DOA Estimation of Coherent Waves by Using Spatial Smoothing Preprocessings in Transmitter

[#] Natsumi ENDO [1], Hiroyoshi YAMADA [2], Yoshio YAMAGUCHI [2]
[1] *Graduate School of Science & Technology, Niigata University*
Ikarashi 2–8050, Niigata-shi, 950–2181 Japan,
natsumi@wave.ie.niigata-u.ac.jp
[2] *Faculty of Engineering, Niigata University*
Ikarashi 2–8050, Niigata-shi, 950–2181 Japan,
{yamada,yamaguch}@ie.niigata-u.ac.jp

Abstract

Direction of arrival estimation in coherent multipath environment by using the superresolution technique often requires decorrelation techniques. Spatial smoothing preprocessings are the most popular scheme as the techniques. In mobile environment, position change of the target/transmitter often brings us decorrelation effect. Multiple transmitting signals by an antenna array like a MIMO transmitter also realize the same effect. These effects can be categorized as a spatial smoothing preprocessing in transmitter. In this report, we analyze the spatial smoothing effect by transmitter in multipath environments. Theoretical and simulation results show that the transmitting-SSP has a good feature in comparison with the conventional (receiving) SSP. Furthermore, when we apply the transmitting-SSP and the receiving-SSP, we can obtain further decorrelation performance.

1. Introduction

Direction of Arrival (DOA) estimation is one of the important applications in signal processing array. The DOA estimation in indoor or urban propagation environments, we must resolve coherent or high correlated multipath waves. There are many algorithms for the DOA estimation. Superresolution techniques, such as MUSIC[1] or ESPRIT[2], become popular recently because of their high resolution capabilities. However, these subspace-based superresolution techniques cannot resolve coherent waves directly. To overcome this difficulty, so-called Spatial Smoothing Preprocessing (SSP)[3] and Forward/Backward-SSP (FB-SSP)[4], [5] are often employed as a decorrelation preprocessing. The methods utilize the overlapped subarrays in the receiving array.

In this report, we analyze the spatial smoothing effect at sources (transmitting targets). We call the scheme as Transmitting-SSP in this report. For DOA estimation of targets in mobile communication environment, the targets may move in snapshots acquisition period. This displacement of the source is equivalent to displacement of the receiving array for fixed source. Displacement of the receiving array is almost corresponds to the subarray partitioning. Clearly, for non-multipath environment, these problems are equivalent and bring us the same decorrelation effect. However, we show in this report that the transmitting-SSP enhances the signal decorrelation effect in multipath environment. In this report, we describe the transmitting-SSP based on the displacement of the sources. The transmitting-SSP can be realized by using closely spaced elements, or an array, whose elements transmit uncorrelated signals each other.

In this report, we analyze characteristics of the proposed transmitting-SSP in multipath environment theoretically. The results show that the proposed scheme has desired decorrelation characteristics; 1) The transmitting-SSP can decorrelate signals effectively for the closely spaced two waves that the conventional-SSP (receiving-SSP) cannot work properly. 2) Further decorrelation performance can be obtained when we apply the transmitting-SSP with the conventional schemes. We derive that the performance of the transmitting-SSP depends on source displacement, direction of the source displacement (relative to the receiving array arrangement and wall) as well as DOAs of waves. The multipath waves can be modeled by the waves from image sources. The same concept can be found in [6] for propagation delay estimation in multipath environment.

By using the proposed transmitting-SSP, we do not have to divide the receiving array into subarrays. That means we can detect L-1 waves with the L-element array. This will be also one of advantages in the transmitting-SSP scheme. In this report, we verify the decorrelation performance of the proposed transmitting-SSP by computer simulations. DOA estimations results of the MUSIC algorithm with each decorrelation scheme are also provided to show the availability of the scheme.

