Simple Calibration Technique for Arrays with Single-mode Elements Based on Mutual Impedance Matrix

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Abstract

Recently, applications of high-resolution Direction-of-Arrival (DOA) estimation with an array have been increasing including position estimation in smart antenna and RF-ID system. To realize high-resolution capability, superresolution technique such as MUSIC algorithm is often adopted. However, precise array calibration is necessary to realize essential performance of the algorithm. In this report, we propose a new simple calibration technique based on impedance matrix of the array. In this technique, number of the adjusted parameters, or unknowns, becomes only one for uniform array when its mutual impedance matrix is known. Performance of the proposed technique is verified by numerically and experimentally.

1. Introduction

Recently, DOA (Direction Of Arrival) estimation with an array has been expanding its application areas. For applications where number of array elements is limited, superresolution techniques, such as the MUSIC algorithm [1], are required. However, to realize their high-resolution capability, array calibration is necessary. The main error factors of arrays are analogue component error of the receiver, phase center (position) error of the elements, and mutual coupling effects among the elements. The former two errors can be decreased by improving component and array precision, however, the letter error is the unavoidable error because it cause by the electromagnetic nature itself.

There are many calibration techniques for mutual couplings of an array. Calibration of the array by using external reference plane waves will be the best calibration technique [2], however, number of the reference waves of the technique is more than number of elements of the array. It would be a hard task in practical applications to carry out such calibration procedures. For the array with single-mode elements, such as half-wavelength dipoles, the mutual coupling effects of the elements can be modeled by a mutual coupling matrix. The simplest derivation of the matrix can be found in [3]. This technique utilizes the mutual impedance matrix of the array. Since measurement of mutual impedance is not so difficult, the technique would be attractive for practical calibrations. However, the calibration accuracy of the matrix is not enough for precise DOA estimation. To improve the accuracy of the approach, some modified techniques have been proposed [4],[5].

In this report, we propose a simple calibration technique by using mutual coupling matrix of the array. The technique is based on the modification of the conventional calibration matrix derived by the mutual impedance matrix. In the technique, required parameter to be adjusted is only one for uniform arrays. It can be easily estimated by using one external (reference) wave. Numerical and experimental results are provided to show availability of the proposed technique.

2. Mutual Couplings in DOA Estimation

For simplicity, we consider the DOA estimation problem of an N-element linear array as shown in Fig.1 (a dipole array with $N = 4$ in this figure). All the elements are assumed to be the same and terminated by Z_L. When a plane wave impinges on the array at angle of θ, the received data vector of the array can be written by

$$\boldsymbol{i} = \boldsymbol{C}\boldsymbol{a}(\theta)s + \boldsymbol{n}, \tag{1}$$

where $\boldsymbol{i}$ and $\boldsymbol{n}$ are the N-dimensional vectors whose elements corresponds to the currents and the noises of the array-elements, respectively. The N-dimensional vector $\boldsymbol{a}(\theta)$ and s are the mode vector and the complex voltage of the incident wave. The $N \times N$ matrix $\boldsymbol{C}$ is the mutual coupling matrix of the array.

The array calibration of the mutual coupling effect can be modeled by the estimation of $\boldsymbol{C}$. It can be precisely estimated when we have a calibration data set of one-wave incidences with different DOAs [2]. Since the $\boldsymbol{C}$ is estimated by the plane wave illumination to the elements, we denotes the matrix as $\boldsymbol{C}_{\text{plane}}$ in the followings.

In this report, we consider how we obtain a good approximated matrix of $\boldsymbol{C}_{\text{plane}}$ by using conventional mutual impedance measurements of the array.

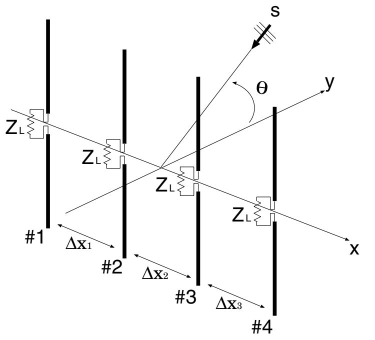


Fig. 1: DOA estimation with an N-element dipole array

3. Calibration Matrix for DOA estimation

A. Impedance matrix and conventional Calibration Matrix

As discussed in [3], induced voltages by the incidence and terminal current can be modeled by the open-circuit voltages and terminal current of the equivalent circuit of the array. When we denotes the voltage and the current vector as $\boldsymbol{v}$ and $\boldsymbol{i}$, they can be related by

$$\boldsymbol{v} = (\boldsymbol{Z} + Z_L \boldsymbol{I})\boldsymbol{i}, \tag{2}$$

where $\boldsymbol{Z}$ is the $N \times N$ mutual impedance matrix of the array, and $\boldsymbol{I}$ is the identity matrix. Clearly, relation of $\boldsymbol{v}$ and $\boldsymbol{i}$ can be determined when we obtain $\boldsymbol{Z}$.

