, the absolute of  $\mathbf{Y}$  should become equal as understood from (9)(10). However, Fig. 2 shows that the data of  $l=3$  is remarkably deteriorated. Here, Fig. 3 shows the absolute of  $\mathbf{Y}$  including a negative area. At this time, the same data appears both in the positive area and in the negative area.

As a principle of the solution, the deteriorated data of  $l=3$  is removed, and only the data of  $l=-2$  to  $2$  is used. This method is conducted by converting  $\mathbf{F}_{NL}$  (Eq.6) into  $\hat{\mathbf{F}}_{NL}$  (Eq.11), which is shown as follows. Here,  $\hat{\mathbf{F}}_{NL}$  is an  $N \times L$  DFT matrix including a negative area. In this case, Eq.10 converts

the element of  $L$  pieces of  $l = -(L-2)/2$  to  $L/2$  which was made a vector and is shown by Eq.12.

$$\hat{\mathbf{F}}_{NL} = \begin{bmatrix} 1 & \dots & 1 & \dots & 1 \\ e^{j2\pi\frac{(L-2)}{2}/N} & \dots & 1 & \dots & e^{-j2\pi\frac{L}{2}/N} \\ \dots & \dots & \dots & \dots & \dots \\ e^{j2\pi\frac{(L-2)}{2}(N-1)/N} & \dots & 1 & \dots & e^{-j2\pi\frac{L}{2}(N-1)/N} \end{bmatrix} \quad L : \text{even number} \quad (11)$$

$$\mathbf{v}(\theta) = [\exp(j\frac{L-2}{2}\theta), \dots, \dots, \exp(-j\frac{L}{2}\theta)] \quad (12)$$

The phase has changed into  $\theta$  between each element in Eq.12. Here, the last deteriorated data is removed, and the SSP by the elements of  $L-1$  piece can distinguish coherent waves.

### 3. DOA Estimation Experiment

#### 3.1 Experimental settings

The experimental settings are shown in Table 1. This experiment was conducted in an anechoic chamber. Yagi-antennas are set toward  $45^\circ$ ,  $90^\circ$ ,  $135^\circ$ ,  $255^\circ$ , and  $270^\circ$  directions from the ESPAR antenna. The same PN sequence is transmitted from each yagi-antenna, and two or three arbitrary yagi-antennas are selected for coherent waves in a DOA estimation experiment. The modulation scheme is BPSK and the input SNR is set to 20 dB.

The number  $L$  is assumed to be six, which is the same as the number  $N$ . F/B (Forward Backward) -SSP is applied to five elements, all except the last element. The number of snapshot is 1000. For simplicity, the number of arriving waves is assumed to be already known. Table 2 shows the reactance sets of the varactor diode forming the  $m$ -th directional pattern. The measured beam pattern toward 0 degrees is shown in Fig.4.

Table 1 Experimental settings.

Items	Settings
Frequency	2.484 GHz
Input SNR	20 dB
Modulation	BPSK
Snapshot	P=1000 symbols
Symbol rate	500 kHz
Element number	7 (N=6)
Frequency Data	L=6
Sub Array	2 (antennas:4) F/B SSP

Table 2 Reactance sets forming directional patterns.

$m$	Pattern	$x_{m1}$	$x_{m2}$	$x_{m3}$	$x_{m4}$	$x_{m5}$	$x_{m6}$
1	$0^\circ$ direction	min	Max	Max	Max	Max	Max
2	$60^\circ$ direction	Max	min	Max	Max	Max	Max
3	$120^\circ$ direction	Max	Max	min	Max	Max	Max
4	$180^\circ$ direction	Max	Max	Max	min	Max	Max
5	$240^\circ$ direction	Max	Max	Max	Max	min	Max
6	$300^\circ$ direction	Max	Max	Max	Max	Max	min

(Max=0( $\Omega$ ) . min=-90( $\Omega$ ))

#### 3.2. Experiment using the proposed solution

Figure 5 shows the experimental result concerning two coherent waves coming from  $90$  and  $270$  degrees. The arrow in Fig.5 indicates the actual DOAs ( $90$ ,  $270$  degrees). Here, “ $l = 0$  to  $5$ ” means the one using the deteriorated data, and “ $l = -2$  to  $2$ ” means the one omitting the deteriorated data. In the former, the peak does not appear in the vicinity of the coming direction at all. In the latter, the peak appears in the direction of  $96$  and  $273$  degrees though slight estimation errors are caused.

