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Simultaneous Measurement of Antenna Gain and Complex Permittivity of Liquid in Fresnel Region and Evaluation Methodology of Uncertainty

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1. Introduction

The mobile communication devices should be evaluated by their SAR (Specific Absorption Rate). In general, the value of SAR can be measured by an electrical field probe in the tissue equivalent liquid [1]. The probe is calibrated to relate the electrical field intensity to its output voltage. In 300MHz – 3GHz, the calibration with the waveguide system is mostly adopted according to the IEC document [1]. However, the mobile communication devices will be used over 3GHz so that the effect of the probe diameter can not be ignored because the wavelength in the liquid is smaller as the frequency is higher.

One solution to this difficulty is to make the probe thin; the other is to propose calibration techniques with no waveguide system [2]. One of the letter techniques developed by the authors is based on the Friis transmission formula in the conducting medium, for example, the liquid [3]. It encounters the difficulty of enormous attenuation in the tissue equivalent liquid so that the data can not be measured in the far-field region of the antenna in the liquid. This means that the precondition of the Friis transmission formula is not satisfied in our measurement. To overcome the difficulty, we proposed to extend this formula to be valid in the Fresnel-field region of the antenna [3]. The merit of the extension is that it is only necessary to include one term or two terms of the asymptotic expansion in regression function and extra equipments, for example, high-power amplifiers, are not required to enhance the dynamic range of the measurement system. On the other hand, estimated gain and dielectric property depend upon the choice of the fitting range as well as the regression function. And one- or two-term asymptotic expansion is somewhat fragile for the regression process.

We have two techniques to determine the gain of the antenna in the liquid. One is to determine the gain after the dielectric property of the liquid is determined by the pre-experiment, for example, the dielectric probe method [4]; the other is to determine the gain and the dielectric property simultaneously [3]. The former is somewhat sensitive to the values of the measured complex permittivity of the liquid so that the Fresnel region gain does not always converge with the far-field gain. On the other hand, the latter can estimate the values of the gain and complex permittivity if curve-fitting works well. In this paper, we introduce the method of weighted least square for the letter technique. The conventional method of least square with no use of the weight does not work below the noise level of the measurement system. The regression with the weight can lessen the effect of the noise in the measurement system. In addition, we can choose wide range of fitting so that the uncertainty of the measurement can be reduced.

2. Measurement Principle and Evaluation Methodology of Uncertainty

To evaluate the value of the absolute gain of the antenna in the liquid is based on the Friis transmission formula. The liquid is conducting medium so that the effect of the attenuation should be included in the formula. As shown in Fig. 1, two identical antennas are assumed to be immersed
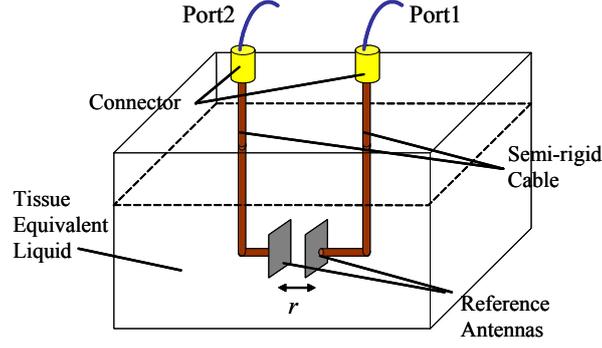
Figure 1: Measurement System to Estimate the Gain of the Antenna in the Liquid

and faced in the liquid with perfectly polarization. The two antennas are connected to the ports of the vector network analyzer via semi-rigid cables. Looking to the ports, the measurement system can be considered as a linear, passive, reciprocal two-port network. The power transmission is a function of the distance between the antennas, r, and is given by

$$|S_{21}(r)|^2 = \left(1 - |S_{11}|^2\right)\left(1 - |S_{22}|^2\right)\left(G/2\beta r\right)^2 \exp(-2\alpha r), \tag{1}$$

