# EFFECT OF ELEMENT SPACING FOR MIMO CHANNEL CHAPACITY IN RAYLEIGH FADING ENVIRONMENT

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## 1 Introduction

High data rates, high level of reliability, and large channel capacity have become important requirements for wireless communication systems recently because of recent progress of multimedia mobile communications. MIMO (Multiple-Input Multiple-Output) communication system which has several antennas as a transmit and a receive array have been attractive in wireless communication to realize large channel capacity [1]. Small sized terminal will often be required in real application. In such a case, we must use the array(s) of closely spaced elements.

In the array of closely spaced elements, mutual coupling among the elements become bigger as well as spatial correlation among their channels. The effects of the mutual coupling on a MIMO system have been reported by several authors. Jungnickel *et al.* have considered the the effect of the couplings by using link capacity in the practical point of view, and shown the validity experimentally [2]. In addition, theoretical and numerical consideration can be found in [3], for $2 \times 2$ MIMO system. However, the theoretical derivation seems to be difficult to extend for conventional $M \times N$ system.

In this report, we provide numerical results of MIMO capacity (link capacity) in the presence of spatial correlation and mutual coupling. As shown in [4],the mutual coupling matrix of the array can be easily obtained both numerically and experimentally. We introduce the mutual coupling matrix in the MIMO capacity evaluation, and show the channel capacity with and without the coupling, especially for the array with closely spaced elements.

## 2 Channel Matrix

In this report, a narrowband flat fading MIMO channel is considered. We only consider the effects of mutual coupling in the receiver. That means transmitting antenna is ideal (no mutual coupling).

The complex baseband representation of channel matrix on entire system is

$$\boldsymbol{H}_{total} = \boldsymbol{C}_a \boldsymbol{H}_s \tag{1}$$

where $\boldsymbol{C}_a \in C^{N\times N}$ is the mutual coupling matrix of the receiving array, and $\boldsymbol{H}_s \in C^{N\times M}$ becomes channel matrix including the effect of spatial correlation. Hence,$\boldsymbol{H}_{total}$ becomes the channel matrix of total MIMO system, which includes the effect of both of mutual coupling and spatial correlation.

Assming that the transmitted power of each branch at the transmitter is the same and the channel matrix can be correctly estimated at the receiver, the channel capacity can be written

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by

$$C = \sum_{i=1}^{m_0} \log_2(1 + \lambda_i \mathrm{SNR}), \quad \mathrm{m_0 = min(M,N)} \tag{2}$$

where $M, N$ denotes number of channels at the transmitter and the receiver, respectively. $\lambda_i$ is the singular value of $\boldsymbol{H}_{total}^H \boldsymbol{H}_{total}/H_0$ or $\boldsymbol{H}_{total}\boldsymbol{H}_{total}^H/H_0$, where the superscript $^H$ denotes complex conjugate transpose. The normalized factor,$H_0$, is defined as [5]

$$H_0 = \sqrt{\frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N} E[|h_{ij}|^2]} \tag{3}$$

where $h_{ij}$ denotes $(i, j)$ element of the channel matrix, and $E[\cdot]$ is the ensemble average. As denoted in [2], the normalized factor should be carefully selected. If we define $h_{ij}$ in (3) as the element of $H_{total}$ in (1), the effects of the path gain variation will be removed. In evaluation of link capacity variation due to mutual coupling (as a function of array spacing), path gain variation will be the important factor because the directivity of each element changes due to the couplings. In this report, therefore, $H_0$ of the ideal array (no mutual coupling and no spatial correlation) is selected.

# 3 Configuration of Simulation

We consider a $4 \times 4$ MIMO system here. 4-element uniform linear array with half-wavelength dipoles is employed as the receiver. Each element is terminated by $Z_0 = 50\Omega$. Element length of the dipoles is 0.464 $\lambda$ and radius of 0.004 $\lambda$, where $\lambda$ is the wavelength. The mutual coupling matrix with the element spacing $d$, $\boldsymbol{C}_a(d)$, is calculated by the Moment method [4].

The channel matrix $\boldsymbol{H}_s$ includes spatial correlation corresponding to the element spacing and the propagation environment. This channel matrix is calculated with a Rayleigh fading model shown in Figure 1. The problem to be considered here is the effect of mutual coupling in channel (link) capacity. Therefore, we adopt the normalized factor $H_0$ of an ideal array including the termination effect of the receiver ($Z_0$). That is,

$$\boldsymbol{H}_0 = \boldsymbol{C}_a(d)|_{d\to\infty}\boldsymbol{H}_{i.i.d} \tag{4}$$

where $\boldsymbol{H}_{i.i.d}$ denotes the i.i.d (independent identically distributed) channel matrix, and $\boldsymbol{C}_a(d)|_{d\to\infty}$ denotes the mutual coupling matrix for an array of enough-spacing dipoles. According to [6], it becomes

$$\boldsymbol{C}_a(d)|_{d\to\infty} = \left(1 + \frac{Z_{11}}{Z_0}\right)\boldsymbol{I} \tag{5}$$

where $Z_{11}$ is the input impedance of the dipole elements, and $\boldsymbol{I}$ is a $4 \times 4$ identity matrix.

