

# DOA ESTIMATION BY USING MUSIC ALGORITHM WITH A 9-ELEMENTS RECTANGULAR ESPAR ANTENNA

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## 1 Introduction

The electronically steerable parasitic array radiator (ESPAR) antenna has several advantage, low-cost and low-consumption of power, etc. However, since the ESPAR antenna has only one output port, the antenna cannot obtain spatial data that can be easily obtained by the ordinary array. So, it was difficult to apply direction-of-arrival (DOA) algorithms for adaptive array antenna to ESPAR antenna directly. However, recently, a calibration technique using a matrix composed of several equivalent weight vectors which forms directional radiation patterns has reported. As a result, it is possible to apply superresolution algorithms (e.g. MUSIC, ESPRIT, etc) to DOA estimation with ESPAR antenna[1].

In this report, we show numerical and experimental results of DOA estimation using MUSIC algorithm with a 9-elements rectangular ESPAR antenna. We carry out DOA estimation of single wave incidence and two coherent waves incidence by numerical simulation with NEC2 and experiment in an anechoic chamber. In the estimation of coherent waves, 2-D spatial smoothing preprocessing(SSP)[2] with uniform rectangular arrays is required. A 9-elements rectangular ESPAR antenna can be divided into four overlapping 4-elements(2×2) rectangular subarrays. Thus, we use SSP with four rectangular subarrays with 4-elements for the coherent wave detection.

## 2 Problem Formulation and Estimation Algorithm

### 2.1 Reactance domain MUSIC algorithm

We assume that  $d$  uncorrelated waves impinge on a 9-elements rectangular ESPAR antenna as shown in Figure 1. A definition of azimuth angle  $\phi$  is shown in Figure 2. When we change the reactance value of each parasite element, we can obtain the output corresponding to the pattern formed by reactances. The measured output vector obtained by  $M$  independent reactance sets is expressed in vector  $\mathbf{y}(t)$  as follows[1]:

$$\mathbf{y}(t) = [y_1(t), y_2(t), \dots, y_M(t)]^T = \mathbf{W} \sum_{k=1}^d \mathbf{a}(\phi_k) s_k(t) + \mathbf{n}(t), \quad (1)$$

$$\mathbf{a}(\psi_k, \phi_k) = [e^{j\alpha_1}, e^{j\alpha_2}, \dots, e^{j\alpha_9}]^T, \quad (2)$$

$$\alpha_l = \frac{2\pi}{\lambda} (x_l \cos \phi_k + y_l \sin \phi_k), \quad (3)$$

where  $\mathbf{W}$  is an equivalent weight matrix,  $\mathbf{a}(\phi_k)$  is steering vector depending on the position of the elements of the modeled ESPAR antenna,  $\mathbf{n}(t)$  is a noise vector,  $\lambda$  is wave length,  $(x_l, y_l)$  is position of  $l$ -th element ( $l = 1, \dots, 9$ ),  $s_k$  and  $\phi_k$  is the complex amplitude and DOA of the  $k$ -th incident wave ( $k = 1, \dots, d$ ), respectively.  $T$  denotes transpose. When  $\mathbf{W}$  in (1) is known, we can apply MUSIC algorithm to transformed covariance matrix  $\mathbf{R}_s$  defined as:

$$\mathbf{R}_s = \mathbf{W}^{-1}(\mathbf{R}_y - \sigma^2 \mathbf{I})(\mathbf{W}^{-1})^H = \mathbf{A} \mathbf{S} \mathbf{A}^H, \quad (4)$$

$$\mathbf{R}_y = E[\mathbf{y} \mathbf{y}^H], \quad (5)$$

$$\mathbf{S} = E[\mathbf{s} \mathbf{s}^H], \quad (6)$$

where  $\mathbf{R}_y$  is a reactance-domain covariance matrix given in (5),  $\mathbf{S}$  is a signal covariance matrix given in (6), and  $\mathbf{s}$  is signal vector composed of  $s_k$ .  $\sigma^2$  is a noise power.  $E[\cdot]$  and  $H$  denotes Ensemble average and complex conjugate transpose, respectively.