where S_{ij}, $i,j=1,2$ are S parameters of the two-port network, G is far-field gain of the antenna, α and β are the attenuation and phase constants in the liquid. Eq. (1) is valid in the far-field region of the antennas. However, the attenuation is enormous in the liquid, for example, $\alpha = 464\text{dB/m}$ at 2.45GHz, so that it is impossible to measure S_{21} in the far-field region. In our previous study, Eq. (1) can be extended in the Fresnel region of the antennas as follows [3]:

$$S_{21}^2(r) \approx \left(1 - |S_{11}|^2\right)\left(1 - |S_{22}|^2\right)\left(G/2\beta r\right)^2 \exp(-2\gamma r)\exp\left(a_1/r + a_2/r^2\right), \tag{2}$$

where $\gamma = \alpha + j\beta$ is the propagation constant in the liquid, and a_i, $i=1,2$ is complex constant which depends upon the geometry and the surrounding medium of the antenna. Decomposing Eq. (2) into the magnitude and phase components, we can find the following regression functions:

$$|S_{21}(r)|_{\text{dB}} + 20\log_{10} r = -8.686\alpha r + A_0 + A_1/r + A_2/r^2 , \tag{3}$$

$$\angle S_{21}(r) = -\beta r + B_0 + B_1/r + B_2/r^2 , \tag{4}$$

where A_i, B_i, $i=0,1,2$ are real constants. If the constants A_0, α and β are determined by the regression analysis, and the far-field gain in dB representation, $G_{\text{dB}}=10\log_{10}G$, is given by

$$G_{dB} = 0.5A_0 + 10\log_{10}(2\beta) - 5\log_{10}\left(1 - |S_{11}|^2\right) - 5\log_{10}\left(1 - |S_{22}|^2\right). \tag{5}$$

As the distance, r, is longer, the value of $|S_{21}|$ is smaller so that the uncertainty of measured S_{21} is larger. Actually, the uncertainty of measurement should be included in the regression process. For example, if the uncertainty of $|S_{21}(r)|_{\text{dB}}$ is denoted as $u(|S_{21}(r)|_{\text{dB}})$, the weight at the distance $r=r_i$ can be selected as $w_i=1/u^2(|S_{21}(r_i)|_{\text{dB}})$. Then, the figure-of-merit function, or, χ^2-squares for this regression analysis can be given by

$$\chi^2 = \sum_i w_i\left(|S_{21}(r_i)|_{\text{dB, measured}} - |S_{21}(r_i)|_{\text{dB}}\right)^2$$

$$= \sum_i w_i\left(|S_{21}(r_i)|_{\text{dB, measured}} + 20\log_{10} r_i + 8.686\alpha r_i - A_0 - A_1/r - A_2/r^2\right)^2 , \tag{6}$$

where $|S_{21}(r_i)|_{\text{dB, measured}}$ is a measurand for $|S_{21}(r_i)|_{\text{dB}}$. If the uncertainty of the variable, r, can be ignored, $20\log_{10} r_i$ in Eq. (3) can not generate the uncertainty itself and can be treated as a known function. Therefore, $|S_{21}(r_i)|_{\text{dB, measured}} + 20\log_{10} r_i$ in Eq. (6) can be considered as a measurand. Then, the constants in Eq. (6), α, A_i, $i=0,1,2$ can be determined by the linear least-squares method. The conditions $\partial\chi^2/\partial\alpha = 0$ and $\partial\chi^2/\partial A_i = 0$ lead to the normal equations to determine the constants. The uncertainty of the solutions, α, A_i, can be given by the square of the diagonal component in the inverse matrix of the coefficient matrix for the normal equations [5].