The mutual impedance matrix can be easily measured without employing external reference plane-waves. yLet us excite only the k-th element by V_0. The voltage vector $\boldsymbol{v}_k$ and corresponding vector $\boldsymbol{i}_k$ can be written by,

$$\begin{aligned} \boldsymbol{v}_k &= V_0 \boldsymbol{u}_i = (\boldsymbol{Z} + Z_L \boldsymbol{I})\boldsymbol{i}_k, \quad k = 1, \cdots, N, && \text{(3a)} \\ \boldsymbol{u}_k &= [\underbrace{0, \cdots, 0}_{k-1}, 1, 0 \cdots, 0]^T, && \text{(3b)} \end{aligned}$$

where T denotes transpose. $\boldsymbol{u}_k$ is the vector whose the k-th element is 1 and remaining elements are zeros. By using separately excited voltages, $\boldsymbol{v}_1 \sim \boldsymbol{v}_k$, we can obtain

$$\begin{aligned} \boldsymbol{Z} + Z_L \boldsymbol{I} &= V_0 \boldsymbol{J}_{\text{all}}^{-1}, && \text{(4a)} \\ \boldsymbol{J}_{\text{all}} &= [\boldsymbol{i}_1, \boldsymbol{i}_2, \cdots, \boldsymbol{i}_N]. && \text{(4b)} \end{aligned}$$

From (2) and (1), we can define the mutual coupling matrix by

$$\boldsymbol{C}_{\text{delta}} = (\boldsymbol{Z} + Z_L \boldsymbol{I})^{-1}. \tag{5}$$

Note that the subscript 'delta' is added to show that the matrix is estimated by the delta-gap feed excitation.

As denoted in [6], the estimated calibration matrix $\boldsymbol{C}_{\text{delta}}$ is slightly biased. In this derivation, the open-circuit voltage of the elements is assumed to be independent from other elements, however, scattered waves by the adjacent elements often affects to the open-circuit voltage of the element.

B. Proposed Calibration Matrix

Main difference in the estimation of $\boldsymbol{C}_{\text{delta}}$ and $\boldsymbol{C}_{\text{plane}}$ is the excitation of the elements. In $\boldsymbol{C}_{\text{plane}}$, each element is excited uniformly by a plane wave. On the other hand, delta-gap feed excitation is employed in $\boldsymbol{C}_{\text{delta}}$. Current distribution of the element slightly changes even in a half-wavelength dipole.

The proposed calibration matrix can be defined by

$$\boldsymbol{C}_{\text{proposed}} = \boldsymbol{C}_{\text{delta}} - \zeta \boldsymbol{I}, \tag{6}$$

where ζ is the modified coefficient to remove the difference due to the excitation. This equation can also be written as follows

$$\boldsymbol{C}_{\text{proposed}} = (\rho \boldsymbol{I}) \odot \boldsymbol{C}_{\text{delta}}, \tag{7}$$

where $\odot$ is the Hadamard matrix product (element-wise multiplication). In this case, ρ can be used as the weight for the diagonal elements. The latter expression of (7) will be preferable in practical use that will be shown later. As shown in each formula, we propose modification of the diagonal elements in $\boldsymbol{C}_{\text{delta}}$ to improve its accuracy. Derivation of (6) and (7) is provided in the next subsection.

