

# An Experimental Study of DOA Estimation with Transmitting-SSP in Coherent Multipath Environment

# Natsumi ENDO<sup>1</sup>, Hiroyoshi YAMADA<sup>2</sup>, Yoshio YAMAGUCHI<sup>2</sup>

<sup>1</sup> Graduate School of Science & Technology, Niigata University

Ikarashi 2-8050, Niigata-shi, 950-2181 Japan,

natsumi@wave.ie.niigata-u.ac.jp

<sup>2</sup> Faculty of Engineering, Niigata University

Ikarashi 2-8050, Niigata-shi, 950-2181 Japan,

{yamada,yamaguch}@ie.niigata-u.ac.jp

## 1. Introduction

Direction of Arrival (DOA) estimation is one of the important applications in signal processing array. For the DOA estimation in indoor or urban propagation environments, we must resolve coherent or high correlated multipath waves. There are many algorithms for the DOA estimation. Superresolution techniques are the popular algorithm among them because of their high resolution capabilities. However, subspace-based superresolution techniques, such as the MUSIC [1] and the ESPRIT, cannot resolve the coherent waves directly. To overcome this difficulty, so-called Spatial Smoothing Preprocessing (SSP) [2] and/or its modified technique called Forward/Backward-SSP (FB-SSP) [3] are often employed as the decorrelation preprocessing scheme. These methods utilize overlapped subarrays in the receiving array. In this report, we analyze the spatial smoothing effect at transmitters, or sources to be estimated for their DOAs.

In this report, we formulate the problem of DOA estimation in the multipath environment including displacement of a transmitter or two closely-spaced transmitters whose signals are uncorrelated. We call this scheme "transmitting-SSP" in this report. Decorrelation performance of the transmitting-SSP is derived theoretically in Ref. [4]. In this report, we verify performance of the proposed scheme by experiments. The experimental results show that the proposed scheme has some desired decorrelation characteristics; 1) The transmitting-SSP can often destroy signal correlation of closely spaced two waves effectively though the conventional SSP at the receiver cannot work well for the waves. 2) Further decorrelation performance can be obtained when we apply the transmitting-SSP and the conventional SSP simultaneously. DOA estimation results of the MUSIC algorithm with each decorrelation scheme are also provided to show the availability of the proposed scheme. Performance of the proposed scheme also depends on displacement directions of the source/transmitter itself and its image source as well as their DOAs.

## 2. Problem Formulation

The DOA estimation environment considered in this report is shown in Fig.1. For simplicity, number of sources/transmitters is assumed to be 1. An  $L$ -element uniform linear array with element spacing of  $\Delta x$  is employed in this analysis.

As shown in Fig.1, we assume that there is a wall whose reflection coefficient is  $\gamma$  near the transmitter, then the array receives a direct wave from the transmitter,  $Tx_1^{(1)}$ , and its reflected wave by the wall. The direct and reflected waves are coherent.

According to the image theory, the reflected wave of  $Tx_1^{(1)}$  can be modeled as a transmitting wave from an image source,  $Tx_2^{(1)}$  as shown in Fig. 1. Here, we also add  $Tx_2^{(i)}$ ,  $Tx_3^{(i)}$ ,  $i = 1, 2, \dots, M_{tr}$ , in the figure to consider the effect of displacement of the transmitter. Angles,  $\psi_1$  and  $\psi_2$ , are the direction of the source displacements, and  $\Delta d$  is the separation between  $Tx_j^{(i)}$  and  $Tx_j^{(i+1)}$ .  $\xi$  is angle of the wall surface,