2. Received Signal Model

The DOA estimation environment considered in this report is shown in Fig.1. For simplicity, number of the sources/transmitters is assumed to be 1. Receiving array is an L-element uniform linear antenna array with element spacing of Δx. As shown in Fig.1, there is a wall near the transmitter, then multipath reflection occurs. The reflection coefficient of

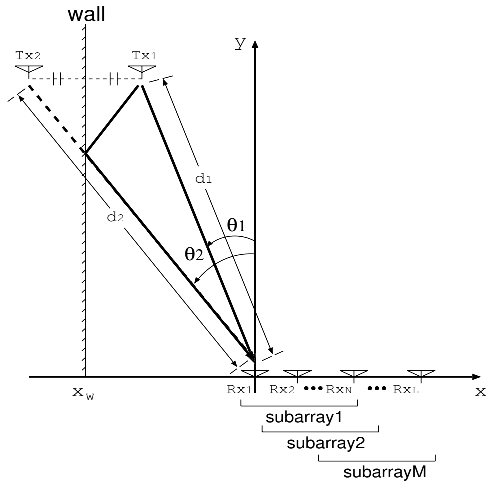

Fig. 1: Antenna Layout and Multipath Signal Model

the wall is γ. These waves, the direct wave from Tx_1 and the reflected wave by the wall, are coherent. According to the image theory, the reflected wave can be modeled as the transmitting wave from an image source, Tx_2. Then, the received signal of each waves at a reference element of the receiving array (Rx$_2$) can be given by

$$s'_1(t) = s(t-\tau_1)e^{-j2\pi\frac{d_1}{\lambda}}, \quad (1a)$$
$$s'_2(t) = \gamma s(t-\tau_2)e^{-j2\pi\frac{d_2}{\lambda}}, \quad (1b)$$

where $s(t)$ is the transmitting signal of Tx_1, λ denotes wave length, and d_1 is the distance between the transmitter and the reference element of the receiving array. d_2 is the propagation distance of the reflected wave. $\tau_i(i=1,2)$ is the propagation delay of each wave. We also assume that the source is located in far-field region. With this assumption dumped term due to wave propagation is constant over the elements of the receiving array, hence the dumped term is omitted in Eq.(1a) and (1b). The received signal vector for the array becomes

$$\boldsymbol{r}(t) = \boldsymbol{A}\boldsymbol{s'}(t) + \boldsymbol{n(t)}, \quad (2)$$

where

$$\boldsymbol{r}(t) = [r_1(t), \cdots, r_L(t)]^T, \quad (3)$$
$$\boldsymbol{A} = [\boldsymbol{a}(\theta_1), \boldsymbol{a}(\theta_2)], \quad (4)$$
$$\boldsymbol{a}(\theta_i) = [1, e^{-j2\pi\frac{\Delta x}{\lambda}\sin\theta_i}, \cdots, e^{-j2\pi\frac{(L-1)\Delta x}{\lambda}\sin\theta_i}]^T, \quad (5)$$
$$\boldsymbol{s'}(t) = [s'_1(t), s'_2(t)]^T, \quad (6)$$
$$\boldsymbol{n}(t) = [n_1(t), \cdots, n_L(t)]^T, \quad (7)$$

θ_1 and θ_2 denote the DOA of the direct wave and the reflect wave, respectively. $n_i(t)$ is the white Gaussian noise having power of σ^2. ^{T} denotes transpose.

3. Decorrelation Preprocessing schemes

A. Conventional Decorrelation Preprocessing scheme

1) Spatial Smoothing Preprocessing: Spatial Smoothing Preprocessing (SSP) is a well-known correlation suppression method for coherent signals. The SSP is defined by the averaged subarray-correlation-matrix. The overlapped subarrays can be defined as shown in Fig.1. The number of the subarrays is assumed to be M in the following discussion (Fig.1). The received signal vectors $\boldsymbol{r}_m$ of the m-th subarray can be written by

$$\begin{aligned}\boldsymbol{r}_m(t) &= \boldsymbol{A}_m\boldsymbol{s'}(t) + \boldsymbol{n}_m(t), \quad (8)\\ &= \boldsymbol{A}_1\boldsymbol{D}^{m-1}\boldsymbol{s'}(t) + \boldsymbol{n}_m(t), m = 1 \sim M, \quad (9)\end{aligned}$$
$$(10)$$

where,

$$\boldsymbol{D} = \begin{bmatrix} e^{-j2\pi\frac{\Delta x\sin\theta_1}{\lambda}} & 0 \\ 0 & e^{-j2\pi\frac{\Delta x\sin\theta_2}{\lambda}} \end{bmatrix}. \quad (11)$$

