



Export Subsidies and Timing of Decision-Making

— An Extension to the Sequential-Move Game of Brander and Spencer (1985) Model —

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Abstract

This paper examines how the timing of decision-making affects strategic trade policy. In this paper, we analyze the relationship between the different timing of decision-making by exporting firms and their subsidizing governments and its impact on export subsidy. The paper aims to extend the analysis of Brander and Spencer (1985) to include the Stackelberg competition and the sequential-move decision on the subsidy choice by governments. Some main results are presented as follows: First, when governments decide simultaneously the export subsidies in advance under the following Stackelberg quantity competition, the original leader firm produces as if it was the follower. Different from the Cournot model, under the Stackelberg model, the subsidy policy by the government that can subsidize the leader firm does not work effectively. Second, under the sequential-move game in which the government that can subsidize the leader firm decides its subsidy level first, the profit of the leader firm is less than that of the follower in the Stackelberg model, although the first-mover advantage of the government is maintained. The result proves that the timing of decision-making affects the results of the export subsidy policy significantly.

JEL classification: D43; F12; L13

Keywords: export subsidy; sequential-move game; Stackelberg competition

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The research for this paper is supported by Grant-in-Aid for Scientific Research (KAKENHI 16730095) from JSPS and MEXT of the Japanese Government.

1 Introduction

This paper examines how the timing of decision-making affects the strategic trade policy. We analyze the relationship between the different timing of decision-making by exporting firms and their subsidizing governments and its impact on the export subsidy policy.

Although WTO reorganized from the GATT in 1995 and the FTAs have been concluded among many countries and tend to increase rapidly nowadays, the export subsidy policy is still being practiced in many countries as a strategic tool to induce more domestic surplus from exportation. In the WTO agreements (Agreement on Subsidies and Countervailing Measures), the export subsidies to the manufactured products are prohibited *per se*, and a reduction of the subsidy rate for the agricultural products is being negotiated by the WTO members. Countervail and anti-dumping measures are officially enshrined in WTO rules; this allows damaged government to countervail the export subsidies. However, in reality, we can easily find many cases about disputes between multinational firms on the export subsidies in the context of the international market competition.

From the theoretical point of view, many studies about export subsidies have been done. Since Brander and Spencer (1985) elucidated the strategic effect of subsidy policy in their seminal paper on strategic trade policy, many studies have been done analyzing the export subsidies in the context of the strategic trade policy. Using the third-country model, Brander and Spencer (1985) analyzed the rent-shifting effect of the export subsidy and the strategic interaction between the export subsidies. They argued that the export subsidy effectively raises domestic welfare, but it implies that the strategic subsidy choices of two governments in the exporting countries fall into the suboptimal excessive competition such as a prisoner's dilemma.

Another pioneer work by Eaton and Grossman (1986) analyzed a more generalized model. They extended the model of Brander and Spencer (1985) to allow the different conjectural variations from the Cournot case, that is, the different competing environments. They showed that under the Bertrand conjecture, the optimal trade policy is the exporting tax imposition to the domestic exporting firm.

Although the two representative papers mentioned above and their successors dealt with a general demand structure and illuminated the strategic aspects on the trade policy, however, those papers have limited their analysis to the situation in which the choices of the strategic variables are made simultaneously by the competitive firms. For example, Brander and Spencer (1985) restricted their analysis to the Cournot quantity competition. Although Eaton and Grossman (1986) generalized the conjectural variations including Cournot, Bertrand, and consistent conjectures, these conjectural variations between firms are identical. The existing literature has usually dealt with only symmetric case between firms, that is, only with the simultaneous-move game on output choice. We consider the Stackelberg leader-follower competition and deal with the asymmetric conjecture as a result. Extending the simultaneous-move game on output choice and also subsidy choice to the sequential one, we present a new perspective about the strategic subsidy policy that is influenced by the timing of decision-making.

In the actual international trade policy, we can imagine many situations in which the timing of decision-making about the trade policies by governments is different. For instance, it may take place that the governments of developed countries determine the subsidy levels in advance of the governments of developing countries. Because of the different abilities of the governments to implement and enforce the trade policy, there exists usually a time lag on the subsidy decisions by governments. On the one hand, whether or not a country has the leading industry may affect

the speed of policy determination positively. On the other hand, to facilitate the infant industry, the government may forestall the rival government and determine the subsidy level in advance.

The paper introduces the difference on the timing of decision-making on output and subsidy levels in the model. We examine how the different timing determining strategic variables impacts on the export subsidy policy under imperfect competitive environments.

The paper extends the analysis of the Cournot model by Brander and Spencer (1985) to the Stackelberg competition and the sequential-move game on the subsidy choice by governments. Although the argument is limited to the linear demand and linear cost model with any loss of generality, it is possible to make a comparative statics with regard to subsidy, output, profit, and welfare levels, in order to clarify the impact of the timing of decision-making by exporting firms and their governments.

Brander and Spencer (1985) presented a well-known result in Proposition 3 (p.89) in their paper:

Proposition 3. The optimal export subsidy, s , moves the industry equilibrium to what would, in the absence of a subsidy, be the Stackelberg leader-follower position in output space with the domestic firm as leader.

Many articles have quoted this proposition. For a recent example, Maggi (1999, p.575) stated as follows: The optimal unilateral subsidy is the one that shifts the domestic firm's reaction function in such a way that it intersects the foreign reaction function $R(q_i)$ at the Stackelberg point. However, there was little contribution that the original quantity competition be in the way of the Stackelberg competition in the context of the strategic trade policy. In this paper, the optimal subsidy policy is reexamined under the Stackelberg leader-follower competition.