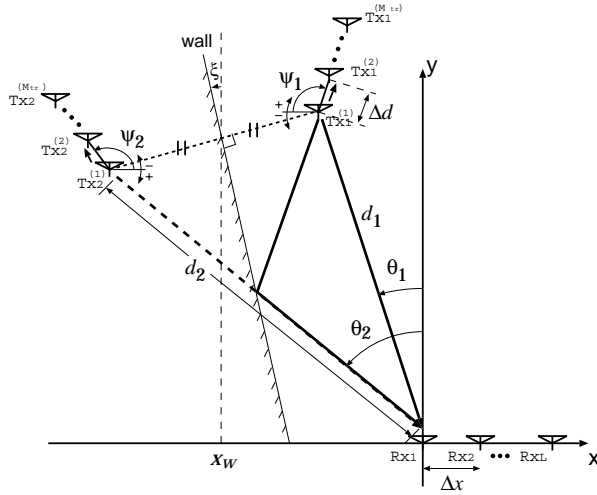


Figure 1: Antenna layout and multipath signal model

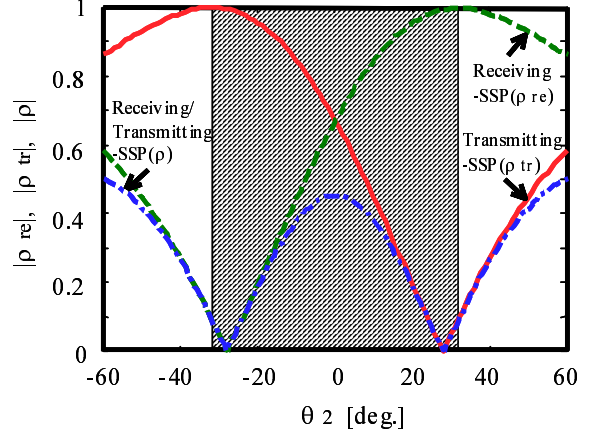


Figure 2: Magnitude of the effective correlation coefficient by variation of reflected waves DOA.  $\theta_1 = 32^\circ$ ,  $\Delta x = \Delta d = \frac{\lambda}{2}$ ,  $\psi_1 = -\psi_2 = 180$ ,  $\xi = 0$ .

where  $\xi = 0$  means that the wall surface is directed parallel to the broadside direction of the receiving array.

The received signal corresponding to each waves at the reference element of the receiving array ( $Rx_1$ ) can be given by

$$s'_1(t) = \frac{k}{d_1} s(t - \tau_1) e^{-j2\pi \frac{d_1}{\lambda}}, \quad s'_2(t) = \gamma \frac{k}{d_2} s(t - \tau_2) e^{-j2\pi \frac{d_2}{\lambda}}, \quad (1)$$

where  $s(t)$  is a transmitting signal of  $Tx_1^{(1)}$ ,  $\lambda$  denotes the wave length, and  $d_1$  is distance between the transmitter and the reference element of the receiving array.  $d_2$  is the propagation distance of the reflected wave.  $\tau_i, i = 1, 2$ , and  $k$  are delay time of each wave and an arbitrary coefficient, respectively. We also assume that the source/transmitter is located in the far-field region. Under these assumptions, the received data vector of the array can be written by

$$\mathbf{r}(t) = \mathbf{A} s'(t) + \mathbf{n}(t), \quad (2)$$

Angles,  $\theta_1$  and  $\theta_2$ , denote the DOA of the direct wave and the reflect wave, respectively.  $n_i(t)$  is the additive white Gaussian noise having power of  $\sigma^2$ .

### 3. Spatial Smoothing Preprocessing at the Transmitter

#### 3.1 Effective Correlation Coefficient of the Proposed Scheme (Transmitting-SSP)

The proposed transmitting-SSP is an decorrelation preprocessing scheme by using displacement of transmitter. The transmitting-SSP is defined by the average among the correlation matrices of the data vector at the receiver for these  $M_{tr}$  transmitting signals,  $s_1^{(m)}(t), m = 1 \sim M_{tr}$ . If the received data contain a multipath wave of the transmitter, we can obtain decorrelation effect by this averaging scheme. The received data vector for the transmitter  $Tx_1^{(i)}, i = 1, 2, \dots, M_{tr}$ , is given by

$$\mathbf{r}_{tr,m}(t) = \mathbf{A}_{tr,1} \mathbf{D}_{tr}^{(m-1)} s'(t) + \mathbf{n}_m(t), \quad m = 1, 2 \dots M_{tr}, \quad (3)$$

where

$$\mathbf{D}_{tr} = \text{diag}\left\{e^{-j2\pi \frac{\Delta d \sin(\theta_1 + \psi)}{\lambda}}, e^{j2\pi \frac{\Delta d \sin(\theta_2 - (\psi + 2\xi))}{\lambda}}\right\}. \quad (4)$$

The preprocessed data correlation matrix by the proposed scheme,  $\bar{\mathbf{R}}_{tr}$  and the signal correlation matrix,  $\bar{\mathbf{S}}_{tr}$ , can be written by

$$\bar{\mathbf{R}}_{tr} = \frac{1}{M_{tr}} \sum_{m=1}^{M_{tr}} \mathbf{R}_m = \frac{1}{M_{tr}} \sum_{m=1}^{M_{tr}} E[\mathbf{r}_{tr,m} \mathbf{r}_{tr,m}^H] = \mathbf{A}_{tr,1} \bar{\mathbf{S}}_{tr} \mathbf{A}_{tr,1}^H + \sigma^2 \mathbf{I}, \quad (5)$$