By using these subarrays, the spatial smoothed data correlation matrix $\overline{\boldsymbol{R}}$ and the decorrelated signal correlation matrix $\overline{\boldsymbol{S}}$ can be given by

$$\begin{aligned}\overline{\boldsymbol{R}} &= \frac{1}{M}\sum_{m=1}^{M}\boldsymbol{R}_m \\ &= \boldsymbol{A}\overline{\boldsymbol{S}}\boldsymbol{A}^H + \sigma^2\boldsymbol{I}, \quad (12)\end{aligned}$$

$$\begin{aligned}\overline{\boldsymbol{S}} &= \frac{1}{M}\sum_{m=1}^{M}\{\boldsymbol{D}^{m-1}\boldsymbol{S}(\boldsymbol{D}^{m-1})^H\} \\ &= \begin{bmatrix} |s'_1|^2 & \rho s'_1 s'^*_2 \\ \rho^* s'^*_1 s'_2 & |s'_2|^2 \end{bmatrix}. \quad (13)\end{aligned}$$

From Eq.(13), the effective correlation coefficient, ρ, between the direct wave and the reflection wave is given by

$$\rho = \frac{\sin(Mu)}{M\sin u}e^{-j(M-1)u} \quad (14a)$$

$$u = \pi\frac{\Delta x}{\lambda}(\sin\theta_1 - \sin\theta_2) \quad (14b)$$

2) Forward/Backward SSP: The method for adding backward averaged correlation matrices to the SSP is called Forward/Backward-SSP (FB-SSP). The backward subarrays can be defined by $\boldsymbol{r}_i = [r_{m+N-1}(t)^*, r_{m+N-2}(t)^*, \ldots, r_m(t)^*]^T, m = 1, 2, \ldots, M$. The correlation matrix for backward array, $\overline{\boldsymbol{R}}_b$, and its signal correlation matrix, $\overline{\boldsymbol{S}}_b$, can be obtained by

$$\begin{aligned}\overline{\boldsymbol{R}}_b &= \frac{1}{M}\sum_{i=1}^{M}E[\boldsymbol{r}^{(b)}\boldsymbol{r}^{(b)H}] = \boldsymbol{J}(\overline{\boldsymbol{R}})^*\boldsymbol{J} \\ &= \boldsymbol{A}\overline{\boldsymbol{S}}_b\boldsymbol{A}^H + \sigma^2\boldsymbol{I}, \quad (15)\end{aligned}$$

$$\boldsymbol{J} = \begin{bmatrix} \boldsymbol{0} & & 1 \\ & \iddots & \\ 1 & & \boldsymbol{0} \end{bmatrix}, \quad (16)$$

$$\overline{\boldsymbol{S}}_b = \frac{1}{M}\sum_{m=1}^{M}\{\boldsymbol{D}^{m-1}\boldsymbol{D}_0\boldsymbol{S}^*\boldsymbol{D}_0^H(\boldsymbol{D}^{m-1})^H\}, \quad (17)$$

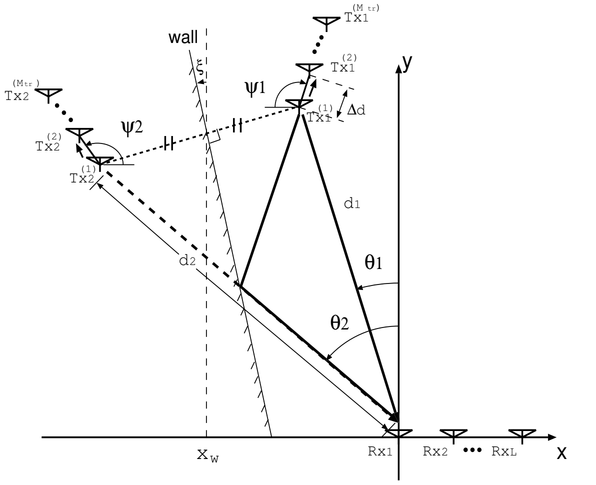

Fig. 2: Antenna Layout for Transmitting-SSP and Multipath Signal Model

where * denotes complex conjugate, and $E[\cdot]$ denotes ensemble average. By using Eq.(12) and Eq.(15), the correlation matrix by the FB-SSP can be defined by

$$\overline{\boldsymbol{R}}_{fb} = \frac{1}{2}(\overline{\boldsymbol{R}} + \overline{\boldsymbol{R}}_b) = \boldsymbol{A}\overline{\boldsymbol{S}}_{fb}\boldsymbol{A}^H + \sigma^2\boldsymbol{I}, \quad (18)$$

$$\overline{\boldsymbol{S}}_{fb} = \frac{1}{2}(\boldsymbol{S} + \boldsymbol{S}_b) \quad (19)$$

$$= \begin{bmatrix} |s_1'|^2 & \rho_{fb}s_1's_2'^* \\ \rho_{fb}^* s_1'^* s_2' & |s_2'|^2 \end{bmatrix}. \quad (20)$$