The main objective is to investigate the effects of the sequential-move between two exporting firms under the Stackelberg model and also the effects of the sequential decision-making between their subsidizing governments on the sizes of subsidy, firm's profit, and national welfare. We pay attention not only to the simultaneous decision on subsidy by governments, which has usually been analyzed by the existing literature, but also to the sequential decision.

For the sequential-move game on strategic trade policy, there are several articles that we should refer to. In the two-country model, Syropoulos (1994) showed that the governments may choose tariffs sequentially under perfect competition. Collie (1994) showed that the domestic government sets tariff at first and then the foreign government sets export subsidy under Cournot quantity competition. In the third-country model, Arvan (1991) concluded that demand uncertainty may cause the sequential-move of the policy choice by governments. Shivakumar (1993) introduced the export quota and showed that the restricted quantity competition and demand uncertainty cause the sequential decision of trade policy by governments. Although the existing literature analyzed the endogenous timing of policy-making by governments, in this paper in which the timing of policy-making is exogenous, we focus on examining the effects of the different timing on decision-making on the effectiveness of trade policy.

Recently, Ohkawa, Okamura, and Tawada (2002) endogenized the timing of government intervention under international oligopoly. Their paper is closely related with our paper in the sense that the sequential-move game by governments is analyzed in the third-country model. Different from our concern, however, they focused on the relationship between the number of firms and the endogenous timing of the policy decision by governments and not dealing with Stackelberg competition between firms. In our work, we introduce the sequential-move game by

firms, that is, the Stackelberg quantity competition.¹

In other related papers, Neary and Leahy (2000) examined optimal trade and industrial policy in dynamic oligopolistic markets. Applying a generalized model, they analyzed the strategic interaction between firms and between governments in a 2-stage game. However, they focused only on the situation in which the economic agents act simultaneously in each stage. Likewise, Balboa, Daughety, and Reinganum (2004) dealt with a 2-stage game which includes both Cournot and Stackelberg competitions at the 2nd stage. Although they obtained independently some of the results that are shown in our paper, they did not deal with the sequential decision-making by the intervening governments.

By comparing the simultaneous and the sequential moves made by firms and governments, we obtain some interesting results. Two main results are as follows: First, under the Stackelberg quantity competition, when the governments decide the export subsidies simultaneously and in advance, the original Stackelberg leader firm produces as if it was the follower. Different from the Cournot model, under the Stackelberg model, the subsidy policy by the government, which can subsidize the leader firm, is almost nullified. Second, under the sequential-move game in which the government, which can subsidize the leader firm, decides the subsidy level in advance, the leader's profit is less than the follower's profit, although the first-mover advantage of the government is maintained and the leader produces more than the follower. The paper presents one of the theoretical foundations on the importance of that the timing of policy decision to the effectiveness of trade policy.

The remainder of the article is organized as follows. Section 2 describes the model. Section

¹In the context of industrial organization, there are many articles that argue the endogenous timing under duopolistic competition. For a representative paper, for example, see Hamilton and Slutsky (1990).

3 derives subsidy, output, profit, and domestic welfare in the equilibrium and analyzes the relationship between the different structures. In Section 4, the calculation results about the variables are summarized under each case and the comparative statics are made with regard to the different structures on timing. Some results on the different timing of decision-making are also presented. Concluding remarks are presented in Section 5.

2 The model

Two identical firms, one from country i and one from country j , produce homogeneous goods and sell in a third country. We consider the imperfect quantity competition model in the third country *à la* Brander and Spencer (1985). It is assumed that since both firms produce only for the third market, there is no consumption effects for the exporting countries.² The firm in country i (j) are denoted by the index i (respectively j). Because the firms are identical, they can be interpreted as interchangeable.

Firm i (firm j) produces quantity q_i (resp. q_j). The total quantity is $Q \equiv q_i + q_j$. The argument is limited to the linear demand and linear cost for simplification of analysis. The inverse demand function is denoted by $P(Q) \equiv a - bQ$ and the constant marginal cost by c_i . It is assumed that $a > c_i$ and $b > 0$.

Government i , that lies in country i , can implement the per unit export subsidy, $s_i \geq 0$, as a means of trade policy. It is defined that $e_i \equiv c_i - s_i$.³

The profit that firm i maximizes is denoted by $\pi^i(q_i, q_j; s_i, s_j) \equiv (P(Q) - c_i + s_i)q_i =$

²This kind of assumption is usual in the context of the strategic trade policy as it simplifies the analysis.

³It is shown that the sign of e_i is indeterminate in the following analysis. The compensation from the government may be higher than the marginal cost.

$(P(Q) - e_i)q_i$. The solution concept is the subgame perfect equilibrium.

The welfare of country i is denoted by $G^i(s_i, s_j)$, which consists of the profit from the exporting firm i minus the cost of the export subsidy: $G^i(s_i, s_j) \equiv \pi^i(q_i, q_j; s_i, s_j) - s_i q_i$. Government i maximizes this welfare.

The timing of the game is as follows:

1st stage: Governments choose subsidy levels simultaneously or sequentially.

2nd stage: Firms choose output levels simultaneously or sequentially.

Subsidy policies can be committed by both governments and can be observed by both firms in advance of the competition stage.

In the next section, the subsidy, the output, the profit, and the welfare in the equilibrium are derived by backward induction. Both the Cournot and the Stackelberg leader-follower duopolistic competition are analyzed.

3 Analysis

In this section, the subsidy, the output, the profit, and the domestic welfare in the equilibrium are derived for all classified cases. In the first step, the subgame at the second stage is solved. In the beginning, the simultaneous output choice, that is, Cournot quantity competition at this subgame is examined. Then we proceed to examine the sequential output choice, that is, the Stackelberg duopoly.