C. Derivation of the Proposed Calibration Matrix

To treat the effect of current distribution on the element, we adopt the method of moments in this section. For simplicity, we employ the N-element dipole array whose element is divided into M segments as an example. The NM-dimensional induced voltage vector $\boldsymbol{v}^{mom}$ and current vector $\boldsymbol{i}^{mom}$ caused by $\boldsymbol{v}^{mom}$ can be given by

$$\begin{aligned} \begin{bmatrix} \boldsymbol{v}_1^{mom} \\ \boldsymbol{v}_2^{mom} \\ \vdots \\ \boldsymbol{v}_N^{mom} \end{bmatrix} &= \begin{bmatrix} \boldsymbol{Z}_{11}^{mom} & \cdots & \boldsymbol{Z}_{1N}^{mom} \\ \boldsymbol{Z}_{21}^{mom} & \cdots & \boldsymbol{Z}_{2N}^{mom} \\ \vdots & \ddots & \vdots \\ \boldsymbol{Z}_{N1}^{mom} & \cdots & \boldsymbol{Z}_{NN}^{mom} \end{bmatrix} \begin{bmatrix} \boldsymbol{i}_1^{mom} \\ \boldsymbol{i}_2^{mom} \\ \vdots \\ \boldsymbol{i}_N^{mom} \end{bmatrix}, \\ \boldsymbol{v}^{mom} &= \boldsymbol{Z}^{mom} \boldsymbol{i}^{mom}, && \text{(8a)} \\ \boldsymbol{Y}^{mom} \boldsymbol{v}^{mom} &= \boldsymbol{i}^{mom}, && \text{(8b)} \end{aligned}$$

where terminal impedance of each element is added in the corresponding element (terminal segment) in $\boldsymbol{Z}_{ii}$

When we define voltage distribution vector of the k-th element, $\boldsymbol{h}_k$, normalized by its voltage of the terminal segment, we can rewrite $\boldsymbol{v}_k^{mon} = \boldsymbol{h}_k v_k$. In addition, we also define the current distribution vector $\boldsymbol{g}_k$ as $\boldsymbol{i}^{mom} = \boldsymbol{g}_k i_i$, where i_k is the corresponding terminal current. Then we have

$$\boldsymbol{i}^{mom} = \boldsymbol{Y}^{mom} \begin{bmatrix} \boldsymbol{h}_1 v_1 \\ \boldsymbol{h}_2 v_2 \\ \vdots \\ \boldsymbol{h}_N v_N \end{bmatrix} = \begin{bmatrix} \boldsymbol{g}_1 i_1 \\ \boldsymbol{g}_2 i_2 \\ \vdots \\ \boldsymbol{g}_N i_N \end{bmatrix}. \tag{9}$$

Therefore, the $N \times 1$ terminal current vector $\boldsymbol{i}$ can be given by

$$\begin{aligned}
\boldsymbol{i} &= [i_1, i_2, \cdots, i_N]^T \\
&= \begin{bmatrix} [\boldsymbol{Y}^{mom}_{11}]_{\text{term}}\boldsymbol{h}_1 & \cdots & [\boldsymbol{Y}^{mom}_{1N}]_{\text{term}}\boldsymbol{h}_N \\ [\boldsymbol{Y}^{mom}_{21}]_{\text{term}}\boldsymbol{h}_1 & \cdots & [\boldsymbol{Y}^{mom}_{2N}]_{\text{term}}\boldsymbol{h}_N \\ \vdots & \ddots & \vdots \\ [\boldsymbol{Y}^{mom}_{N1}]_{\text{term}}\boldsymbol{h}_1 & \cdots & [\boldsymbol{Y}^{mom}_{NN}]_{term}\boldsymbol{h}_N \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} \\
&= \boldsymbol{C}\boldsymbol{v}, \qquad (10)
\end{aligned}$$

where $[\boldsymbol{Y}_{kk}]_{\text{term}}$ denotes the row corresponding to the terminal of the k-th element. Estimation of $\boldsymbol{C}_{\text{plane}}$ is done with $\boldsymbol{h}_i = [1, 1, \cdots, 1]^T = \mathbf{1}$, while the estimation of $\boldsymbol{C}_{\text{delta}}$ is done with $\boldsymbol{h}_i = \boldsymbol{u}_{term}$. As reported in several papers, voltage distribution difference on the elements causes the difference between $\boldsymbol{C}_{\text{plane}}$ and $\boldsymbol{C}_{\text{delta}}$.