The uncertainty of the far-field gain, $u(G_{dB})$, should be estimated from the uncertainty of the constant A_0 as seen from Eq. (5).

$$u^2(G_{dB}) = \left(\frac{\partial G_{dB}}{\partial A_0}\right)^2 u^2(A_0) + \left(\frac{\partial G_{dB}}{\partial \beta}\right)^2 u^2(\beta) + \left(\frac{\partial G_{dB}}{\partial |S_{11}|}\right)^2 u^2(|S_{11}|) + \left(\frac{\partial G_{dB}}{\partial |S_{22}|}\right)^2 u^2(|S_{22}|)$$

$$= \left(0.5u(A_0)\right)^2 + \left(\frac{4.343u(\beta)}{\beta}\right)^2 + \left(\frac{4.343|S_{11}|u(|S_{11}|)}{1-|S_{11}|^2}\right)^2 + \left(\frac{4.343|S_{22}|u(|S_{22}|)}{1-|S_{22}|^2}\right)^2 \quad (7)$$

The fractional uncertainty of the gain, $u(G)/G$, can be also estimated as follows:

$$u(G)/G = 0.2303u(G_{dB}). \tag{8}$$

Needless to say, the above discussion is valid for the lower order asymptotic expansion in Eqs. (3) and (4). That is, we can treat the above estimation with $A_2=0$ or $A_1=A_2=0$. The latter corresponds to the procedure to determine the gain and dielectric property of the liquid in the far-field region as we discussed before [3]. The estimated far-field gain is sensitive to the choice of the asymptotic terms as well as the fitting range. The term of A_2/r^2 in Eq. (3) expresses the contribution of the extremely near-field of the antenna so that it is effective for curve-fitting extremely in the neighborhood of the antenna whereas some ill-conditioned behaviors can be observed in the far-field region. Therefore, we should examine the choice of the asymptotic terms and fitting range.

3. Some Experiment Results for Estimating Gain and Uncertainty

As shown in Fig. 1, a rectangular tank is filled with about 50l of the tissue equivalent liquid which has a relative permittivity of 39.2 and conductivity of 1.84S/m at 2.45GHz. Two offset dipole antennas with a length of 13mm are used in our measurement. S_{21} are measured in the range of $r=0$mm to 150mm in increment of 0.5mm, which is different from the fitting range. In our previous study, $|S_{21}|$ can not be measured at the level of about -80dB with the factory preset of the network analyzer (Agilent N5230A). This limitation can be observed at the distance of 70mm and the measured data are greatly fluctuated beyond the distance of 90mm at 2.45GHz [3].

Table 1 shows some estimated far-field gains and corresponding fractional uncertainties for various regression functions and fitting ranges. For the description of S_{21} in the table, n means that we choose α, A_i, $i=0,\ldots,n$ and β, B_i, $i=0,\ldots,n$ as the parameters of the regression functions in Eqs. (3) and (4). The uncertainties of S parameters can be evaluated by the worksheet provided by the manufacturer of the instrument. We also list the far-field gains and fractional uncertainties for the regression analysis based on the Fresnel region gain [4] in the table and we can see that the estimated gain has a typical value of -2.4dBi irrespective of the choice of the fitting range. However, we can also find that the estimated gain from the S_{21} regression greatly depends upon the number of n and the fitting range. As one of the reasons of these scattering results, the regression function should be selected with careful regard to its behavior in the fitting range. For $n=2$, the fitting range should include the contribution of the extremely near-field and exclude the contribution of the far-field effect because the asymptotic expansion is valid in the limited range considered. In this case, we should choose 10mm as the start of the fitting range, r_{start}, and $n=2$. Furthermore, the uncertainty is larger as n is larger and r_{start} is larger. Especially, the choice of r_{start} is serious problem affecting the uncertainty as seen from Table 1.