3.1 Subgame at the second stage

3.1.1 Cournot competition

Given the subsidies (s_i, s_j) , both firms maximize their profits. The first-order condition for firm i to maximize its profit is as follows: $\pi_i^i = (a - b(q_i + q_j) - e_i) - bq_i = 0$.⁴ The reaction function of firm i is $q_i = R_i(q_j) = \frac{a - bq_j - e_i}{2b}$. In order to obtain the output levels under the Cournot duopolistic competition, the intersection of the reaction functions is solved as follows:

$$(q_i^C(s_i, s_j), q_j^C(s_i, s_j)) = \left(\frac{a - 2e_i + e_j}{3b}, \frac{a - 2e_j + e_i}{3b} \right). \quad (1)$$

If there is no subsidy, the Cournot outcome is as follows: $(q_i^C(0, 0), q_j^C(0, 0)) = \left(\frac{a - 2c_i + c_j}{3b}, \frac{a - 2c_j + c_i}{3b} \right)$.

The total quantity is $Q^C = \frac{2a - e_i - e_j}{3b}$, the price is $P(Q^C) = \frac{a + e_i + e_j}{3}$, and the profit margin is $P(Q^C) - e_i = \frac{a - 2e_i + e_j}{3} = bq_i^C$. The profit levels under the Cournot competition are calculated as follows:

$$(\pi^{Ci}(s_i, s_j), \pi^{Cj}(s_i, s_j)) = (b(q_i^C)^2, b(q_j^C)^2) = \left(\frac{(a - 2e_i + e_j)^2}{9b}, \frac{(a - 2e_j + e_i)^2}{9b} \right). \quad (2)$$

3.1.2 Stackelberg competition

In the sequential-move game, under the Stackelberg competition, suppose that firm i is the Stackelberg leader and firm j is the follower without loss of generality. Firm i , anticipating the reaction of firm j to its own output choice q_i , that is, $q_j = R_j(q_i)$, maximizes the profit function $\pi^i(q_i, q_j)$. That is, the following maximization problem is solved: $\max_{q_i} \pi^i(q_i, R_j(q_i))$. Note that

$$R'_j(q_i) = -\frac{1}{2}.$$

⁴The subscript i of the profit denotes the partial derivative by q_i , that is, $\pi_i^i \equiv \frac{\partial \pi^i}{\partial q_i}$. The second-order condition is satisfied because $\pi_{ii}^i = -2b < 0$.

⁵For the output to be positive, it must be assumed that $a - 2e_i + e_j > 0$. When there is no subsidy, it is assumed that $a - 2c_i + c_j > 0$.

The f.o.c. is $\pi_i^i + \pi_j^i R_j'(q_i) = ((a - b(q_i + R_j(q_i)) - e_i) - bq_i) - bq_i(-\frac{1}{2}) = 0$.⁶ The Stackelberg output pairs are derived as follows:

$$(q_i^S(s_i, s_j), q_j^S(s_i, s_j)) = \left(\frac{a - 2e_i + e_j}{2b}, \frac{a - 3e_j + 2e_i}{4b} \right). \quad (3)$$

If there is no subsidy, the Stackelberg outcome is as follows: $(q_i^S(0, 0), q_j^S(0, 0)) = \left(\frac{a - 2c_i + c_j}{2b}, \frac{a - 3c_j + 2c_i}{4b} \right)$.

As a well-known fact in the oligopoly theory, if the subsidy pairs (s_i, s_j) are identical, $q_i^C(s_i, s_j) < q_i^S(s_i, s_j)$ and $q_j^C(s_i, s_j) > q_j^S(s_i, s_j)$ are satisfied.⁸

The total quantity is $Q^S = \frac{3a - 2e_i - e_j}{4b}$ and the price is $P(Q^S) = \frac{a + 2e_i + e_j}{4}$. $Q^S > Q^C$ and $P(Q^C) > P(Q^S)$ are satisfied. The profit margin is $P(Q^S) - e_i = \frac{a - 2e_i + e_j}{4} = \frac{b}{2}q_i^S$ and $P(Q^S) - e_j = \frac{a - 3e_j + 2e_i}{4} = bq_j^S$.

The profit levels under the Stackelberg competition are calculated as follows:

$$(\pi^{Si}(s_i, s_j), \pi^{Sj}(s_i, s_j)) = \left(\frac{b}{2}(q_i^S)^2, b(q_j^S)^2 \right) = \left(\frac{(a - 2e_i + e_j)^2}{8b}, \frac{(a - 3e_j + 2e_i)^2}{16b} \right). \quad (4)$$

It is satisfied that $\pi^{Ci}(s_i, s_j) < \pi^{Si}(s_i, s_j)$ and $\pi^{Cj}(s_i, s_j) > \pi^{Sj}(s_i, s_j)$, $\forall (s_i, s_j)$.

For the following analysis, the results of the comparative statics are presented as follows:

$$\frac{\partial q_i^C(s_i, s_j)}{\partial s_i} = \frac{2}{3b} > 0, \frac{\partial q_i^C(s_i, s_j)}{\partial s_j} = -\frac{1}{3b} < 0, \frac{\partial q_i^S(s_i, s_j)}{\partial s_i} = \frac{1}{b} > 0, \frac{\partial q_i^S(s_i, s_j)}{\partial s_j} = -\frac{1}{2b} < 0, \frac{\partial q_j^S(s_i, s_j)}{\partial s_j} = \frac{3}{4b} > 0, \text{ and } \frac{\partial q_j^S(s_i, s_j)}{\partial s_i} = -\frac{1}{2b} < 0.$$

⁶The s.o.c. is satisfied because $\pi_i^i + \pi_{ij}^i R_j'(q_i) + (\pi_j^i + \pi_{jj}^i R_j'(q_i))R_j'(q_i) + \pi_j^j R_j''(q_i) = -2b + b/2 + b/2 = -b < 0$.

