# Angular Resolution Improvement of Ocean Surface Current Radar


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## 1. Introduction

Ocean surface current radar is a Doppler radar to observe oceanographic information using the Bragg scattering resonance mechanism [1]. In this paper, we consider angular resolution enhancement of the ocean surface current radar. The radar employs an antenna array with FM-i-CW operation, then it can resolve angular distribution by Digital Beam Forming (DBF) method and distance can be discriminated by the FM-i-CW radar. In order to obtain sufficient angular resolution, array length should be expanded by increasing the number of elements of the array. Recently, a signal processing technique which can extend the aperture length virtually has proposed, that is called the Khatri-Rao (KR) product array. The angular resolution can be improved without changing the physical aperture length of the array antenna by using this method [2]. The ocean surface current radar is operated in very low frequency. Therefore it is the very important factor to achieve required angular resolution with narrower array length. In this paper, to ensure that the KR product array processing is valid in angular resolution enhancement for the ocean surface current radar, we apply the KR product array to actual data set of the radar and verify the performance .

## 2. 2-dimensional Correlation Matrix

The obtained data by using the actual ocean surface current radar is a 3-dimensional data for array-element, beat-signal of each FM pulse, and time-series of the pulses. In this paper, we first estimate the distance in advance, by applying a Fourier transform to each beat-signal. Since this is the FM-i-CW based radar, the beat-frequency can be related to the distance from the radar. Then, we can pick up a beat-frequency component in each signal at every elements to make 2-dimensional data vector as shown in (1). This vector includes information of the speed and direction of arrival (DOA) of the surface current at the distance. Speed of the current can be estimated by the Doppler shift component. Fig.1 illustrates the signal model. In (1), we assume that number of reflected wave is $K$ that coming from the direction of $\theta_K$, and $s_{rk}(0)$ shows each complex amplitude. Also $\boldsymbol{n}$ is the additive noise vector. The vector $\boldsymbol{a}(\theta_k)$ in (2) and $\boldsymbol{a}(\overline{f_d})$ in (3) show the mode vector for angle and Doppler frequency, respectively, where $[\cdot]^T$ denotes the transpose. The 2-dimensional correlation matrix of (1) is defined by (4), where $[\cdot]^H$ is the complex conjugate transpose, $E[\cdot]$ is the ensemble averaging. $M$ is the total number of sweep samples (pulses), $L$ is The number of elements, and $\otimes$ denotes a operator of the Kronecker product. $\boldsymbol{A}_{2d}$ is a $LM \times K$ mode matrix whose column is given by $\boldsymbol{a}(\overline{f_{dk}}) \otimes \boldsymbol{a}(\theta_k)$. Also, $\boldsymbol{R}_{LM}$ is the noise correlation matrix.

$$\boldsymbol{x}_{2d} = \sum_{k=1}^{K}(\boldsymbol{a}(\overline{f_{dk}}) \otimes \boldsymbol{a}(\theta_k))s_{rk}(0) + \boldsymbol{n} = \boldsymbol{A}_{2d}\boldsymbol{s}_r + \boldsymbol{n}, \tag{1}$$

$$\boldsymbol{a}(\theta) = [1, e^{-j\frac{2\pi}{\lambda}\Delta d \sin\theta}, \cdots, e^{-j\frac{2\pi}{\lambda}(L-1)\Delta d \sin\theta}]^T, \tag{2}$$

$$\boldsymbol{a}(\overline{f_d}) = [1, e^{j2\pi\overline{f_d}}, \cdots, e^{j2\pi(M-1)\overline{f_d}}]^T, \quad (3)$$

$$\boldsymbol{R}_{xx}^{(2d)} = E[\boldsymbol{x}_{2d}\boldsymbol{x}_{2d}^H] = \boldsymbol{A}_{2d}\boldsymbol{S}_r\boldsymbol{A}_{2d}^H + \boldsymbol{R}_{LM}. \quad (4)$$

# 3. Khatri-Rao Product Array Processing

The Khatri-Rao product can be defined by two matrices having the same column order. The KR product array processing is a new correlation matrix $\hat{\boldsymbol{R}}_{xx}^{(2d)}$ which can be constructed by using KR product by the two-dimensional correlation matrix $\boldsymbol{R}_{xx}^{(2d)}$ as shown in (5). In this equation, $[\cdot]^*$ denotes the complex conjugate, $\odot$ is the KR product operator, and vec$(\cdot)$ is the operation which stacks columns of the argument matrix to make a vector. In addition, $\boldsymbol{s}_{eff}$ is a K-dimensional column vector consisting of the diagonal elements of $\boldsymbol{S}_r$. This extended data vector can be obtained when we assume that the each incident signals in (4) is uncorrelated with each other because $S_r$ becomes a diagonal matrix. The equation (5) can be replaced by (6). It turns out that the equation (6) has the same form as that in (1) except for the dimension. The Degree-of-Freedom (DOF) of the array can be improved by the KR product array processing [3]. As reported in [3], this transform unable us to increase effective aperture. Effective aperture becomes almost twice for the uniform linear array. Furthermore, we can resolve up to $2(L-1)$ waves by using the transformed data vector in (5) when all of the signals are uncorrelated.