$$\bar{\mathbf{S}}_{tr} = \frac{1}{M_{tr}} \sum_{m=1}^{M_{tr}} \{ \mathbf{D}_{tr}^{m-1} \mathbf{S} (\mathbf{D}_{tr}^{m-1})^H \} = \begin{bmatrix} |s'_1|^2 & \rho_{tr} s'_1 s'_2{}^* \\ \rho_{tr}^* s'_1{}^* s'_2 & |s'_2|^2 \end{bmatrix}, \quad (6)$$

where  $E[\cdot]$  denotes ensemble average, and  $\rho_{tr}$  is the effective correlation coefficient.  $H$  is complex conjugate transpose. The effective correlation coefficient,  $\rho_{tr}$ , by the proposed transmitting-SSP can be derived by

$$\rho_{tr} = \frac{\sin(M_{tr}u_{tr})}{M_{tr} \sin u_{tr}} e^{-j(M_{tr}-1)u_{tr}}, \quad u_{tr} = \pi \frac{\Delta d}{\lambda} \{ \sin(\theta_1 + \psi) + \sin(\theta_2 - (\psi + 2\xi)) \}, \quad (7)$$

where we use the property  $\psi = \psi_1 = -\psi_2$ .

### 3.2 Effective Correlation Coefficient of the Combined Scheme

The conventional SSP (at the receiving array) is a well-known decorrelation preprocessing scheme for uniform linear array. This SSP is defined by the averaged data correlation matrix of the overlapping subarrays. We call this preprocessing as receiving-SSP in the following discussion.

The transmitting-SSP and receiving-SSP are independent preprocessings, then we can apply there preprocessings simultaneously. Effect of this combined preprocessing can be derived easily.

The effective correlation coefficient  $\rho_{re}$  between the direct wave and reflected wave by the receiving-SSP, is given by [2]

$$\rho_{re} = \frac{\sin(Mu)}{M \sin u} e^{-j(M-1)u}, \quad u = \pi \frac{\Delta x}{\lambda} (\sin \theta_1 - \sin \theta_2), \quad (8)$$

where  $M$  is number of the subarrays.

As shown in this equation, only DOAs of incident waves are the parameters which relate to the effective correlation coefficient for a given receiving array configuration. DOAs of the incident waves are assumed to be unchanged by the transmitting-SSP, hence the effective decorrelation coefficient by the combined scheme,  $\rho$ , can be easily derived by  $\rho = \rho_{re} \rho_{tr}$ .

Then the signal correlation matrix  $\bar{\mathbf{S}}$  obtained by the combined scheme can be given by

$$\bar{\mathbf{S}} = \begin{bmatrix} |s'_1|^2 & \rho s'_1 s'_2{}^* \\ \rho^* s'_1{}^* s'_2 & |s'_2|^2 \end{bmatrix} = \begin{bmatrix} |s'_1|^2 & \rho_{re} \rho_{tr} s'_1 s'_2{}^* \\ \rho_{re}^* \rho_{tr}^* s'_1{}^* s'_2 & |s'_2|^2 \end{bmatrix}. \quad (9)$$

This means that we can obtain further decorrelation effect by this combined preprocessing scheme. We denote the combined scheme by ‘‘transmitting/receiving-SSP’’ in the next section. Similarly, we can also apply the forward/backward averaging [3] in addition to the combined scheme [4]. The theoretical values of the magnitude of effective correlation coefficients by each preprocessing are plotted in Fig.2.

## 4. Experiments

### 4.1 Experimental Environment

In this section, we provide the DOA estimation results of coherent 2 waves by the MUSIC algorithm with the decorrelation preprocessings. In this experiment, we adopt the antenna layout and multipath signal environment as shown in Fig.1. A four-element uniform linear array (ULA,  $L = 4$ , element: monopole) and a five-element ULA ( $L = 4$ ) with element spacing of  $\Delta x = 0.5\lambda$  ( $\lambda = 12.5\text{cm}$  (frequency  $f=2.4\text{GHz}$ )) are employed. Displacement of the transmitting antenna in the transmitting-SSP is  $\Delta d = \frac{\lambda}{2}$ , whose direction is  $x$  ( $\psi_1 = 180^\circ$ ). In this experiment, an metallic plate is placed parallel to  $y$ -axis ( $\xi = 0$ ) as a wall, whose size is  $1.5\text{m} \times 1.5\text{m}$ . Distance  $d_1$  in Fig.1 is selected as  $2\text{m}$ .