⁷For the output to be positive, it must be assumed that $a - 3e_j + 2e_i > 0$. In the case of no subsidy, it is assumed that $a - 3c_j + 2c_i > 0$.

⁸And also it is well-known that $q_i^S - q_i^C = \frac{a - 2e_i + e_j}{6b} > q_j^C - q_j^S = \frac{a - 2e_i + e_j}{12b}$, $\forall (s_i, s_j)$. That is, the total quantity expands under the Stackelberg competition.

3.2 Subsidy decision at the first stage

At the first stage, government i maximizes the welfare in country i as follows: $\max_{s_i \geq 0} G^i(s_i, s_j) \equiv \pi^i(q_i, q_j; s_i, s_j) - s_i q_i$. The f.o.c. for government i to maximize its welfare is as follows:

$$\frac{\partial G^i(s_i, s_j)}{\partial s_i} = \frac{\partial \pi^i(q_i, q_j; s_i, s_j)}{\partial s_i} - q_i - s_i \frac{\partial q_i}{\partial s_i} = 0, \quad (5)$$

if $s_i \geq 0$ (interior solution). If $\frac{\partial G^i(s_i, s_j)}{\partial s_i} < 0$, the solution is $s_i = 0$ (corner solution).^{9 10}

Finally, the different timing of decision-making among firms and governments are classified into five cases. In Case A and Case B, the unilateral and the bilateral intervention by government(s) under the Cournot competition are examined. In Case C and Case D, the unilateral and the bilateral intervention by government(s) under the Stackelberg competition are examined. In Case E, the situation in which all players sequentially decide is analyzed. In the following subsection, we investigate all cases in sequence. See Figure 1. The superscripts, C and S stand for Cournot and Stackelberg equilibrium respectively for notational convenience.

< Figure 1 around here >

⁹This is derived from the Kuhn-Tucker slackness condition.

¹⁰Under the following analysis, the s.o.c. is satisfied and the solution is interior, unique, and stable. These are confirmed by tedious calculation since the demand and cost are linear.

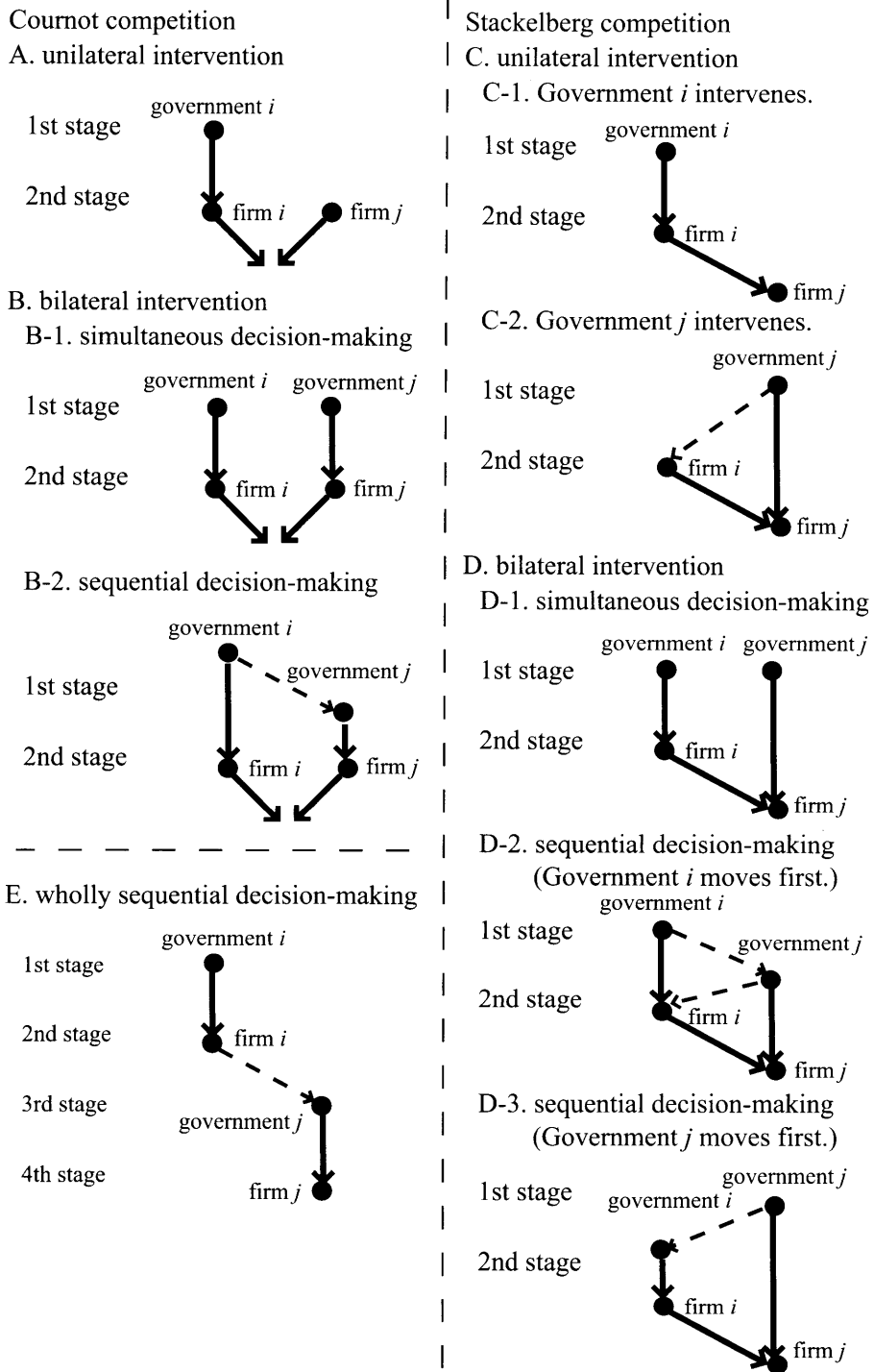


Figure 1: Nodes of decision-making

A. Unilateral intervention under Cournot competition

First, we examine the unilateral intervention case in which only government i subsidizes under the Cournot competition.