## 2.2 SSP for the rectangular array

In this section, we briefly describe the SSP for  $\mathbf{R}_s$  given in (4). Since this is 9-elements( $3 \times 3$ ) rectangular ESPAR antenna, it can be divided into four rectangular subarrays with  $2 \times 2$  elements.  $\mathbf{R}_{11}$  is subarray covariance matrix composed of 1,2,4,5-th element,  $\mathbf{R}_{12}$ ,  $\mathbf{R}_{21}$ , and  $\mathbf{R}_{22}$  is the subarray covariance matrix defined as (#2,#3,#5,#6), (#4,#5,#7,#8), and (#5,#6,#8,#9) elements. Here, spatially smoothed covariance matrix  $\bar{\mathbf{R}}_s$  can be defined as the average of these four subarray covariance matrices.

$$\bar{\mathbf{R}}_s = \frac{1}{4} \sum_{m=1}^2 \sum_{n=1}^2 \mathbf{R}_{mn} = \mathbf{A} \bar{\mathbf{S}} \mathbf{A}^H, \quad (7)$$

where  $\bar{\mathbf{S}}$  is a spatial smoothed signal covariance matrix. In this definition, the subarrays shifted to x-direction and y-direction are included, therefore DOA dependence of the decorrelation is decreased.

## 3 Numerical and Experimental Result

### 3.1 Simulation and experiment at setup

First, we show parameters of simulation by NEC2. We modeled the ESPAR antenna as a 9-elements uniform rectangular array by parameters of dipoles. Radius of the wire is 0.5[mm] and length is 60[mm] for the dipole elements. Each element is divided by 11 segments. Loading impedance of the 5-th dipole is 100[ $\Omega$ ], and those of the other parasite dipole are the corresponding reactances. Azimuth angle of incident wave is varied every  $5^\circ$  with range from  $-180^\circ$  to  $180^\circ$  in each simulation. Frequency of incident wave is 2.484[GHz]. Element spacing is 3.0[cm]. The reactance value of parasite elements wire is set -201.10[ $\Omega$ ] or -27.67[ $\Omega$ ].

We also carried out experiment in an anechoic chamber. Transmitting antenna is horn antenna, and the 9-elements rectangular ESPAR antenna is set on the azimuth table. Azimuth angle of incident wave is measured every  $5^\circ$  with range from  $-180^\circ$  to  $180^\circ$  in each experiment. Frequency of incident wave is 2.484[GHz]. Element spacing is 3.0[cm]. Here we obtain only 1 snapshot.

In this report, we use 9 independent reactance sets listed in Table 1 to make  $\mathbf{W}$ , and 16 DOAs listed in Table 2 for estimation of  $\mathbf{W}$  both simulation and experiment.

Table 1: 9 independent reactance sets (unit:[ $\Omega$ ])

pattern	#1	#2	#3	#4	#6	#7	#8	#9
0	-27.67	-27.67	-27.6	-27.6	-27.6	-27.6	-27.6	-27.6
1	-201.10	-27.67	-27.6	-27.6	-27.6	-27.6	-27.6	-27.6
2	-27.67	-201.10	-27.6	-27.6	-27.6	-27.6	-27.6	-27.6
3	-27.67	-27.67	-201.10	-27.6	-27.6	-27.6	-27.6	-27.6
4	-27.67	-27.67	-27.6	-201.10	-27.6	-27.6	-27.6	-27.6
5	-27.67	-27.67	-27.6	-27.6	-201.10	-27.6	-27.6	-27.6
6	-27.67	-27.67	-27.6	-27.6	-27.6	-201.10	-27.6	-27.6
7	-27.67	-27.67	-27.6	-27.6	-27.6	-27.6	-201.10	-27.6
8	-27.67	-27.67	-27.6	-27.6	-27.6	-27.6	-27.6	-201.10

Table 2: Calibration DOAs for estimation of  $\mathbf{W}$

Azimuth angle [deg.]	-150,-135,-120,-90,-60,-45,-30,0,30,45,60,90,120,135,150,180
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### 3.2 DOA estimation of single wave incidence

In this section, we show results of DOA estimation of single wave incidence. Figure 3(a) and (b) show MUSIC spectrums where the wave impinges on  $-60^\circ$ ,  $0^\circ$ ,  $90^\circ$ . Three MUSIC spectrums are plotted in each figure. Sharp peaks can be observed in both simulation and experiment. Figure 4(a) and (b) show the result of simulation and experiment, respectively.

In result of simulation, the MUSIC works properly in all directions with almost no bias. In result of experiment, however, estimation error about  $\pm 5^\circ$  can be observed. These estimation error become small in  $0^\circ$ ,  $\pm 90^\circ$  or  $\pm 180^\circ$ . These error may be carried by the calibration and/or alightment error of the antennas.