The following channel matrices are evaluated in the next section to show the effects of mutual couplings.

- Channel matrix including the effect of both of mutual coupling and spacial correlation($\boldsymbol{C}_a\boldsymbol{H}_s$)
- Channel matrix including the effect of spatial correlation only ($\boldsymbol{C}_a(d)|_{d\to\infty}\boldsymbol{H}_s$)

# 4 Result and Consideration of Simulation

Here, we define channel capacity loss $C_{loss}$ as a function of the ratio of the channel capacity with and without mutual coupling. It is defined as spacing of $d$.

$$C_{loss} = 1 - \frac{C_{co}(d)}{C_{sp}(d)} \tag{6}$$

where $C_{co}(d)$ and $C_{sp}(d)$ denote the channel capacity with and without mutual coupling, respectively. Note that the spatial correlation is included in both evaluations. For example, when $d = 0.2\lambda$, the channel capacity loss to this element spacing is $C_{loss} = 1 - C_{co}(0.2\lambda)/C_{sp}(0.2\lambda)$. Moreover, $C_{loss} < 0$ represent the increase of channel capacity by effect of mutual coupling. The channel capacity loss due to element spacing is plotted in Fig.2 for several SNRs. When the element spacing becomes small (close to 0), loss of the channel capacity due to the mutual coupling is increased in almost the all SNRs except for the high SNR(s) in this case. The loss becomes minus ($C_{loss} < 0$) around $d = 0.03 \sim 0.18$ at SNR=40dB, that means the capacity is increased by the coupling.

Figure 3 shows the results of Fig.2 as a function of SNR. Channel capacity loss increase as SNR decrease, and the capacity is monotonously decreased (loss is increased) as the element spacing becomes small. Interestingly, we can obtain capacity gain ($C_{loss} < 0$) in high SNRs.

Here, we use new three figures to explain the effects appeared in Figs.2 and 3. Figure 4 shows eigenvalue characteristic as the function of element spacing. Figure 5 and figure 6 show the difference of each channel-path capacity defined by each eigenvalue at SNR=40 dB and SNR=0 dB. Here we define the channel capacity difference as

$$C_{diff} = C_{sp}(d, \lambda_i) - C_{co}(d, \lambda_i) \tag{7}$$

where $C_{sp}(d, \lambda_i)$ is channel capacity by the i-th eigenvalue of $\boldsymbol{C}_a(d)|_{d\to\infty}\boldsymbol{H}_s$, and $C_{co}(d, \lambda_i)$ is channel capacity by the i-th eigenvalue of $\boldsymbol{C}_a\boldsymbol{H}_s$. From Fig.4, it can be seen that the 1st eigenvalue of the matrix $\boldsymbol{C}_a(d)|_{d\to\infty}\boldsymbol{H}_s$ increases and becomes dominant in small spacing. Clearly, this is the effect of spatial correlation. On the other hand, the 1st eigenvalue of the matrix $\boldsymbol{C}_a\boldsymbol{H}_s$ decreases in small spacings. Since the channel capacity defined by using eigenvalues is the function of logarithm as shown in (2), the magnitude change of the eigenvalue highly affects its channel capacity for low SNRs in comparison with those for high SNRs. This causes the large channel capacity loss in low SNR. In high SNR environments, the channel capacity is not sensitive to variation of the magnitude of eigenvalues, therefore, the channel capacity loss becomes small. Moreover, we can see that the $C_{loss} < 0$ is caused by the effect of the third and fourth eigenvalues from Figs 5 and 6. The total capacity increase in high SNR is caused by the increased gains of the third and fourth path by the effect of mutual coupling.

## 5 Conclusion

In this report, we analyze the effect of the mutual coupling for MIMO system. Simulation results show that the performance deterioration by the coupling is relatively small in medium SNR, moreover, we obtain gain in high SNR environments. However, the capacity loss due to the coupling becomes large in low SNR. We show that the effects can be explained by the capacity corresponding to each path-eigenvalue.