As $s_j = 0$, that is, $e_j = c_j$, government i maximizes the following objective: $\max_{s_i \geq 0} G^{Ci}(s_i, 0) = \pi^i(q_i^C(s_i, 0), q_j^C(s_i, 0); s_i, 0) - s_i q_i^C(s_i, 0)$. The f.o.c. for government i is as follows: $\frac{\partial G^{Ci}(s_i, 0)}{\partial s_i} = \pi_i^i \frac{\partial q_i^C}{\partial s_i} + \pi_j^i \frac{\partial q_j^C}{\partial s_i} + \frac{\partial \pi^i}{\partial s_i} - q_i^C - s_i \frac{\partial q_i^C}{\partial s_i} = 0$.¹¹ It is calculated that $\pi_j^i = -bq_i$ and $\frac{\partial \pi^i}{\partial s_i} = q_i$, and $\pi_i^i = 0$ is satisfied by the f.o.c. of the Cournot equilibrium. By substituting them into the f.o.c., it is obtained that $(-bq_i)(-\frac{1}{3b}) + q_i - q_i - s_i \frac{2}{3b} = 0$, that is, $s_i = \frac{b}{2} q_i^C$. Arranging this equation, the optimal subsidy level, s_i^{uC} is derived as follows:

$$s_i^{uC} = \frac{a - 2c_i + c_j}{4}. \quad (6)$$

In this case, the Cournot output is calculated as follows:

$$(q_i^C(s_i^{uC}, 0), q_j^C(s_i^{uC}, 0)) = \left(\frac{a - 2c_i + c_j}{2b}, \frac{a - 3c_j + 2c_i}{4b} \right) [= (q_i^S(0, 0), q_j^S(0, 0))]. \quad (7)$$

This result is summarized in the following proposition.

Proposition 1. *Under the Cournot competition, the unilateral intervention by government i changes the market structure from the Cournot duopoly to the Stackelberg one in which firm i is the leader.*

This proposition is just a corollary of Proposition 3 in Brander and Spencer (1985, p.89). The optimal subsidy has the profit-shifting effect and moves the Cournot competition to the Stackelberg leader-follower position. This result is well-known in the context of strategic subsidy policy.¹² As a result of the unilateral subsidy, the profit of firm i , which is subsidized by the

¹¹The s.o.c. is satisfied because $\frac{\partial^2 G^{Ci}(s_i, s_j)}{\partial s_i^2} = \pi_{ii}^i \frac{\partial q_i^C}{\partial s_i} + \pi_{ji}^i \frac{\partial q_j^C}{\partial s_i} - \frac{\partial q_i^C}{\partial s_i} = (-2b)(\frac{2}{3b}) + (-b)(-\frac{1}{3b}) - \frac{2}{3b} = -(1 + \frac{2}{3b}) < 0$.

¹²It is shown that $q_i^C(s_i^{uC}, 0) > q_i^C(0, 0)$ and $q_j^C(s_i^{uC}, 0) < q_j^C(0, 0)$.

government, rises. That is, $\pi^{Ci}(s_i^{uC}, 0)(= \pi^{Si}(0, 0)) > \pi^{Ci}(0, 0)$ and $\pi^{Cj}(s_i^{uC}, 0)(= \pi^{Sj}(0, 0)) < \pi^{Cj}(0, 0)$. Also, it is evident that the subsidy of government i expands the welfare in country i (j) (resp. contracts), that is, $G^{Ci}(s_i^{uC}, 0)(= \max_{s_i} G^{Ci}(s_i, 0)) > G^{Ci}(0, 0)$ and $G^{Cj}(s_i^{uC}, 0)(= \pi^{Cj}(s_i^{uC}, 0)) < G^{Cj}(0, 0)(= \pi^{Cj}(0, 0))$.¹³

B. Bilateral intervention under Cournot competition

Next, we analyze the bilateral intervention case in which both governments subsidize under the Cournot competition. The simultaneous and the sequential decision of subsidy are examined in sequence.

B-1. Simultaneous decision of subsidy

Consider that the simultaneous decision of subsidies (s_i, s_j) by both governments has the similar timing of decision as the Cournot quantity competition. Given s_j , government i maximizes the welfare with regard to its own subsidy as follows: $\max_{s_i \geq 0} G^{Ci}(s_i, s_j)$

$$= \pi^i(q_i^C(s_i, s_j), q_j^C(s_i, s_j); s_i, s_j) - s_i q_i^C.$$

The f.o.c. is as follows:

$$\frac{\partial G^{Ci}(s_i, s_j)}{\partial s_i} = \pi_i^i \frac{\partial q_i^C}{\partial s_i} + \pi_j^i \frac{\partial q_j^C}{\partial s_i} + \frac{\partial \pi^i}{\partial s_i} - q_i - s_i \frac{\partial q_i^C}{\partial s_i} = 0. \quad (8)$$

Like Case A, the f.o.c. is arranged as $s_i = \frac{b}{2} q_i^C = \frac{a-2e_i+e_j}{6}$. The reaction function is derived as $s_i = R_i(s_j) = \frac{-s_j+a-2c_i+c_j}{4}$.

