# Study on DOA/DOD Estimation using a Quasi MIMO System

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## 1. Introduction

In recent years, the MIMO (Multiple-Input Multiple-Output) radar have been attracting attention. The MIMO radar system is expected to increase the maximum number of targets that can be uniquely identified and to detect target/multipath waves separately [1]. The MIMO radar uses arrays both in transmitter and receiver, and it can separate direction of departure (DOD) of waves as well as direction of arrival (DOA). For this reason, MIMO radar can discriminate targets from multipath waves.

In this report, we consider DOA/DOD estimation performance of a MIMO system. As discussed in [2], effectiveness of the MIMO radar compared with traditional radars is not improved under same transmitting power or on a term of hardware. However, application of the MIMO to the short-range radar is still attractive because of its high adaptivity of beam/null steering property. In this report, we consider performance a quasi MIMO system by using switching transmit array as a fundamental study. This system is equivalent to channel sounders for DOA/DOD estimation. We discuss a characteristic using a quasi MIMO system with two typical DOA/DOD estimation methods, beamformer method and Capon method.

## 2. The Receiving Data Model

We consider a DOA/DOD estimation using a quasi MIMO system with $M_t$-element transmitting uniform linear array (ULA) and $M_r$-element receiving ULA. Figure 1 shows the array model. Each of the transmitting elements transmits $N_s$ samples of signal $s_i(t)$ alternately. For simplicity, let $s_1(t) = s_2(t) = ... = s_{M_t}(t) = s(t)$, this means that the same probing signal $s(t)$ is transmitted repeatedly. $\theta_t$ denotes DOD, and $\theta_r$ denotes DOA. In this study, we consider monostatic radar configuration. However, this is not an essential problem even because bistatic radar can also apply to DOA/DOD parameter estimation. Under the assumption that the target is in far field, the receiving data vector of $K$-incident wave can be written by

$$\boldsymbol{x}(t) = \sum_{k=1}^{K} \sigma_s^{(k)} s^{(k)}(t) \boldsymbol{a}_t(\theta_{t,k}) \otimes \boldsymbol{a}_r(\theta_{r,k}) + \tilde{\boldsymbol{n}}(t), \quad (1)$$

$$\boldsymbol{a}_t(\theta_t) = [1, e^{j\frac{2\pi}{\lambda} d_t \sin\theta_t}, \cdots, e^{j\frac{2\pi}{\lambda}(M_t-1) d_t \sin\theta_t}]^T, \quad (2)$$

$$\boldsymbol{a}_r(\theta_r) = [1, e^{-j\frac{2\pi}{\lambda} d_r \sin\theta_r}, \cdots, e^{-j\frac{2\pi}{\lambda}(M_r-1) d_r \sin\theta_r}]^T, \quad (3)$$

where $\lambda$ is wavelength, $\boldsymbol{a}_t(\theta_t)$ is transmitting mode vector, $d_t$ is element spacing of transmitter, $\boldsymbol{a}_r(\theta_r)$ is receiving mode vector, $d_r$ is element spacing of receiver, $\sigma_s$ is a complex coefficient containing target radar cross section (RCS) and propagation loss, $\tilde{\boldsymbol{n}}(t)$ is white noise (average 0, variance $\sigma_n^2$), $^T$ is transpose, and $\otimes$ is Kronecker product. The $\{\boldsymbol{a}_t(\theta_t) \otimes \boldsymbol{a}_r(\theta_r)\}$ is MIMO virtual mode vector. If a MIMO system consists of $M_t$ transmitting and $M_r$ receiving, maximum degrees of freedom (DOF) of MIMO can be $M_t M_r$ with a suitable array parameter. For example, Fig. 2 shows the image in the case of $M_t = 3, M_r = 4$. However, it depends on element spacing of ULA whether $M_t M_r = 12$ data are effective for target separation or not [1]. The received data correlation matrix can be written by

$$\boldsymbol{R}_{xx} = \mathrm{E}[\boldsymbol{x}(t)\boldsymbol{x}(t)^H] \quad (4)$$

where $\mathrm{E}[\cdot]$ denotes ensemble averaging, and $^H$ is complex conjugate transpose.