$$z_{2d} = \text{vec}(\boldsymbol{R}_{\boldsymbol{xx}}^{2d}) = \text{vec}(\boldsymbol{A}_{2d}\boldsymbol{S}_r\boldsymbol{A}_{2d}^H) + \text{vec}(\boldsymbol{R}_{LM}) = (\boldsymbol{A}_{2d}^* \odot \boldsymbol{A}_{2d})\boldsymbol{s} + \text{vec}(\boldsymbol{R}_{LM}), \quad (5)$$

$$z = (\boldsymbol{A}_{2d})_{eff}\boldsymbol{s}_{eff} + \boldsymbol{c}. \quad (6)$$

In the case of the ocean surface current radar, the number of reflected signals $K$ exceeds to the number of element in (5). That means we can hardly apply superresolution algorithm, such as Capon and MUSIC. However, enhancement of the array aperture by the KR product array processing realizes narrower beam by the array. Therefore improvement of angular resolution can be realized only by the signal processing in the given radar system.

# 4. Experimental Study

In this section, we analyze actual data set obtained by the ocean surface current radar, and show validity of the KR product array processing. The data used in this analysis is provided by the Nagano Japan Radio Co., Ltd., Japan. The data is observed in the Ariake Ocean on October 22, 2006. Table 1 shows the specifications of the data set.

Table 1: Measured parameters

| DOA estimation method | Beamformer |
|---|---|
| Array shape | ULA |
| Number of elements | 4(with KR), 7, 8 |
| Array element spacing | 7 [m] |
| Target distance | 15 [km] |
| Target speed | -3.0656 [m/s] |
| Center frequency | 24.515 [MHz] |
| Frequency bandwidth | 100 [kHz] |
| Sweep time | 500 [ms] |
| The total number of sweeps | 1024 |
| Number of points in each sweep | 512 |

## 4.1 Experimental Results

First, we'd like to show the estimated 2-D spectrum in Doppler frequency and angle in the given range bin (15 km) by the 8-element array. Fig. 2 shows the result without the KR product array processing, and 3 shows that with the KR product array processing. A vertical axis expresses the Doppler frequency shift and the horizontal axis expresses the Direction of Arrival (DOA) of the wave . Figures 2 and 3 show the wave Doppler frequency components at about 0.5 [Hz] and -0.5 [Hz]. The Doppler components around 0 [Hz] could be the system noise. Figures 4 and 5 show the angular spectrum of the Doppler components at -0.500 [Hz] shown in Figures 2 and 3, respectively. The estimated angular spectrum in Fig. 4 is not so sharp. This is the actual angular resolution obtained by the conventional Beamformer. By applying the KR product array processing, we can obtain better angular spectrum as shown in Fig. 5. The half power beam width (HPBW) becomes almost half. The dominant surface current(s) can be estimated peak(s) of the spectrum. The estimated velocity and angle of the current are also listed in Table 2.

Table 2: Estmated surface current parameters results

| The presence or absence of KR product | Estimated velocity | Estimationd DOA |
|---|---|---|
| With KR product expansion array processing | -3.0656 [m/s] | $-9°$ |
| Without KR product expansion array processing | -3.0656 [m/s] | $-3°$ |

### 4.2 Comparison of a real array and a virtual array

When KR product extension array processing is used, degree-of-freedom of the array becomes $2L-1$ for the $L$-element (physical) uniform linear array (ULA) [3]. Therefore, when the KR product extension array processing is performed using a 4-element ULA, it can be said that performance of the 7-element can be acquired. Since the 8-element array data is available in this data, we can directly compare the results of the physical 7-elemet array and the 4-element KR processing array. Theoretically these two has the same performance. To confirm the angular spectrum of the virtual 7-element array (4-element array with the KR processing) and the physical 7-element array, we calculate the spectrums of the arrays. As shown in Figures 6 and 7, we can pick up 5 and 2 subarrays in spectrum estimation. Therefore we evaluate the angular spectrum by using avaraged data correlation matrices among them in each estimation. The examples of the angular spectrums of the physical 7-elemet array and the 4-element KR processing array are shown in Fig. 8. Since this is the experimental data and they contains calibration errors and so forth, slight difference can be seen. However, these results are almost coincide with each other. Therefore, we can say that the proposed KR product array processing enable us to enhance effective array aperture twice. This means that the same angular resolution can be realized with almost half physical aperture array.