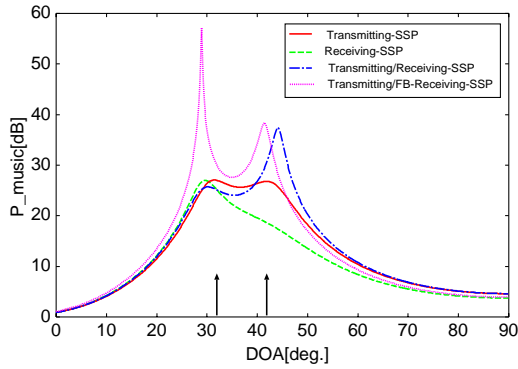


Figure 3: MUSIC Spectrum.  $L = 5$ ,  $\theta_1 = 32^\circ$ ,  $\theta_2 = 42^\circ$ ,  $\Delta x = \Delta d = \frac{\lambda}{2}$ ,  $\psi_1 = -\psi_2 = 180^\circ$ ,  $\xi = 0$ ,  $d_1 = 2m$ .

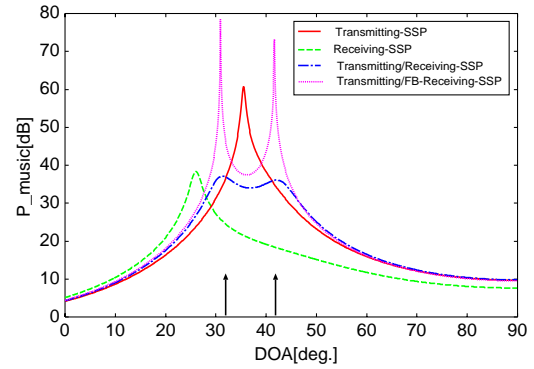


Figure 4: MUSIC Spectrum.  $L = 4$ ,  $\theta_1 = 32^\circ$ ,  $\theta_2 = 42^\circ$ ,  $\Delta x = \Delta d = \frac{\lambda}{2}$ ,  $\psi_1 = -\psi_2 = 180^\circ$ ,  $\xi = 0$ ,  $d_1 = 2m$ .

## 4.2 DOA Estimation Results

Figure 3 and 4 show experimental results of the DOA estimation by the MUSIC algorithm [1] with each preprocessing. The estimation results for the waves having  $\theta_1 = 32^\circ$  and  $\theta_2 = 42^\circ$  are shown as examples in this figure. In Fig. 3, since the effective signal correlation between the waves by the receiving-SSP is still high, we cannot resolve the DOAs by the conventional scheme. When we apply the transmitting-SSP, we can almost resolve these two waves. The MUSIC spectrum can be further improved when we employ the combined preprocessing (transmitting/receiving-SSP). From these results, we can conclude that the transmitting-SSP is available for DOA estimation to enhance decorrelation effect in coherent multipath environment. As shown in Fig. 4, when the number of receive element is four, both the receiving-SSP and the transmitting-SSP cannot resolve two waves. The combined schemes can give us the well-resolved spectrum also in this case.

## 5. Conclusions

In this report, the experimental results of DOA estimation with the proposed transmitting-SSP scheme are shown. The transmitting-SSP can be realized by using moving targets/transmitter during snapshot acquisition and/or closely spaced transmitters which transmit independent signals. From these results, it can be said that the proposed scheme often remove difficulty of the conventional SSP and realize superior decorrelation performance. Although the decorrelation effect by the transmitting-SSP is varied by the direction of the wall and transmitter's displacement, the decorrelation performance can be enhanced when we apply the "transmitting-SSP" scheme and the conventional scheme simultaneously.

## References

- [1] R. O. Schmidt, "Multiple emitter location and signal parameter estimation," IEEE Trans. Antennas and Propagat., vol. AP-34, no. 3, pp. 276-280, March 1986.
- [2] T. J. Shan, M. Wax, and T. Kailath, "On spatial smoothing for direction-of-arrival estimation of coherent signals," IEEE Trans. Acoust., Speech and Signal Processing, vol. ASSP-33, no. 4, pp. 806-811, Aug. 1985.
- [3] S. U. Pillai, and B. H. Kwon., "Forward/backward spatial smoothing techniques for coherent signal identification," IEEE Trans. Acoust., Speech and Signal Processing, vol. ASSP-37, no. 1, pp. 8-15, January 1989.
- [4] N. Endo, H. Yamada, and Y. Yamaguchi, "DOA estimation of coherent waves by using spatial smoothing preprocessings in transmitter," International Symposium on Antennas and Propagation, CD-ROM, Nov 2006.