In order to examine the simultaneous decision on the subsidy levels, the intersection of the reaction functions of both governments is solved. The subsidy level in the equilibrium is obtained as follows:

$$s_i^{bCC} = \frac{a-3c_i+2c_j}{5}. \quad (9)$$

¹³As $s_i^{uC} = \frac{b}{2} q_i^C$, the welfare of country i is $G^{Ci}(s_i^{uC}, 0) = b(q_i^C)^2 - s_i^{uC} q_i^C = \frac{b}{2}(q_i^C)^2 = \frac{(a-2c_i+c_j)^2}{8b} (> G^{Ci}(0, 0) = b(q_i^C)^2 = \frac{(a-2c_i+c_j)^2}{9b})$.

By substituting the subsidy level, s_i^{bCC} , the Cournot output levels in the equilibrium are obtained under the simultaneous decision of subsidy.

$$(q_i^C(s_i^{bCC}, s_j^{bCC}), q_j^C(s_i^{bCC}, s_j^{bCC})) = \left(\frac{2(a - 3c_i + 2c_j)}{5b}, \frac{2(a - 3c_j + 2c_i)}{5b} \right). \quad (10)$$

First, comparing the subsidy levels under the unilateral and the bilateral cases, the following lemma can be stated.

Lemma 1. (*comparison of the subsidy levels under Case A and Case B-1*)

The subsidy under the unilateral intervention is larger than under the bilateral intervention.

That is, $s_i^{uC} > s_i^{bCC}$.¹⁴

This lemma implies that under the bilateral intervention, there is a strategic interaction between governments about the subsidy setting and, as a result, the impact of the subsidy on the output of the firm is smaller than the impact under the unilateral intervention.

Then we proceed to compare the output levels under the unilateral and bilateral intervention.

Proposition 2. (*comparison of the output levels under Case A and Case B-1*)

Under the Cournot competition,

(a) the output level of firm i (j) under the bilateral intervention is smaller (resp. larger) than under the unilateral intervention by government i . That is,

$$q_i^C(s_i^{uC}, 0)(= q_i^S(0, 0)) > q_i^C(s_i^{bCC}, s_j^{bCC}) \text{ and } q_j^C(s_i^{uC}, 0)(= q_j^S(0, 0)) < q_j^C(s_i^{bCC}, s_j^{bCC}).$$

(b) When the marginal costs of the firms are almost identical, the output level under the bilateral intervention is larger than under nonintervention. That is,

$$\text{if } a - 8c_j + 7c_i > 0,^{15} \text{ then } q_i^C(0, 0) < q_i^C(s_i^{bCC}, s_j^{bCC}).$$

¹⁴ $s_i^{uC} - s_i^{bCC} = \frac{a - 2c_i + c_j}{4} - \frac{a - 3c_i + 2c_j}{5} = \frac{a - 3c_j + 2c_i}{20} > 0$.

¹⁵ If the firms have identical marginal cost, $c \equiv c_i = c_j$, this condition is satisfied, because $a - 8c_j + 7c_i = a - 3c_j + 2c_i + 5(c_i - c_j) > 0$.

Due to strategic substitutes on output, subsidizing by rival government results in the output reduction of the own firm. As the reaction functions of both firms shift outwards by subsidizing bilaterally, as a result, the output competition under the bilateral intervention becomes more severe than without intervention. This is an example of the prisoner's dilemma.

As for the firm's profit, $\pi^{Ci} = b(q_i^C)^2$, the relation about the profit size is obtained from the previous relation about the output size. From Proposition 2, it is obtained that $\pi^{Ci}(s_i^{uC}, 0) > \pi^{Ci}(s_i^{bCC}, s_j^{bCC})$, and $\pi^{Cj}(s_i^{uC}, 0) < \pi^{Cj}(s_i^{bCC}, s_j^{bCC})$. When firms are almost identical, $\pi^{Ci}(0, 0) < \pi^{Ci}(s_i^{bCC}, s_j^{bCC})$. This implies that the subsidy from government i (j) makes the profit of firm i larger (resp. smaller) and the bilateral intervention makes the profits of both firms larger than without intervention.

Finally, the effect of the subsidy on the welfare is examined. By direct calculation, it is obtained that $G^{Ci}(0, 0) = \frac{(a-2c_i+c_j)^2}{9b}$, $G^{Ci}(s_i^{uC}, 0) = \frac{(a-2c_i+c_j)^2}{8b}$, $G^{Cj}(s_i^{uC}, 0) = \pi^{Cj}(s_i^{uC}, 0) = \frac{(a-3c_j+2c_i)^2}{16b}$, and $G^{Ci}(s_i^{bCC}, s_j^{bCC}) = \frac{2(a-3c_i+2c_j)^2}{25b}$.¹⁶ Hence $G^{Ci}(s_i^{uC}, 0) > G^{Ci}(0, 0)$ and $G^{Cj}(s_i^{bCC}, s_j^{bCC}) > G^{Cj}(s_i^{uC}, 0)$.¹⁷ If the costs are almost identical, then $G^{Ci}(0, 0) > G^{Ci}(s_i^{bCC}, s_j^{bCC})$.¹⁸ That is, the bilateral intervention falls into the prisoner's dilemma for both governments. This is just a corollary of Proposition 5 in Brander and Spencer (1985, p.95). This result is restated in the following proposition. See also Table 1.