The derived current $\boldsymbol{i}^{mon}$ by the method of moments is the steady state current. It can be regarded as the sum of initial current (without mutual coupling) $\boldsymbol{i}^{(mom-0)}$, the first-order coupled current $\boldsymbol{i}^{(mom-1)}$, and the higher-order coupled currents $\boldsymbol{i}^{(mom-\ell)}, \ell = 2, 3, \cdots, \infty$. This can be also understood as the multiple reflections among the elements. Then, the current can be written by

$$\boldsymbol{i}^{mom} = \boldsymbol{i}^{(mom-0)} + \boldsymbol{i}^{(mom-1)} + \cdots + \boldsymbol{i}^{(mom-\ell)} + \cdots \quad (11)$$

This concepts can also be applied to the terminal current $\boldsymbol{i}$.

$$\begin{aligned}
\boldsymbol{i} &= \boldsymbol{i}^{(0)} + \boldsymbol{i}^{(1)} + \cdots + \boldsymbol{i}^{(\ell)} + \cdots \\
&= \boldsymbol{i}^{(0)} + \sum_{\ell=1}^{\infty} \boldsymbol{i}^{(\ell)} = \boldsymbol{i}^{i} + \boldsymbol{i}^{c} \qquad (12)
\end{aligned}$$

where $\boldsymbol{i}^i (= \boldsymbol{i}^{(0)})$ is the initial terminal-current vector without mutual coupling, and $\boldsymbol{i}^c$ is the coupled terminal-current vector caused by the initial excited current $\boldsymbol{i}^{(0)}$.

When we apply the expansion in (12) to (4b), and substitute it for (4a) and (5), then we obtain that the $\boldsymbol{C}_{\text{delta}}$ can be written by

$$\begin{aligned}
\boldsymbol{C}_{\text{delta}} &= (\boldsymbol{Z} + Z_L\boldsymbol{I})^{-1} = V_0^{-1}(\boldsymbol{J}^i_{\text{delta}} + \boldsymbol{J}^c_{\text{delta}}) \quad (13a) \\
\boldsymbol{J}^i_{\text{delta}} &= [\boldsymbol{i}^{(0)}_1, \boldsymbol{i}^{(0)}_2, \cdots, \boldsymbol{i}^{(0)}_N] \quad (13b) \\
\boldsymbol{J}^c_{\text{delta}} &= [\boldsymbol{i}^{c}_1, \boldsymbol{i}^{c}_2, \cdots, \boldsymbol{i}^{c}_N] \quad (13c)
\end{aligned}$$

where $\boldsymbol{i}^{(0)}_k$ and $\boldsymbol{i}^c_k$ are the initial current vector and its coupled current vector in the k-th element excitation. The constant coefficient V_0^{-1} in (13a) can be omitted in the calibration, then we assume $V_0 = 1$ in the followings. Note that $\boldsymbol{J}^i_{\text{delta}}$ is a diagonal matrix and its diagonal terms are the same value.

On the other hand, when we apply the expansion to $\boldsymbol{C}_{\text{plane}}$ we can obtain almost the same results. In the ordinary estimation for $\boldsymbol{C}_{\text{plane}}$, all of the elements are excited simultaneously, then the initial current term, $\boldsymbol{J}^i_{\text{plane}}$, is not a diagonal matrix. However, any current distributions can be expressed by the sum of current distributions obtained by the single-element-excitation at each element. Therefore, we can also expand the $\boldsymbol{C}_{\text{plane}}$ as

$$\boldsymbol{C}_{\text{plane}} = V_p^{-1}(\boldsymbol{J}^i_{\text{plane}} + \boldsymbol{J}^c_{\text{plane}}) \quad (14)$$

Again, the constant term V_p^{-1} can be ignored because it does not affect calibration accuracy. In the followings, we assume that $V_p = 1$.

In comparison (13a) and (14), it can be found that $\boldsymbol{J}^c_{\text{delta}} \simeq \boldsymbol{J}^c_{\text{plane}}$ holds. Because 1) they are excited term by the external sources, and 2) initial current distribution on the source-element is almost the same due to the single-mode assumption. As the results we can say that the difference between $\boldsymbol{J}^i_{\text{plane}}$ and $\boldsymbol{J}^i_{\text{delta}}$ is dominant. Since the diagonal term of $\boldsymbol{J}^i_{\text{delta}}$ are the same value, then the compensation of the difference can be easily done by selecting an adequate value of ζ. It can be given by

$$\begin{aligned}
\boldsymbol{C}_{\text{plane}} &\propto \boldsymbol{J}^i_{\text{plane}} + \boldsymbol{J}^c_{\text{plane}} \\
&\simeq (\boldsymbol{J}^i_{\text{delta}} - \zeta\boldsymbol{I}) + \boldsymbol{J}^c_{\text{delta}} \\
&= \boldsymbol{C}_{\text{delta}} - \zeta\boldsymbol{I} \qquad (15)
\end{aligned}$$

