Functions Measuring the Centrality (or Mediality) of a point in a Network

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Abstract:

    A new theory of functions measuring the centrality (or mediality) of
a point in a network is developed.

Summary:

    It is one of the fundamental problems in network theory with applications
to study the centrality (or mediality) of a point in a network and properties
measuring the centrality (or mediality).   In this paper, a new theory of
functions mesuring the centrality (or mediality) of a point in a network is
developed.   Kajitani and Maruyama's theory and our old theory are a special
case of the theory developed herein.

    Consider a connected network N with vertex set $V(N)$ and edge set $E(N)$.
The network may be either directed or undirected.   With each edge e of N,
two kinds of non-negative real numbers $l(e)$ and $c(e)$, called the edge-length
and the edge-capacity of e, respectively, are associated, and with each vertex
v of N, a non-negative real number $\sigma(v)$, called the vertex-weight of v, is
associated.   Let S be a set of points of N where a point of N can be either
a vertex or an interior point of an edge.   For any two points s_i and s_j in
S, we define two kinds of non-negative real numbers $\rho(s_i, s_j)$ and $\gamma(s_i, s_j)$,
called the di-distance from s_i to s_j and the di-capacity from s_i to s_j,
respectively, by using the underlying graph, edge-lengths and edge-capacities
of N.   A typical example of a di-distance from a point s_i to a point s_j in N
is the length of a shortest path from s_i to s_j in N and a typical example of

a di-capacity from a point s_i to a point s_j in N is the value of the maximum flow from s_i to s_j in N. The concepts of di-distance and di-capacity are different but are fundamentally important in evaluating the degree of closeness of one point to another. Next, we introduce the concept of monotone modification as a natural unification of network modifications such as adding new edges, coalescencing some verties, shortening edge-lengths and increasing edge-capacities. The monotone modification consists of two kinds of network modifications, called a monotone contraction and a monotone expansion, where the monotone contraction is a network modification with respect to di-distance and the monotone expansion is a network modification with respect to di-capacity. Next, in order to measure the centrality (or mediality) of a point r in N, we consider a real-valued function $f(r,\rho,\gamma,\sigma)$ defined on N where $\rho \mid S \times S \to \bar{R}_+$, $\gamma \mid S \times S \to \bar{R}_+$ and $\sigma \mid V(N) \to \bar{R}_+$ and characterize the centrality of a point r in N by using the tendency of the change of the functional value of $f(r,\rho,\gamma,\sigma)$ under a monotone modification of N. Next, in the case where we restrict the form of f to

$$ f(r,\rho,\gamma,\sigma) = \sum_{s_i \in V(N) \subseteq S} \psi(\rho(r,s_i),\gamma(r,s_i),\sigma(s_i)), $$

we give a necessary and sufficient condition for f to be a function measuring the centrality (or mediality) of a point in N in our sense, and study some relationships between the monotone modification of N and the convexity or concavety of ψ.

A precise description of this paper will be included in a full paper of the coming Trans. of IECE of Japan.

References:

[1] Kajitani, Y. and Maruyama, T.: Functional expression of the centrality of a vertex in a graph, Trans. IECE of Japan, Vol.59A, 531-538, 1976.

[2] Shinoda, S. and Sengoku, M.: Axiomatic foundations of the theories of functions expressing the mediality of a point in a metric space, ibid., Vol.J66-A, 352-359, 1983.