## 5. Conclusion

In this paper, in order to show that the KR product array processing is effective in the ocean surface current radar, we provide some experimental results. The angle resolution improvement by the KR product extension array processing can be verified experimentally. Moreover, since the almost comparable result was obtained in 7-element ULA with 4-element ULA KR product extension array processing, it can be said that KR product extension array processing for the ocean surface current radar is effective.

## References


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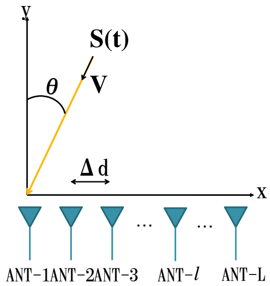


Figure 1: L-element array and signal model

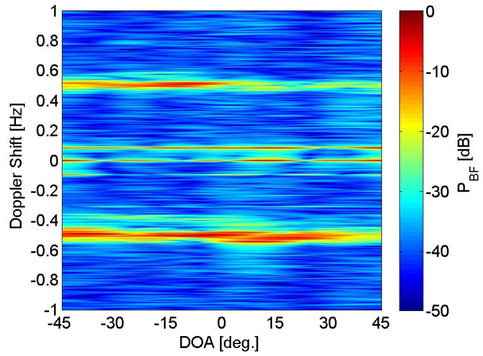


Figure 2: Estimated 2-D spectrum without the KR product expansion array processing

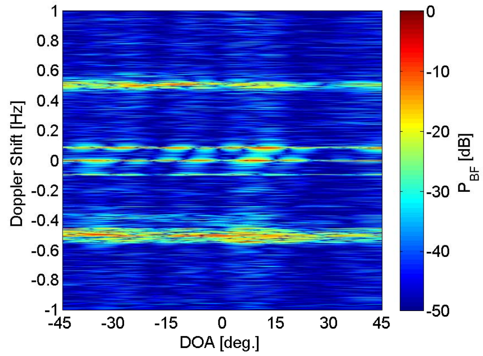


Figure 3: Estimated 2-D spectrum with the KR product expansion array procesinsg

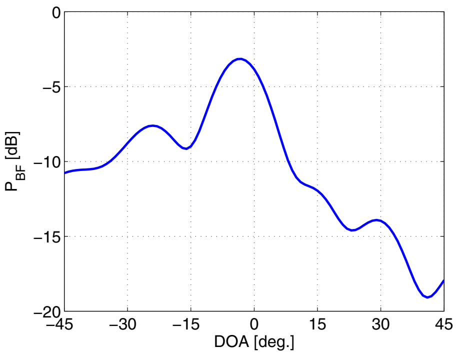


Figure 4: Estimated angular spectrum without the KR product expansion array processing(Doppler frequency shift at -0.500 [Hz])

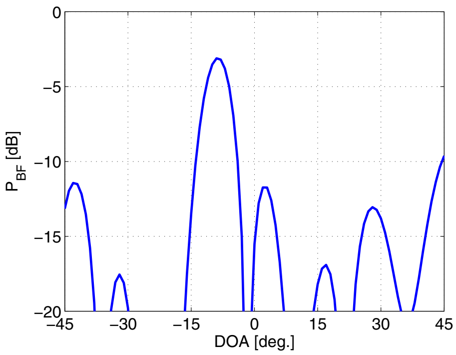


Figure 5: Estimated angular spectrum with the KR product expansion array processing(Doppler frequency shift at -0.500 [Hz])

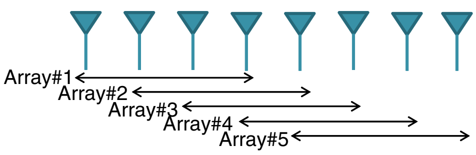


Figure 6: Selected 4-element array in the evaluation

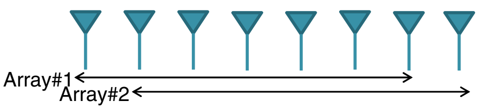


Figure 7: Selected 7-element array in the evaluation

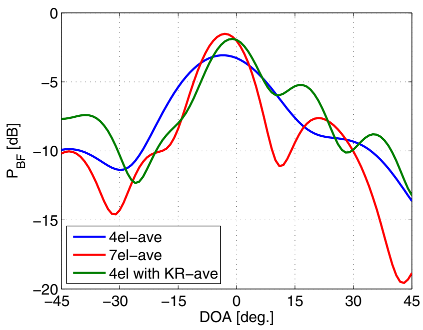


Figure 8: Estimated by angular spectrum 4-element ULA, 7-element ULA, and 4-element array with KR processing