This corresponds to expression of the proposed calibration matrix in (6). In this expression we should carry out two-dimensional estimation for ζ (amplitude and phase, or real and imaginary value). This can be further simplified as follows. Since we employ (quasi-)single-mode elements, change of the initial current distribution on the elements by each excitation will be small, that is $|\zeta| \ll 1$. Therefore, it will be easy to use the next expression.

$$\boldsymbol{C}_{\text{plane}} \propto (\rho\boldsymbol{I}) \odot \boldsymbol{C}_{\text{delta}} \quad (16)$$

This corresponds to expression of the proposed calibration matrix in (7). For (quasi-)single-mode elements such as dipoles, we can approximate $|\rho| \simeq 1$ because of $|\zeta| \ll 1$. Then only the phase adjustment of $\boldsymbol{C}_{\text{delta}}$ will bring us a better calibration performance than the conventional $\boldsymbol{C}_{\text{delta}}$.

Next problem to be solved is the how we determine the suitable value of ρ. In ideal situation where SNR is high enough or large number of snapshots is available, ρ will be estimated as well as the DOAs of incoming waves. When we apply the MUSIC algorithm, we can define the search function by

$$P_{\text{MUSIC}}(\theta, \rho) = \frac{\boldsymbol{a}(\theta)^H \boldsymbol{C}_{\text{est}}(\rho)^H \boldsymbol{C}_{\text{est}}(\rho)\boldsymbol{a}(\theta)}{\boldsymbol{a}(\theta)^H \boldsymbol{C}_{\text{est}}(\rho)^H \boldsymbol{E}_N \boldsymbol{E}_N^H \boldsymbol{C}_{\text{est}}(\rho)\boldsymbol{a}(\theta)} \quad (17)$$

The MUSIC spectrum is sensitive to calibration errors, then maximum of the peaks will appear at the adequate ρ and true θs of the waves. When $|\rho| \simeq 1$ holds, the estimation problem can be further simplified by the approximation of $\rho = |\rho|e^{j\phi_\rho} \simeq e^{j\phi_\rho}$.

Practically, SNR and number of snapshots may be limited and we cannot realize the ideal situation. Peak values of the MUSIC spectrum are also sensitive to SNR and number of snapshots, in addition to the calibration error, then the estimation of ρ by its peak(s) detection will become difficult. In such a case, it will be the simplest scheme to use only one reference signal of known θ_0. We can estimate the suitable ρ by the maximum of $P_{\text{MUSIC}}(\theta_0, \rho)$.

TABLE 1: ARRAY PARAMETERS

Frequency (Wavelength:λ)	y2.4 GHz (λ=12.5 cm)
Element length	5.8 cm (0.464 λ)
Element radius	0.5 mm
Terminal Impedance (Z_L)	50 Ω
Number of the elements (N)	4

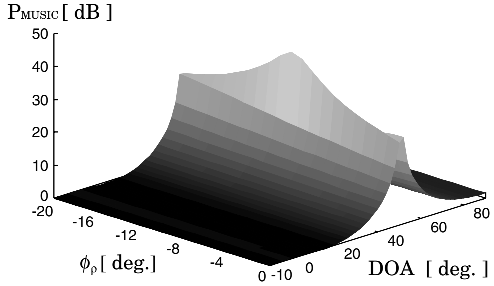


Fig. 2: DOA and Calibration coefficient estimation by $P_{\mathrm{MUSIC}}(\theta, \phi_\rho)$. $N = 4, \Delta x_1 = \Delta x_2 = \Delta x_3 = 0.48\lambda, \theta_0 = 50°$, SNR $= \infty$(no noise).

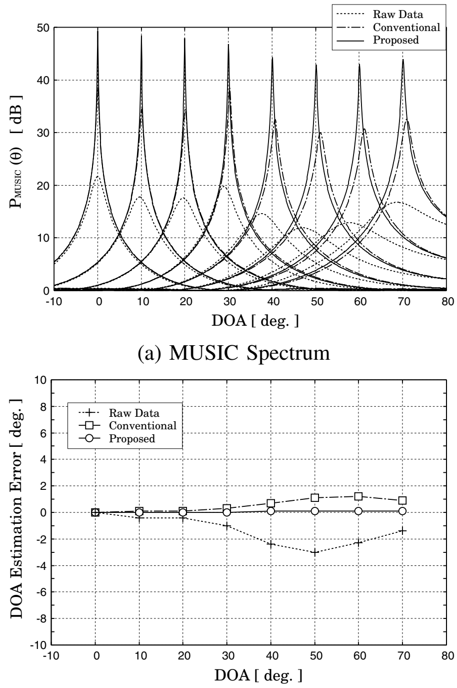
4. NUMERICAL AND EXPERIMENTAL RESULTS

In this section, we show some numerical results and experimental results to verify the availability of the proposed calibration technique.