### 3.3 DOA estimation of two coherent waves incidence

In this section, we show results of DOA estimation of two coherent waves incidence. DOA of one wave is fixed to  $0^\circ$ , and DOA of another wave is varies from  $-180^\circ$  to  $180^\circ$ . Here, we denote “WAVE 1” as the former wave, and “WAVE 2” as the latter one. Figure 5(a) and (b) show the MUSIC spectrums with and without the SSP. In these examples, DOAs of WAVE 1 and WAVE 2 are  $0^\circ$  and  $90^\circ$ , respectively. Two peaks can be clearly resolved with the SSP both numerically and experimentally. Next, we show results that DOA of WAVE 2 varies every  $5^\circ$  with range from  $-180^\circ$  to  $180^\circ$  where DOA of WAVE 1 is fixed to  $0^\circ$ . If two waves can be separated, estimation error of both WAVE 1 and WAVE 2 are plotted. Figure 6(a) and (b) show result of simulation and experiment, respectively. In result of simulation, the narrower the angle difference becomes, the larger the estimation error of both WAVE 1 and WAVE 2 becomes. Two waves can be resolved even when WAVE 2 is  $5^\circ$  with error less than  $\pm 8^\circ$  in simulation. In result of experiment, the two waves can be separated when angle difference is larger than  $25^\circ$ . Although, resolution threshold of the experimental result is worse than that of simulation, DOA of two waves can be estimated with error less than  $\pm 8^\circ$  in experiment.

## 4 Conclusions

In this report, we showed DOA estimation results using MUSIC algorithm with a 9-elements rectangular ESPAR antenna. We carried out DOA estimation of single wave and two coherent wave incidence by numerical simulation with NEC2 and experiment in anechoic chamber. For coherent wave detection, we employed MUSIC algorithm with SSP of 4 rectangular subarrays. In result of simulation, we showed MUSIC algorithm with 9-elements rectangular ESPAR antenna can estimate single wave without error, and two coherent waves even when angle difference is  $5^\circ$  with error less than  $\pm 8^\circ$ . In result of experiment, resolution performance was slightly deteriorated due to experimental error (calibration and/or alightment), however we could separate two waves when angle difference was  $25^\circ$ .

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## References

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- [2] Yih-Min Chen, IEEE Trans. on Signal Processing, “On Spatial Smoothing for Two-Dimensional Direction-of-Arrival Estimation of Coherent Signals,” vol.45, No.7, pp.1689–1696, July 1997.

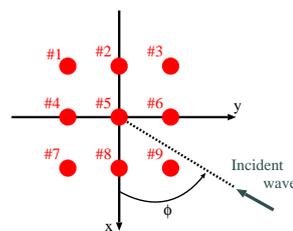
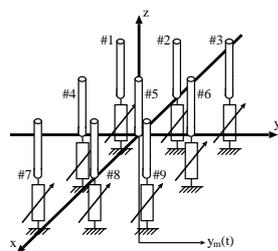
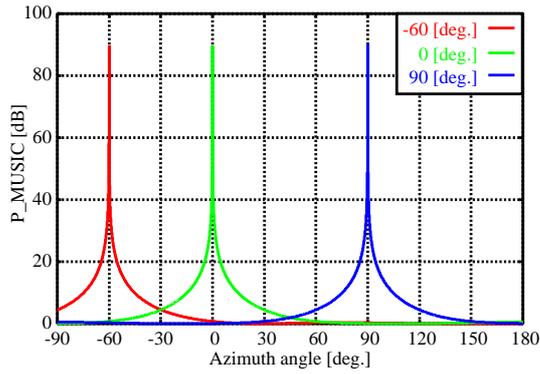
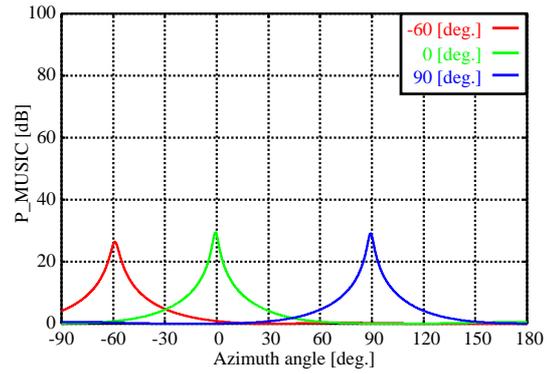