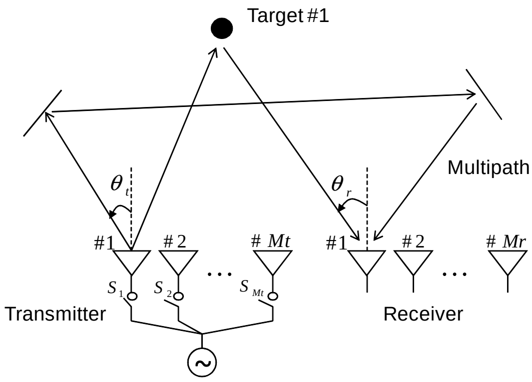


Figure 1: array model

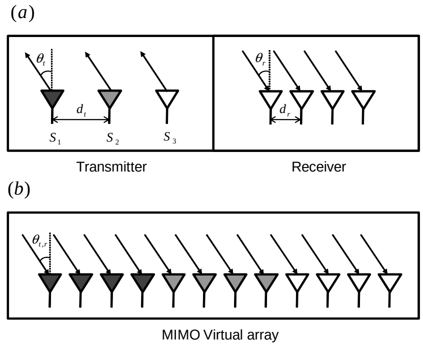


Figure 2: (a) ULA MIMO system image ($M_t = 3$, $M_r = 4$) (b) Corresponding MIMO virtual image

# 3. DOD/DOA Estimation

## 3.1 Beamformer Method

The beamformer method is simple DOA/DOD estimation method. The spatial spectrum by the beamformer is estimated by

$$P_{BF}(\theta_t, \theta_r) = \frac{\{\boldsymbol{a}_t(\theta_t)\otimes\boldsymbol{a}_r(\theta_r)\}^H \boldsymbol{R}_{xx}\{\boldsymbol{a}_t(\theta_t)\otimes\boldsymbol{a}_r(\theta_r)\}}{\{\boldsymbol{a}_t(\theta_t)\otimes\boldsymbol{a}_r(\theta_r)\}^H\{\boldsymbol{a}_t(\theta_t)\otimes\boldsymbol{a}_r(\theta_r)\}}. \tag{5}$$

The beamformer method can apply directly even when incident waves are coherent. Resolution of the method depends on beamwidth of main lobe. Namely, DOD and DOA resolution depends on aperture of transmitting and receiving array, respectively.

## 3.2 Capon Method

The spatial spectrum by the Capon method is estimated by

$$P_{Capon}(\theta_t, \theta_r) = \frac{1}{\{\boldsymbol{a}_t(\theta_t)\otimes\boldsymbol{a}_r(\theta_r)\}^H \boldsymbol{R}_{xx}^{-1}\{\boldsymbol{a}_t(\theta_t)\otimes\boldsymbol{a}_r(\theta_r)\}}. \tag{6}$$

The method has higher resolution capability than the beamformer method. However, in the case of coherent wave incidence, it cannot apply directly. In such a case, spatial smoothing preprocessing (SSP) or Forward/Backward SSP (FB-SSP) is necessary to destroy signal coherence.

## 3.3 Spatial Smoothing Preprocessing (SSP)

The SSP and the FB-SSP are effective preprocessing methods to reduce signal coherence and able to resolve coherent signals. In this report, SSP is employed for subarrays of the MIMO virtual array and averaging these correlation matrices. We call a averaging process by transmitting subarrays $\{\boldsymbol{a}_t^{sub} \otimes \boldsymbol{a}_r\}$ *transmitting SSP*, and a averaging process by receiving subarrays $\{\boldsymbol{a}_t \otimes \boldsymbol{a}_r^{sub}\}$ *receiving SSP*. The number of elements of the MIMO virtual subarray should be greater than the number of incident waves $K$. The correlation matrix with SSP can written by

$$\boldsymbol{R}^f_{\mathrm{SSP}} = \frac{1}{L}\sum_{l=1}^{L} \boldsymbol{R}^f_l, \tag{7}$$

$$\boldsymbol{R}^f_l = E[\boldsymbol{x}^f_l(t)\boldsymbol{x}^f_l(t)^H], \tag{8}$$

where $L$ is the number of subarrays, and $\boldsymbol{x}^f_l(t)$ is the $l$-th receiving data vector. $L$ should be equal to or greater than $K$. Figure 3 and 4 show the transmitting SSP and the receiving SSP subarray configuration, respectively where $M_t = 3, M_r = 4$, and $L = 2$. Transmitting SSP is effective in DOD domain, and receiving SSP is effective in DOA domain. Transmitting SSP can not separate same DOD waves, to the contrary receiving SSP can not separate same DOA waves. Therefore, it has to apply both transmitting SSP and receiving SSP when multipath waves caused by a target.