A. Numerical Results

Numerical verification of the proposed technique is carried out by the method of moments. Dipole Linear array as shown in Fig.1 is used in this verification. Number of the elements is 4 (N=4). Array parameters are listed in Table.1.

The first example is the uniform linear array (ULA) with element separation of 0.48λ ($\Delta x_1 = \Delta x_2 = \Delta x_3 = 0.48\lambda$ in Fig.1). The $\boldsymbol{C}_{\mathrm{delta}}$ is derived numerically by the method of moments by the procedure described in Sec.3A. Figure 2 shows an example of $P_{\mathrm{MUSIC}}(\theta, \rho)$ spectrum in (17). Here we use the data for one incident wave from $\theta_0 = 50°$ in this estimation. We also assume that $|\rho| \simeq 1$, then we carried out 2-D peak search on θ and ϕ_ρ. The estimated peak in Fig.2 is located at $(\theta, \phi_\rho) = (50°, -9.3°)$. As shown in here, the maximum appears at the true DOA of the wave. DOA estimation results of one-wave incidence at several DOAs and their DOA estimation errors are shown in Figs.3(a) and (b), where 'Raw Data', 'Conventional', and 'Proposed' show the results of estimation results without calibration, with $\boldsymbol{C}_{\mathrm{delta}}$ calibration, and with $\boldsymbol{C}_{\mathrm{proposed}}$ calibration, respectively. In the calibration with $\boldsymbol{C}_{\mathrm{proposed}}$, estimated value of $\rho = e^{-j\phi_\rho}$ in Fig.2 is used throughout the evaluations. As shown in these figures, the peak property of the MUSIC algorithm is improved and the DOA estimation error becomes almost zero by the proposed calibration technique. Since the proposed calibration matrix $\boldsymbol{C}_{\mathrm{proposed}}$ is derived by approximation, then estimated ϕ_ρ slightly changes by incident angle (θ_0) of the wave. However, the estimated spectrum and DOA bias were almost unchanged when we employed the wave having DOA of $20° \leq \theta_0 \leq 60°$. Then, we may say that the technique is also robust.



(a) MUSIC Spectrum

(b) DOA Estimation Error

Fig. 3: DOA estimation results and estimation errors by the MUSIC algorithm. $N = 4, \Delta x_1 = \Delta x_2 = \Delta x_3 = 0.48\lambda$, SNR $= \infty$(no noise).

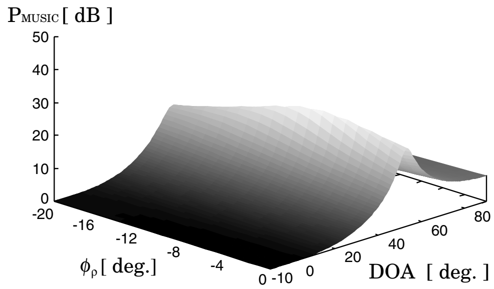
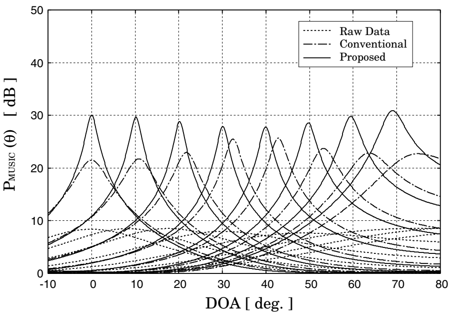


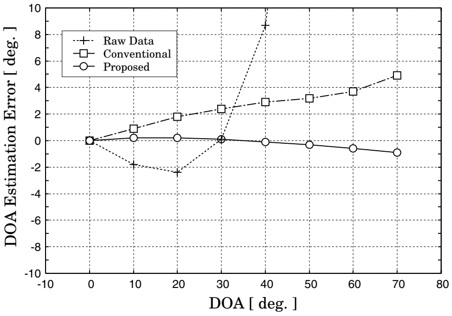
Fig. 4: DOA and Calibration coefficient estimation by $P_{\mathrm{MUSIC}}(\theta, \phi_\rho)$. $N = 4, \Delta x_1 = \Delta x_2 = \Delta x_3 = 0.2\lambda, \theta_0 = 50°$, SNR $= \infty$(no noise).