Proposition 3. (*comparison of the welfare under nonintervention and Case B-1*)

Under the Cournot competition, when firms have almost identical marginal costs, the welfare under the bilateral intervention is smaller than under nonintervention. That is, the bilateral

¹⁶It is shown that $G^{Ci}(s_i^{uC}, 0) > G^{Ci}(s_i^{bCC}, s_j^{bCC})$, if $9a + 13c_j - 22c_i > 0$. This condition is sufficiently satisfied when the costs are identical, $c_i = c_j$, because $G^{Ci}(s_i^{uC}, 0) - G^{Ci}(s_i^{bCC}, s_j^{bCC}) = \frac{(a-2c_i+c_j)^2}{8b} - \frac{2(a-3c_i+2c_j)^2}{25b} = \frac{(a-3c_j+2c_i)(9a+13c_j-22c_i)}{200b} > 0$, if $9a + 13c_j - 22c_i = 9(a - 3c_j + 2c_i) + 40(c_j - c_i) > 0$.

¹⁷ $G^{Cj}(s_i^{bCC}, s_j^{bCC}) - G^{Cj}(s_i^{uC}, 0) = \frac{2(a-3c_j+2c_i)^2}{25b} - \frac{(a-3c_j+2c_i)^2}{16b} = \frac{7}{400} \frac{(a-3c_j+2c_i)^2}{b} > 0$.

¹⁸ $G^{Ci}(s_i^{bCC}, s_j^{bCC}) - G^{Ci}(0, 0) = \frac{450(c_i-c_j)^2 - (7a-11c_j+4c_i)^2}{7 \times 225b} < 0$, if $c_i = c_j$.

intervention falls into the prisoner's dilemma for both governments.

		government j	
		nonintervention	intervention
government i	nonintervention	$G^{Ci}(0, 0), G^{Cj}(0, 0)$	$G^{Ci}(0, s_j^{uC}), G^{Cj}(0, s_j^{uC})$
	intervention	$G^{Ci}(s_i^{uC}, 0), G^{Cj}(s_i^{uC}, 0)$	$G^{Ci}(s_i^{bCC}, s_j^{bCC}), G^{Cj}(s_i^{bCC}, s_j^{bCC})$

If the costs are almost identical, $G^{Ci}(0, s_j^{uC}) < G^{Ci}(s_i^{bCC}, s_j^{bCC}) < G^{Ci}(0, 0) < G^{Ci}(s_i^{uC}, 0)$.

		government j	
		nonintervention	intervention
government i	nonintervention	$\frac{(a-2c_i+c_j)^2}{9b}, \frac{(a-2c_j+c_i)^2}{9b}$	$\frac{(a-3c_i+2c_j)^2}{16b}, \frac{(a-2c_j+c_i)^2}{8b}$
	intervention	$\frac{(a-2c_i+c_j)^2}{8b}, \frac{(a-3c_j+2c_i)^2}{16b}$	$\frac{2(a-3c_i+2c_j)^2}{25b}, \frac{2(a-3c_j+2c_i)^2}{25b}$

Table 1: Welfare under nonintervention, Case A, and Case B-1

B-2. Sequential decision of subsidy

Next, we consider the sequential decision of subsidy (s_i, s_j) as a sequential-move game between both governments. First, government i decides the subsidy level, s_i , and then, after observing s_i , government j decides s_j . That is, government i (j) acts as the Stackelberg leader i (resp. follower j) with regard to the subsidy choice.

Since s_i is given, the follower government j decides the subsidy $s_j = R_j(s_i) = \frac{-s_i + a - 2c_j + c_i}{4}$.

Note that $R'_j(s_i) = -\frac{1}{4} < 0$. This reaction function is induced by the leader government who solves the following maximization problem:

$$\max_{s_i \geq 0} G^{Ci}(s_i, R_j(s_i)) = \pi^i(q_i^C(s_i, R_j(s_i)), q_j^C(s_i, R_j(s_i)); s_i, R_j(s_i)) - s_i q_i^C(s_i, R_j(s_i)).$$

The f.o.c. is as follows:

$$\begin{aligned} \frac{\partial G^{Ci}(s_i, R_j(s_i))}{\partial s_i} &= \frac{\partial \pi^i(q_i^C(s_i, R_j(s_i)), q_j^C(s_i, R_j(s_i)); s_i, R_j(s_i))}{\partial s_i} - q_i(s_i, R_j(s_i)) - s_i \left(\frac{\partial q_i}{\partial s_i} + \frac{\partial q_i}{\partial s_j} R'_j(s_i) \right) \\ &= \pi_i^i \left(\frac{\partial q_i}{\partial s_i} + \frac{\partial q_i}{\partial s_j} R'_j(s_i) \right) + \pi_j^i \left(\frac{\partial q_j}{\partial s_i} + \frac{\partial q_j}{\partial s_j} R'_j(s_i) \right) + \left(\frac{\partial \pi^i}{\partial s_i} + \frac{\partial \pi^j}{\partial s_j} R'_j(s_i) \right) - q_i - s_i \left(\frac{\partial q_i}{\partial s_i} + \frac{\partial q_i}{\partial s_j} R'_j(s_i) \right) = 0. \quad (11) \end{aligned}$$