Next, we consider the effect of the likely introduction of the weighted least squares. Figs. 2 and 3 show the estimated far-field gain and corresponding fractional uncertainty as a function of the stop of the fitting range, r_{stop}, when the start is fixed as $r_{start}=10$mm. For comparison, the figures include the results for the regression with/without weight. In the equal-weighted regression, the uncertainty of measurand is selected as one at the center of the fitting range. As shown in Fig. 2, when r_{stop} changes, the estimated gain is fluctuated and not converge with its true far-field gain for the equal-weighted regression, whereas little variation of the estimated gain can be observed for the weighted regression. In practice, the fluctuation of measurand can not be exactly estimated so that the weighted regression is effective for our gain estimation in the liquid. As shown in Fig. 3, the introduction of the weight into the regression analysis hardly influences the uncertainty of the

Table 1: Estimated Far-Field Gain and Fractional Uncertainty Based on the Method of Weighted Least Squares for measured S_{21} and Fresnel Region Gain

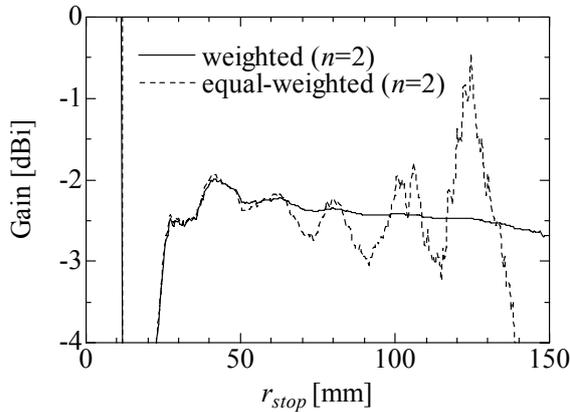| Regression Function		Fitting Range		
		10mm – 100mm	20mm – 100mm	30mm – 100mm
S_{21}	$n=0$	-4.10dBi (0.23%)	-3.67dBi (0.36%)	-3.44dBi (0.60%)
	$n=1$	-2.79dBi (0.76%)	-2.63dBi (1.91%)	-2.92dBi (4.41%)
	$n=2$	-2.42dBi (2.22%)	-3.00dBi (7.82%)	-2.87dBi (23.5%)
Fresnel Region Gain		-2.40dBi (1.28%)	-2.39dBi (2.20%)	-2.28dBi (3.51%)

Figure 2: Variation in Estimated Far-Field Gain as a Function of Upper Limit of the Fitting Range, r_{stop}, When Lower Limit is Fixed as r_{start} =10mm.
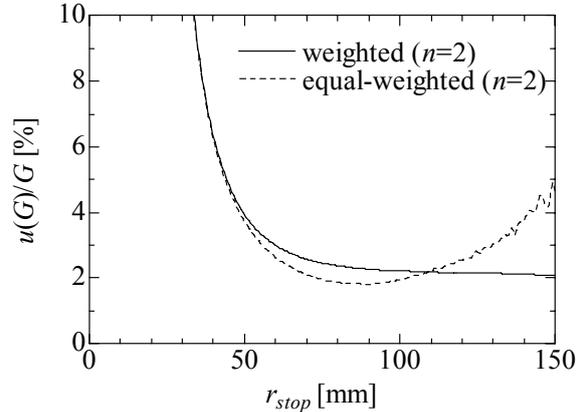
Figure 3: Variation in Uncertainty as a Function of Upper Limit of the Fitting Range, r_{stop}, When Lower Limit is Fixed as r_{start} =10mm.

estimated gain for $r_{stop} > 70$mm. Of course, the uncertainty for equal-weighted regression tends to increase when r_{stop} is longer because the uncertainty of the measurement system degrades when the level of $|S_{21}|$ is small.

4. Conclusion

The probes used in the standard SAR evaluation for the mobile communication devices can be calibrated by the procedure with no use of the waveguide. It is required to calibrate the gain of the reference antenna in the liquid. In this paper, we evaluate the gain as well as dielectric property in the liquid according to extended Friis transmission formula with the method of weighted least squares. If the start of the fitting range is carefully selected, the introduction of the weight in the regression has the estimated gain little fluctuated for various selections of the stop of the range. In the regression for S_{21}, it is also important to select the regression function. And we estimate the uncertainty of the estimated gain determined by the weighted regression.

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