Assumption of similarity of the coupled current terms, $\boldsymbol{J}^c_{\mathrm{plane}} \simeq \boldsymbol{J}^c_{\mathrm{delta}}$, in $\boldsymbol{C}_{\mathrm{plane}}$ and $\boldsymbol{C}_{\mathrm{delta}}$ is the key in the derivation of $\boldsymbol{C}_{\mathrm{proposed}}$. For arrays with narrow element spacings, this assumption will be violated. Figures 4 and 5 shows the results of the 4-element ULA with element spacings of 0.2λ. Although the errors of the DOAs are decreased in comparison with those by the conventional calibration, there still remain DOA biases. Also peaks of the MUSIC spectrum are not so improved, which is also the effect by the approximation. As shown in this example, the proposed calibration technique is not suitable for ULAs with closely spaced elements. In the numerical verifications, we found that the proposed algorithm works well for ULA with element spacing of $\Delta x \geq 0.3$.

One feature of the proposed calibration technique to be noted is that the proposed technique can be applied to arbitrary

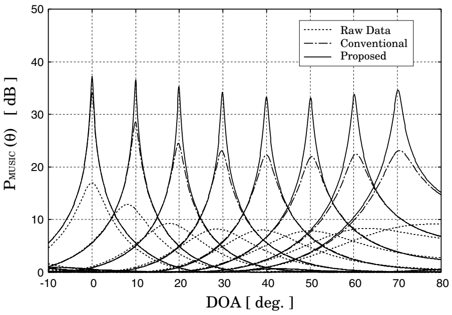


(a) MUSIC Spectrum

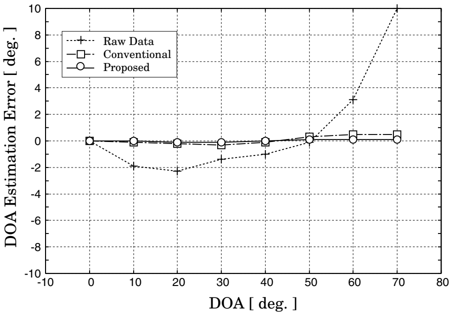


(b) DOA Estimation Error

Fig. 5: DOA estimation results and estimation errors by the MUSIC algorithm. $N = 4, \Delta x_1 = \Delta x_2 = \Delta x_3 = 0.2\lambda, \mathrm{SNR} = \infty$(no noise).



(a) MUSIC Spectrum



(b) DOA Estimation Error

Fig. 6: DOA estimation results and estimation errors by the MUSIC algorithm. $N = 4, \Delta x_1 = \Delta x_3 = 0.65\lambda, \Delta x_2 = 0.2\lambda, \mathrm{SNR} = \infty$(no noise).

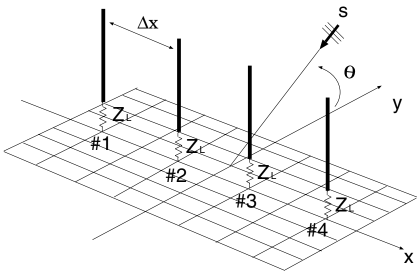


Fig. 7: DOA estimation with 4-element monopole array

TABLE 2: ARRAY PARAMETERS FOR THE EXPERIMENT

Frequency (Wavelength:λ)	y2.4 GHz (λ=12.5 cm)
Element length	3.11 cm (0.249 λ)
Element radius	0.5 mm
Terminal Impedance (Z_L)	50 Ω
Number of the elements (N)	4
Element Separation (Δx)	6.22 cm (0.498λ)

array with single-mode elements. The method in (7) can be applied to non-uniform arrays with no modifications. Figures 6(a) and (b) show the results of MUSIC spectrum and estimated DOA errors by the 4-element array with $\Delta x_1 = \Delta x_3 = 0.65\lambda$ and $\Delta x_2 = 0.2\lambda$. As shown in this example, we can obtain a good estimation of precise calibration matrix $\boldsymbol{C}_{\mathrm{plane}}$ by the proposed $\boldsymbol{C}_{\mathrm{proposed}}$. The diagonal weight coefficient ϕ is also constant all over the elements. This is because all of the elements are the same in this array.