#### 3.3. Experiment for two coherent waves with a close angle distance

Figure 6 shows the DOA estimation result of two coherent waves coming from  $45$  and  $90$  degrees. The peaks appear toward  $37$  and  $94$  degrees, which are in the vicinity of the coming direction. It is shown to be possible to estimate two coherent waves with an angle distance which arrive more than  $45$  degrees apart from each other.

#### 3.4. Experiment for three coherent waves

Figure 7 shows the DOA estimation result of three coherent waves coming from  $45$ ,  $135$ , and  $255$  degrees. The peaks appear toward  $40$ ,  $142$ , and  $250$  degrees, which are in the vicinity of the coming direction. It is shown that DOA can be estimated in the case of three coherent waves.

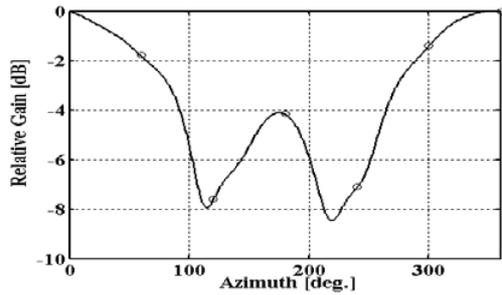


Fig. 4 Measured beam pattern  $|b(n\theta_0)|$ .

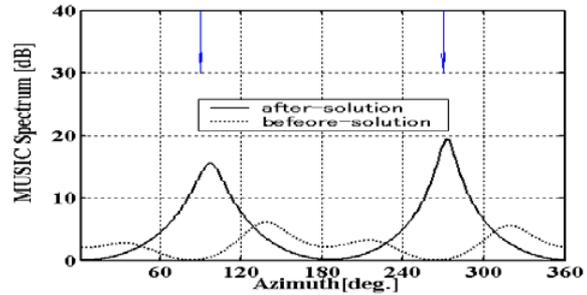


Fig. 5 DOA estimation experiment concerning two coherent waves arriving from 90 and 270 degrees. (Estimated direction : 96 and 273 degrees)

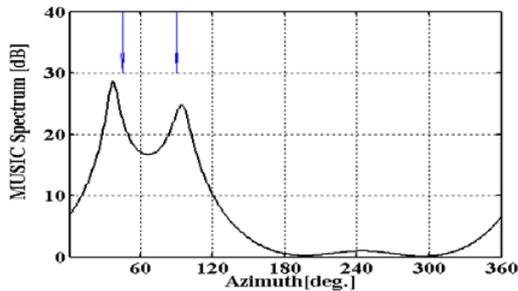


Fig. 6 DOA estimation experiment of two coherent waves from 45 and 90 degrees. (Estimated direction : 37 and 94 degrees)

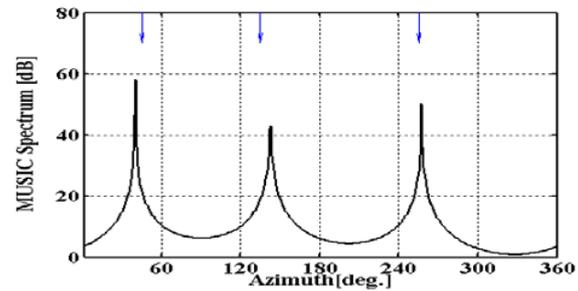


Fig. 7 DOA estimation experiment of three coherent waves from 255, 135 and 45 degrees. (Estimated direction: 250, 142 and 40 degrees)

### 3. Conclusion

It was difficult to apply CUBA-MUSIC to the 7-element circular ESPAR antenna, because of data with remarkably deteriorated accuracy. As a solution, SSP was used to make the coherent waves separate after the deteriorated data was shifted to the right end and removed.

This paper shows the experimental result of an anechoic chamber in which RD-CUBA was applied to a 7-element circular ESPAR antenna. It was thus shown that it is possible to estimate the DOAs with two coherent waves up to the angle distance of 45 degrees coming and three coherent waves coming.

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