Figure 1: 9-Elements rectangular ESPAR antenna

Figure 2: Definition of azimuth angle

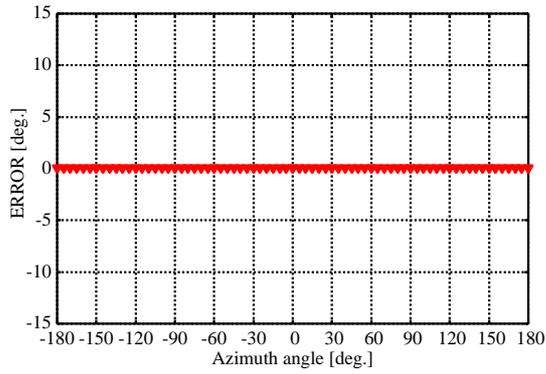


(a) Simulation results

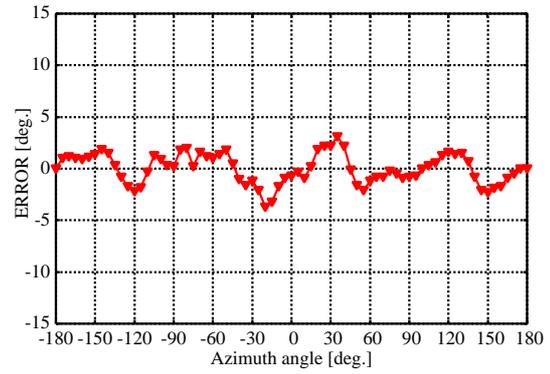


(b) Experimental result

Figure 3: MUSIC spectrum (single wave)

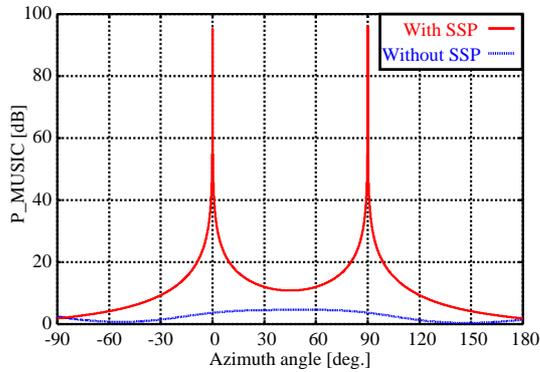


(a) Simulation results

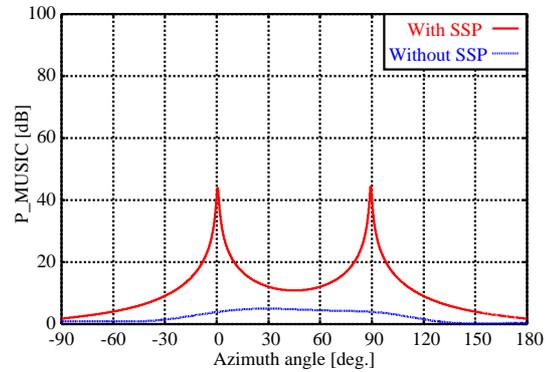


(b) Experimental result

Figure 4: DOA estimation error for single wave incidence

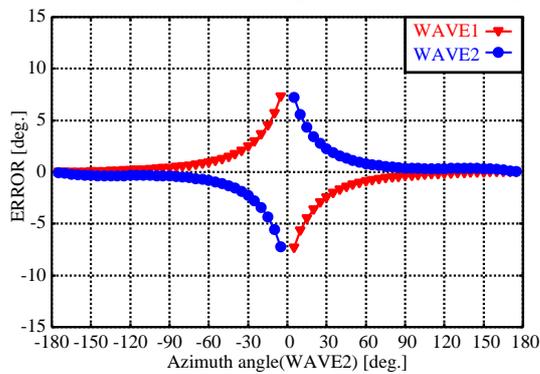


(a) Simulation results

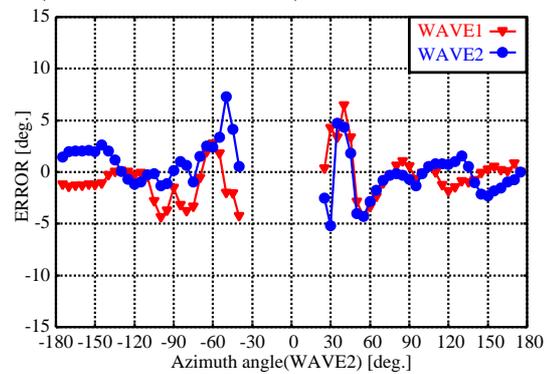


(b) Experimental result

Figure 5: MUSIC spectrum (two coherent waves)



(a) Simulation results



(b) Experimental result

Figure 6: DOA estimation error for two coherent waves incidence