B. Experimental Results

Availability of the proposed calibration was also verified by experiments. The experiment was done with a network analyzer in an echoic chamber. The array employed in the experiment was a 4-element uniform monopole array on an infinite ground plane as shown in Fig.7. The array parameter is also listed in Table.2. The size of the ground plane was $2\lambda \times 3.5\lambda$ whose edges were rounded so as to decrease the edge and corner scatterings.

Figure 8 shows the estimated $P_{\mathrm{MUSIC}}(\theta, \phi_\rho)$ spectrum for the one-wave incidence from $\theta_0 = 50°$ with 1 snapshot. The peak is detected at around $(\theta, \phi_\rho) \simeq (50.6°, -26.0°)$. Since the estimation was carried out with only one snapshot data, then the estimated peak became dull and discriminated peak location would be biased due to noise. It will be improved when many snapshots are available. Small scattering by edges of the ground plate may also cause bias of the peak. When these affection cannot be negligibly small, we should use a reference wave of known DOA and derived ϕ_ρ by $P_{\mathrm{MUSIC}}(\theta_0, \phi_\rho)$ to decrease bias. When we employed $P_{\mathrm{MUSIC}}(\theta_0, \phi_\rho)$, the estimated ϕ_ρ became $\phi_\rho = -24°$. In the MUSIC estimation, we used this value.

Estimated MUSIC spectrum and DOA errors are shown in Figs.9(a) and (b). As shown in Fig.9(b), DOA estimation error can be decreased effectively. Improvement of the peaks by the proposed calibration is not so dominant. Although the

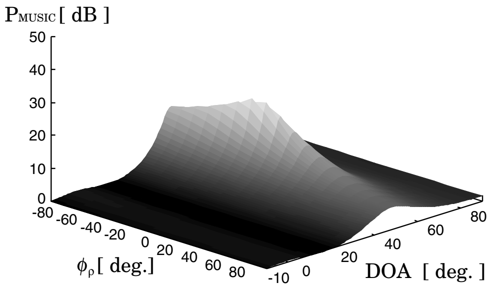
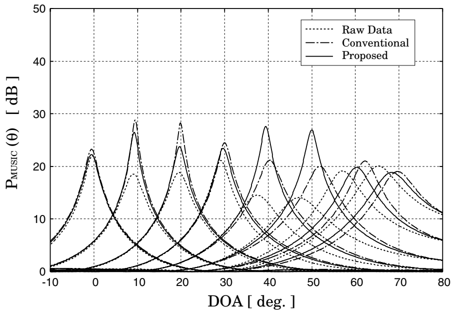
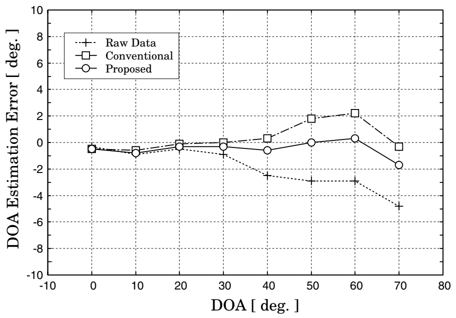


Fig. 8: DOA and Calibration coefficient estimation by $P_{\mathrm{MUSIC}}(\theta, \phi_\rho)$. $N = 4, \Delta x = 0.498\lambda, \theta_0 = 50°$, 1 snaphot.



(a) MUSIC Spectrum



(b) DOA Estimation Error

Fig. 9: DOA estimation results and estimation errors by the MUSIC algorithm. $N = 4, \Delta x = 0.498\lambda$, 1 snapshot.

array was made precisely as we could, element imbalance and position error (manufacturing error) will still remain. Since the proposed technique is available for calibration of mutual couplings, the technique cannot remove these errors. Also, noise and small number of snapshots yield small peaks. They will affect the peak performance of the calibrated spectrum.

5. Conclusions

In this report, we propose a simple array calibration technique based on the mutual impedance matrix of the array. We examine relation between correct mutual coupling matrix and the matrix derived by the eqivalent circuit of an array, and show that accuracy of the mutual coupling matrix by the equivalent circuit of array can be easily improved by suitable diagonal weighting.

Numerical and experimental results of 4-element arrays are provided to show validity of the proposed calibration technique. The propose calibration is valid for arrays with single-mode elements such as half-wavelength dipoles and a $\lambda/4$-monopoles, that are the important elements of arrays in DOA estimation.

References

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