Then the effective correlation coefficient for the FB-SSP is can be written as

$$\rho_{fb} = \frac{\rho + \rho_b}{2}. \quad (21)$$

Hence

$$|\rho_{fb}| = |\rho| \cdot |\cos(\phi_{12} - (L-1)u)|, \quad (22)$$

where $s_1' = |s_1'|e^{j\phi_1}$, $s_2' = |s_2'|e^{j\phi_2}$, and $\phi_{12} = \phi_1 - \phi_2$.

B. Proposed Decorrelation Preprocessing scheme in Transmitter

1) Spatial Smoothing Preprocessing in Transmitter: The proposal scheme can be thought as the spatial smoothing preprocessing in the transmitter (source). We just refer the scheme as "Transmitting-SSP" in the followings. Transmitting-SSP is the correlation suppression method using the transmitting antenna's movement, or closely spaced antennas whose transmitted waves are uncorrelated each other. Since we assume the wave source exists in far-field, the DOA changes due to the antenna movement (or DOAs of closely spaced two antennas) can be negligibly small when the displacement (antenna separation) is small. Fig.2 shows an antenna arrangement for the transmitting-SSP. When the transmitted waves from $Tx_1^{(1)}$ and $Tx_1^{(2)}$ are described by $s_1^{(1)}(t)$ and $s_1^{(2)}(t)$ respectively, these waves hold $E[s_1^{(1)}(t)s_1^{(2)*}(t)] = 0$ by the assumption. This can be easily applied to $M_{tr}\}$ sources. The antenna displacement is Δd. This transmitting antenna's image locates on the position symmetrical to the wall surface as shown in Fig.2.

The transmitting-SSP is defined by the average of the correlation matrices of the data vector for these M_{tr} transmitting signals, $s_1^{(m)}(t), m = 1 \sim M_{tr},$. If the received data contain multipath waves, we can obtain decorrelation effect by the averaging scheme. Hence, we derive the effective correlation coefficient of the transmitting-SSP. For simplicity, we assume that the transmit antenna is moved to the direction $\psi_1 = \psi_2 = 0$ and the wall's angle normal to the receiving array is $\xi = 0$. Then the received signal vectors for $Tx_1^{(1)}$ to $Tx_1^{(M_{tr})}$ (including their multipath, $Tx_2^{(1)}$ to $Tx_2^{(M_{tr})}$) are given by

$$\boldsymbol{r}_{tr,m}(t) = \boldsymbol{A}_{tr,1}\boldsymbol{D}_{tr}^{(m-1)}\boldsymbol{s}'(t) + \boldsymbol{n}_m(t), \quad (23)$$

$$m = 1 \sim M_{tr}, \quad (24)$$

$$\boldsymbol{D}_{tr} = \begin{bmatrix} e^{-j2\pi\frac{\Delta d \sin\theta_1}{\lambda}} & 0 \\ 0 & e^{j2\pi\frac{\Delta d \sin\theta_2}{\lambda}} \end{bmatrix}, \quad (25)$$

where displacement of the transmitting antenna is Δd. Then the data correlation matrix and signal correlation matrix for the transmitting-SSP is

$$\overline{\boldsymbol{R}}_{tr} = \frac{1}{M_{tr}}\sum_{m=1}^{M_{tr}} \boldsymbol{R}_m = \boldsymbol{A}_{tr1}\overline{\boldsymbol{S}}_{tr}\boldsymbol{A}_{tr1}^H + \sigma^2\boldsymbol{I}, \quad (26)$$

$$\overline{\boldsymbol{S}}_{tr} = \frac{1}{M_{tr}}\sum_{m=1}^{M_{tr}}\{\boldsymbol{D}_{tr}^{m-1}\boldsymbol{S}(\boldsymbol{D}_{tr}^{m-1})^H\} = \begin{bmatrix} |s_1'|^2 & \rho_{tr}s_1's_2'^* \\ \rho_{tr}^* s_1'^* s_2' & |s_2'|^2 \end{bmatrix}. \quad (27)$$