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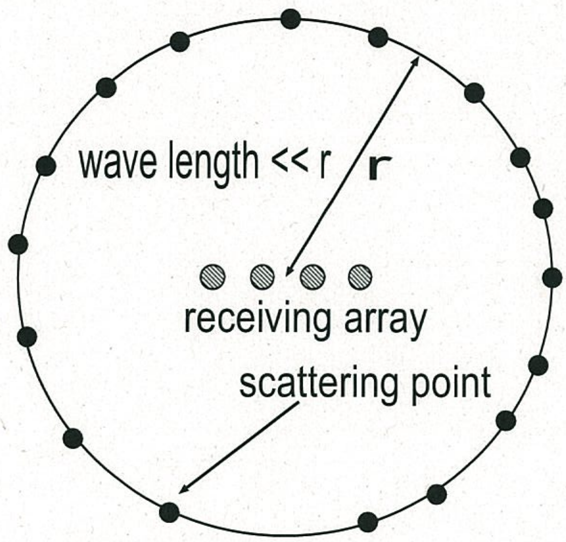


Figure 1: Rayliegh fading environment model

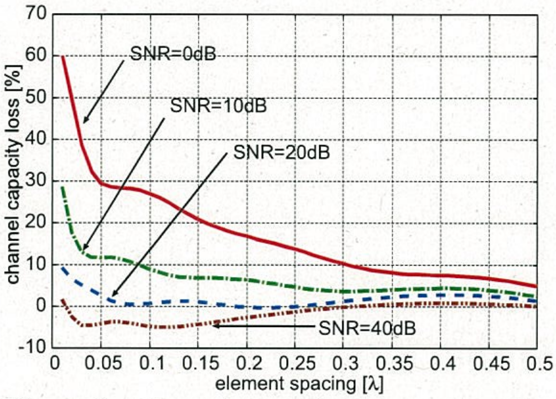


Figure 2: channel capacity loss versus element spacing

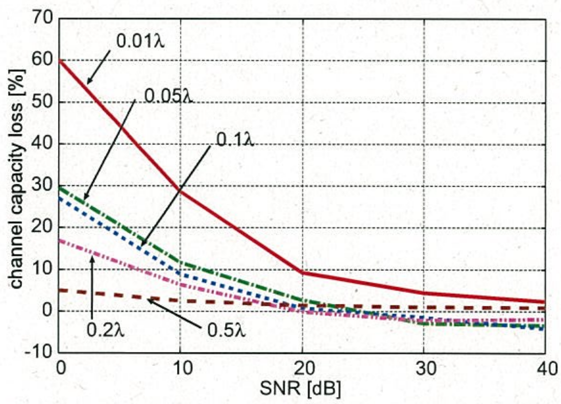


Figure 3: channel capacity loss versus SNR

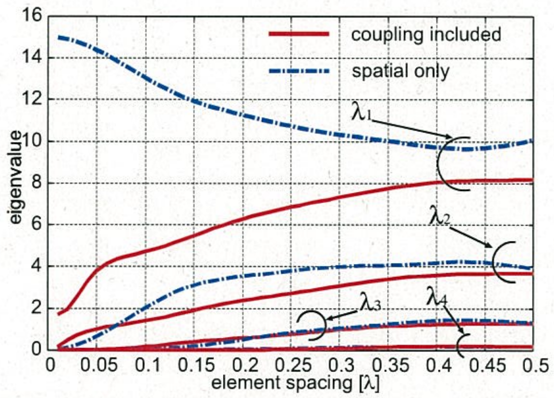


Figure 4: eigenvalue versus element spacing

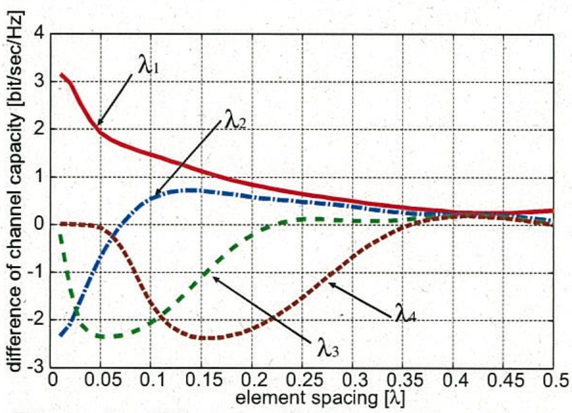


Figure 5: difference of channel capacity at SNR=40dB

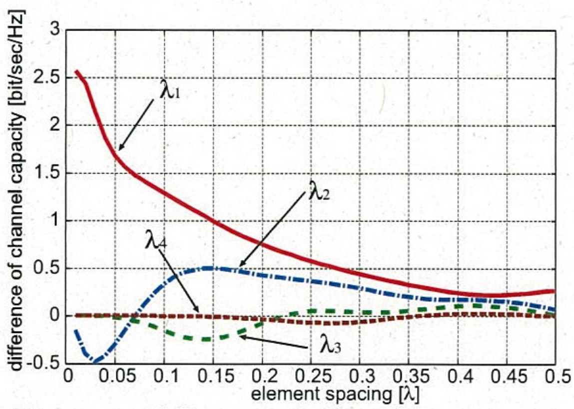


Figure 6: difference of channel capacity at SNR=0dB