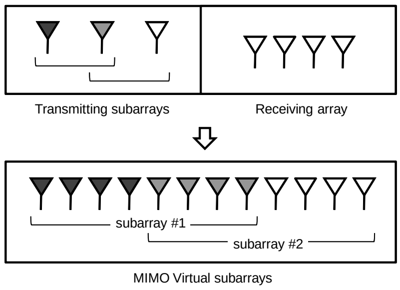


Figure 3: Transmitting SSP subarrays ($M_t = 3, M_r = 4, L = 2$)

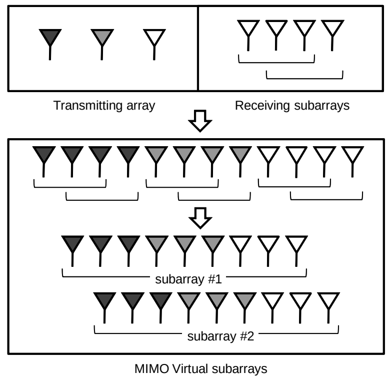


Figure 4: Receiving SSP subarrays ($M_t = 3, M_r = 4, L = 2$)

## 4. Simulation

In this section, we show computer simulation results by a quasi MIMO system. First, we show simulation results of detection and separation target/multipath waves. Simulation parameters are listed in Table 1. Figure 5 shows the 2D (DOA/DOD) spatial spectrum by the beamformer. Figure 6 shows the 2D spatial spectrum by Capon method with transmitting SSP and receiving SSP. The number of subarrays of the transmitting SSP is 2, and that of the receiving SSP is 2. In these two methods, target/multipath waves are detected and separated correctly.

Next, we show difference of $\gamma = d_t/d_r$. Other simulation parameters are the same in Table 2. Figure 7 and 8 show the 1D (DOA=DOD) spatial spectrum by the beamformer under $\gamma = 1$ and $\gamma = M_r = 5$ respectively. DOF of MIMO becomes maximum when $\gamma = M_r$ in the case of monostatic configuration with ULAs. Accordingly the MIMO virtual aperture length can be increased by wide element spacing of transmitter. However, it has to note that grating lobes will of often be caused in DOD domain. This property is the same in the case of the Capon method.

Table 1: Simulation parameters 1

| | |
|---|---|
| Array form (both Trans. and Rec.) | ULA |
| Number of array elements $M_t$ (Trans.) | 5 |
| Number of array elements $M_r$ (Rec.) | 5 |
| Element spacing of transmitter $d_t$ | 0.5 $\lambda$ |
| Element spacing of receiver $d_r$ | 0.5 $\lambda$ |
| Target angle ([DOD, DOA]) | [0, 0] (deg.) |
| Multipath wave angle | [0, 45] [45, 0] (deg.) |

Table 2: Simulation parameters

| | |
|---|---|
| Array form (both Trans. and Rec.) | ULA |
| Number of array elements $M_t$ (Trans.) | 5 |
| Number of array elements $M_r$ (Rec.) | 5 |
| $\gamma = d_t/d_r$ | 1, 5 |
| Element spacing of transmitter $d_t$ | $0.5\lambda$, $2.5\lambda$ |
| Element spacing of receiver $d_r$ | $0.5\lambda$ |
| Target angle ([DOD=DOA]) | [0, 20, 45] (deg.) |
| Multipath wave angle | - |

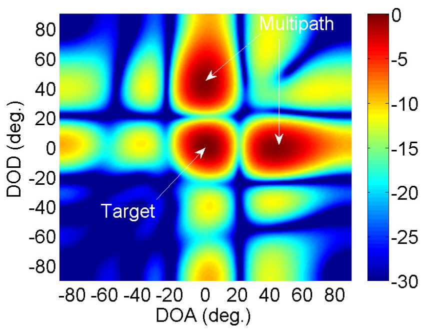


Figure 5: Beamformer 2D Spatial Spectrum

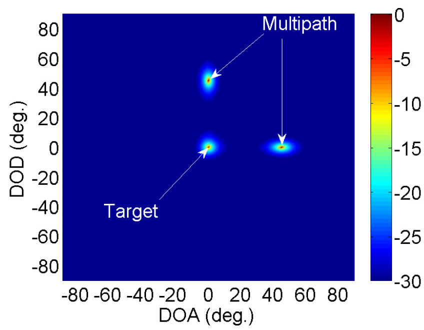


Figure 6: Capon 2D Spatial Spectrum

## 5. Experimental results of Target/Multipath Wave Detection

Experiment of detection and separation target/multipath waves using the quasi MIMO system was carried out in a radio anechoic chamber. The experimental setup and measurement parameters are shown