Then the effective correlation coefficient for the Transmitting-SSP can be written as

$$\rho_{tr} = \frac{\sin(M_{tr}u')}{M_{tr}\sin u'}e^{-j(M_{tr}-1)u'}, \quad (28a)$$

$$u' = \pi\frac{\Delta d}{\lambda}(\sin\theta_1 + \sin\theta_2). \quad (28b)$$

When we take the direction of antenna displacement (ψ_1, ψ_2) and the wall's gradient (ξ), into account as shown in Fig.2. Eq.(28a) can be rewritten as

$$\rho_{tr} = \frac{\sin(M_{tr}u_2')}{M_{tr}\sin u_2'}e^{-j(M_{tr}-1)u_2'}, \quad (29a)$$

$$u_2' = \pi\frac{\Delta d}{\lambda}\{\sin(\theta_1 + \psi) + \sin(\theta_2 - (\psi + 2\xi))\} \quad (29b)$$

For wave reflection, $\psi_2 = -\psi_1$ (i.e. $\psi = \psi_1 = -\psi_2$) holds because of the image source.

2) Combined scheme (Transmitting and Receiving-SSP): When we combine the conventional SSP at receiving array and the transmitting-SSP, we can obtain further signal decorrelation effects. We denote the method which combines the SSP and the Transmitting-SSP by "Transmitting-SSP+Receiving-SSP", and the FB-SSP and the Transmitting-SSP by "Transmitting-SSP+Receiving-FB-SSP". Performance

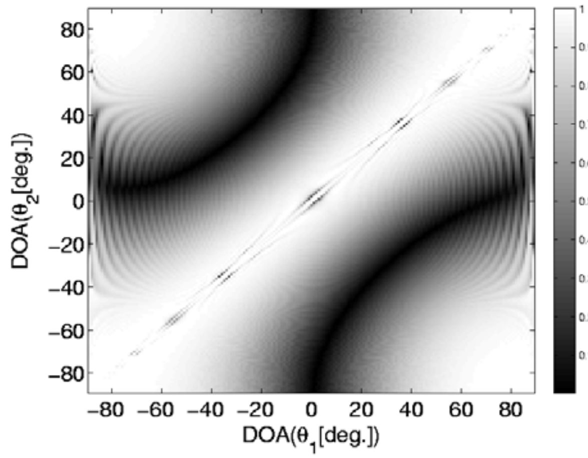

Fig. 3: Magnitude of the effective correlation coefficient(Receiving-SSP)

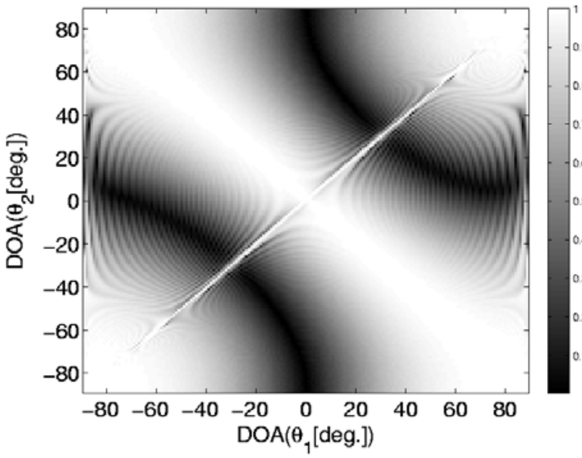

Fig. 4: Magnitude of the effective correlation coefficient(Transmitting-SSP)

of these combined schemes is examined by simulation in the next section.

4. COMPUTER SIMULATION

A. Simulation Parameters

Using an L-element uniform linear array with the element spacing of $\Delta x = 0.5\lambda$, we will show the value of effective correlation coefficient and DOA estimation results by the MUSIC algorithm [1]. In the simulations, we adopt the antenna layout and multipath waves as shown in Fig.2. The distance d_1 in Fig.2 is 5m. We evaluate the combination of various angles by changing angle of the transmitting antenna (θ_1) and the position of the wall (x_w). Displacement of the transmitting antenna is $\Delta d = \frac{\lambda}{2}$, and its direction is $-x$ ($\psi_1 = 0$). Also we assume the wall is parallel to y-axis ($\xi = 0$) and its reflection coefficient is $\gamma = -1$. The frequency is 2.4GHz. DOA of $Tx_1(\theta_1)$ is slightly changes due to the displacement for the transmitting-SSP. In this situation maximum change of θ_1 by the displacement in only $0.69°$. It can be acceptably small. Also, the attenuation of the signal is omitted.