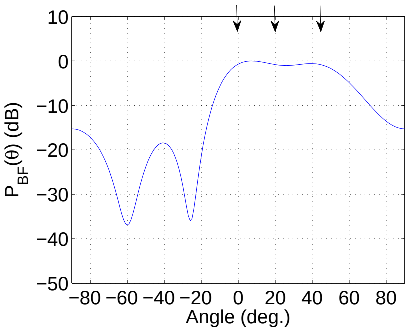


Figure 7: Beamformer 1D Spatial Spectrum ($\gamma$ = 1)

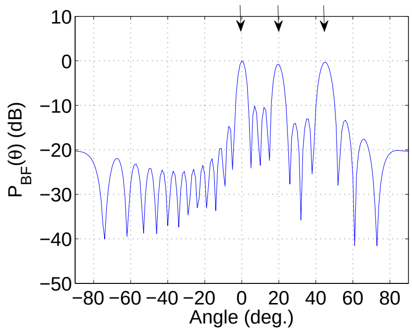


Figure 8: Beamformer 1D Spatial Spectrum ($\gamma$ = 5)

in Fig. 9 and Table 3, respectively. Figure 10 shows the 2D spatial spectrum by the beamformer. Figure 11 shows that of the Capon 2D with transmitting SSP and receiving SSP. The number of subarrays of transmitting SSP is 2, and the number of subarrays of receiving SSP is 2. The beamformer method does not need SSP. However, resolution of the method limited by the beamwidth of main lobe. The Capon method needs SSP, therefore effective aperture becomes smaller than that of the beamformer method. However, as shown in Fig. 11 the Capon method has higher resolution capability than the beamformer method.

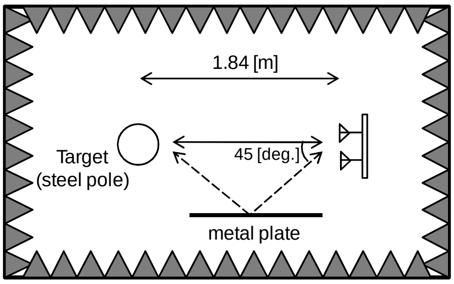


Figure 9: Experimental setup

Table 3: Measurement parameters

| | |
|---|---|
| Array form (both Trans. and Rec.) | ULA |
| Number of array elements $M_t$ (Trans.) | 4 |
| Number of array elements $M_r$ (Rec.) | 4 |
| Element spacing of transmitter $d_t$ | 0.5 $\lambda$ |
| Element spacing of receiver $d_r$ | 0.5 $\lambda$ |
| Target angle ([DOD, DOA]) | [0, 0] (deg.) |
| Multipath wave angle | [0, 45] [45, 0] (deg.) |

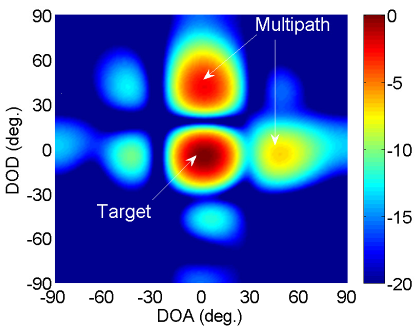


Figure 10: Beamformer 2D Spatial Spectrum

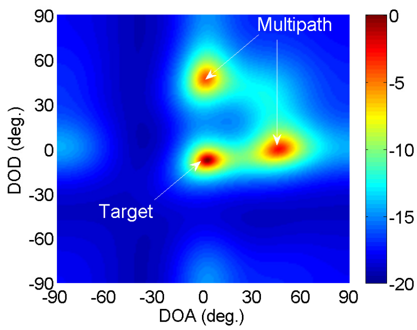


Figure 11: Capon 2D Spatial Spectrum

## 6. Conclusion

In this report, we considered DOA/DOD estimation using a quasi MIMO system, and discussed a characteristic using a quasi MIMO system with beamformer method and Capon method. Furthermore, we showed experiment of target/multipath wave detection.

## References

[1] J. Li, P. Stoica, MIMO Radar Signal Processing, John Wiley & Sons, Inc., 2008.

[2] F. Daum, J. Huang, "MIMO Radar: Snake Oil or Good Idea?," IEEE A&E System Magazine, pp.8-12, May, 2009.