B. Simulation Results

The magnitude of effective correlation coefficients for various DOAs of two coherent waves by each preprocessing scheme are shown in Figs.3∼7. Figures.3 and 4 show the magnitude of effective correlation coefficients obtained by the conventional (receiving) SSP and the proposed (transmitting) SSP. When we use the receiving-SSP, the value ot the coefficients are almost 1 for the angles $\theta_1 \simeq \theta_2$ as shown in Fig.3. The white area in the figure shows the area where we cannot obtain sufficient decorrelation effects. In other words, the two waves are still coherent or highly correlated. This result shows that we cannot obtain effective decorrelation effects for the closely spaced two coherent waves by the receiving-SSP. As shown in Eq.(14a), decorrelation efficiency is almost depends on angle difference, not on DOA itself. On the other hand, magnitude of the correlation coefficient of the proposed transmitting-SSP becomes small even when the angle difference is small except for the broadside direction of the receiving array ($\theta_1 \approx \theta_2 \approx 0°$) as shown in Fig.4. Note that decorrelation performance of the transmitting-SSP depends not only on the displacement of transmitting antenna, but also on displacement direction ψ and orientation of the wall ξ.

Magnitude of the correlation coefficient for Transmitting-SSP+Receiving-SSP for various θ_1 and θ_2 is shown in Fig.5. This pattern corresponds to the pattern in Fig.3 multiplied by the pattern in Fig.4. Therefore this method gives us further decorrelation effects than the effects when we apply the transmitting-SSP or the receiving-SSP alone.

When further decorrelation performance is required, we can also apply the Forward/Backward averaging scheme to the data correlation matrix. Fig.6 shows the decorrelation performance of the Forward/Backward averaging alone with no subarray partitioning in the receiving array. Magnitude of the correlation coefficients changes rapidly in DOAs. This is caused by the behavior of Forward/Backward averaging scheme whose decorrelation performance depends on the phase difference of incident waves as shown in Eq.(21). Since the received signals, $s'_1(t)$ and $s'_2(t)$, can be defined by Eq.(1a) and (1b), respectively, and $d_1 = 5$ m in this simulation, distance d_2 for the reflected wave becomes function of θ_1 and θ_2. Therefore the phase difference between $s'_1(t)$ and $s'_2(t)$ changes in accordance with the transmitting and receiving SSP is shown in Fig.7. This correlation pattern of Figs.5 and 6. Clearly, the decorrelation performance is further enhanced. As discussed above, displacement of the transmitter (source) in multipath environment bring the decorrelation effect to the correlation matrix of the receiving array without any subarray pertitioning. When we adopt this concept for the DOA estimation, we just select adequate snapshot acquisition time so as to move the target (source) at desired distance. For DOA estimation of fixed target, closely spaced antennas transmitting uncorrelated signals each other can also realize the same decorrelation effects.

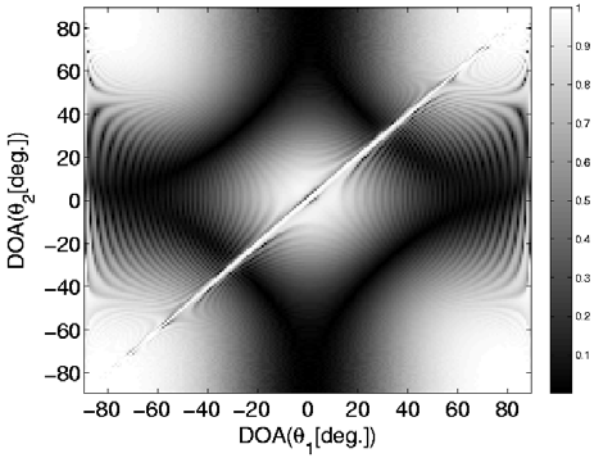

Fig. 5: Magnitude of the effective correlation coefficient(Transmitting-SSP+Receiving-SSP)

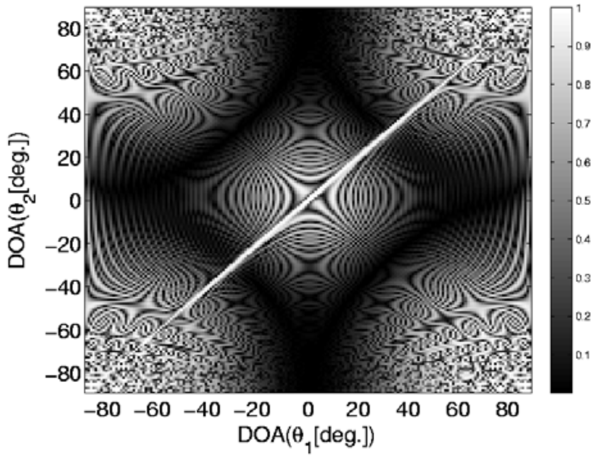

Fig. 7: Magnitude of the effective correlation coefficient(Transmitting-SSP+Receiving-FB-SSP)

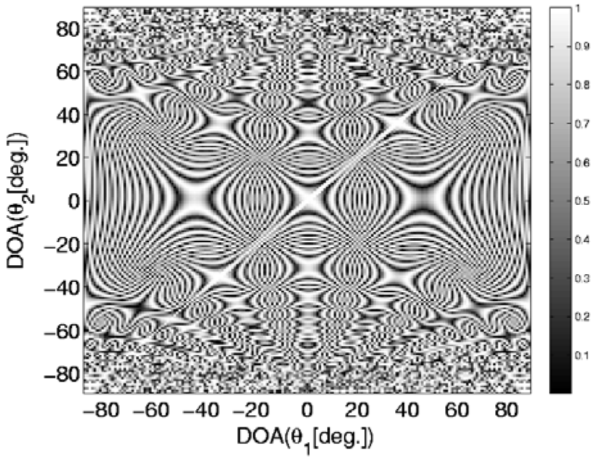

Fig. 6: Magnitude of the effective correlation coefficient(Receiving-FB)

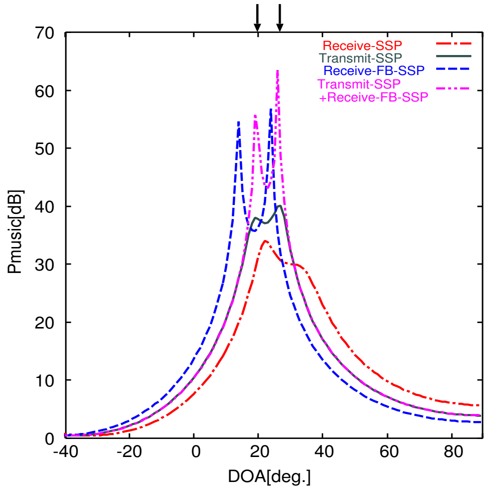

Fig. 8: MUSIC spectrum ($\theta_1 = 20°, \ \theta_2 = 27°$)

Examples of DOA estimation by the MUSIC altorithm are shown in Fig.8. We show the MUSIC spectrum for Receive-SSP, Transmit-SSP, Receive-FB-SSP, Transmit-SSP+Receive-FB-SSP respectively (Fig.8). The DOAs of θ_1 and θ_2 are $20°$ and $27°$, respectively, SNR is 20dB, and number of the snapshots is 100. In this simulation, angular difference of the incident waves is small ($\theta_2 - \theta_1 = 7°$), we cannot resolve two waves by the receiving SSP. While the transmitting-SSP can almost detect these two waves. The spectrum of the Receiving-FB-SSP can separate the waves, however, the estimated DOAs are biased. When we apply the transmitting-SSP with them (Transmitting-SSP+Receiving-FB-SSP), the DOAs are almost correctly estimated. From these results, we can say that the transmitting SSP is available for DOA estimation to enhance decorrelation effect in coherent multipath environment.

5. Conclusions

In this report, we proposed the transmitting-SSP scheme as a correlation suppression method and analyze its performance theoretically. Availability of the scheme is also evaluated by computer simulations. From these results, we show that the proposed scheme has superior performance to the conventional-SSP especially for closely spaced waves. We also show that the decorrelation performance can be enhanced when we apply the proposed "Transmitting-SSP" scheme to the conventional scheme.

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