Like Case A, it is satisfied that $\pi_i^i = 0, \pi_j^i = -bq_i, \frac{\partial \pi^i}{\partial s_i} = q_i$, and $\frac{\partial \pi^i}{\partial s_j} = 0$. Arranging the f.o.c., $\pi_j^i(\frac{\partial q_j}{\partial s_i} + \frac{\partial q_j}{\partial s_j} R_j'(s_i)) - s_i(\frac{\partial q_i}{\partial s_i} + \frac{\partial q_i}{\partial s_j} R_j'(s_i)) = 0$. That is, $s_i = \frac{2b}{3} q_i(s_i, R_j(s_i)) = \frac{2b}{3} \frac{a-2(c_i-s_i)+(c_j-R_j(s_i))}{3b}$ is derived. The subsidy levels are obtained as follows:

$$s_i^{bSC} = \frac{a-3c_i+2c_j}{3}, s_j^{bSC} = R_j(s_i^{bSC}) = \frac{a-4c_j+3c_i}{6}. \quad (12)$$

By substituting (s_i^{bSC}, s_j^{bSC}) into the Cournot output levels, the following equations are derived:

$$(q_i^C(s_i^{bSC}, s_j^{bSC}), q_j^C(s_i^{bSC}, s_j^{bSC})) = (\frac{a-3c_i+2c_j}{2b}, \frac{a-4c_j+3c_i}{3b}).^{19} \quad (13)$$

First, comparing the subsidy levels under the unilateral and the bilateral cases, the following lemma can be stated.

Lemma 2. (*comparison of the subsidy levels under Case A and Case B-2*)

When firms have almost identical marginal costs, the subsidy under the unilateral intervention is smaller than that of the leader government under the bilateral intervention. The subsidy under the unilateral intervention is larger than that of the follower government under the bilateral intervention. That is, if $a-6c_i+5c_j > 0$, then $s_i^{uC} < s_i^{bSC}$ and $s_j^{uC} > s_j^{bSC}$.²⁰

Different from Case B-1, when the sequential decision of subsidy is made by governments under the bilateral intervention, the subsidy of the first-mover government is larger and the subsidy of the follower government is smaller compared to the unilateral case.

Then we proceed to compare the output levels under the unilateral and the bilateral intervention.

¹⁹For the output and the subsidy to be positive, it is assumed that $a-4c_j+3c_i > 0$ throughout the following analysis.

²⁰ $s_i^{uC} - s_i^{bSC} = -\frac{a-6c_i+5c_j}{12} < 0$, if $a-6c_i+5c_j = a-c_i-5(c_i-c_j) > 0$. $s_j^{uC} - s_j^{bSC} = \frac{a-3c_i+2c_j}{12} > 0$.

Proposition 4. (comparison of the output levels under nonintervention, Case A, and B-2)

Under the Cournot competition.

(a) whether the output level of firm i under the bilateral sequential intervention is smaller than that under the unilateral intervention by government i , depends on the relative sizes of the marginal costs of the firms. That is, $q_i^C(s_i^{uC}, 0) \gtrless q_i^C(s_i^{bSC}, s_j^{bSC}) \Leftrightarrow c_i \gtrless c_j$.²¹

When the marginal costs are almost identical, the output level of firm j under the bilateral intervention is smaller than that under the unilateral intervention either by government i or j .

That is, if $a - 7c_j + 6c_i > 0$, then $q_j^C(s_i^{uC}, 0) < q_j^C(s_i^{bSC}, s_j^{bSC})$ or if $a - 6c_i + 5c_j > 0$, then $q_j^C(0, s_j^{uC}) > q_j^C(s_i^{bSC}, s_j^{bSC})$.²²

(b) When the marginal costs are almost identical, the output level of firm i under the bilateral intervention is larger than that under nonintervention. That is, if $a - 5c_i + 4c_j > 0$, then $q_i^C(0, 0) < q_i^C(s_i^{bSC}, s_j^{bSC})$.²³

Whether the output level of firm j under the bilateral intervention is smaller than that under nonintervention, depends on the relative sizes of the marginal costs of the firms. That is, $q_j^C(0, 0) \gtrless q_j^C(s_i^{bSC}, s_j^{bSC}) \Leftrightarrow c_i \lesseqgtr c_j$.²⁴

As a corollary of this proposition, if the costs are identical, that is, $c_i = c_j$, then $q_i^C(s_i^{uC}, 0) = q_i^C(s_i^{bSC}, s_j^{bSC})$ and $q_j^C(0, 0) = q_j^C(s_i^{bSC}, s_j^{bSC})$. When the costs are identical, the output level of firm i under the bilateral intervention by the leader government i is equal to that under the unilateral intervention by government i . And also the output level of firm j is the same, whether under the bilateral intervention by the follower government j or under nonintervention. The subsidy policy by the government has two effects on output. The first is to shift the reaction

²¹ $q_i^C(s_i^{uC}, 0) - q_i^C(s_i^{bSC}, s_j^{bSC}) = \frac{c_i - c_j}{2b} \gtrless 0 \Leftrightarrow c_i \gtrless c_j$.

²² $q_j^C(s_i^{bSC}, s_j^{bSC}) - q_j^C(s_i^{uC}, 0) = \frac{a - 7c_j + 6c_i}{12b} > 0$, if $a - 7c_j + 6c_i > 0$.

$q_j^C(0, s_j^{uC}) - q_j^C(s_i^{bSC}, s_j^{bSC}) = \frac{a - 6c_i + 5c_j}{6b} > 0$, if $a - 6c_i + 5c_j > 0$.

²³ $q_i^C(s_i^{bSC}, s_j^{bSC}) - q_i^C(0, 0) = \frac{a - 5c_i + 4c_j}{6b} > 0$, if $a - 5c_i + 4c_j > 0$.

²⁴ $q_j^C(0, 0) - q_j^C(s_i^{bSC}, s_j^{bSC}) = -\frac{c_i - c_j}{b} \gtrless 0 \Leftrightarrow c_i \lesseqgtr c_j$.

function outwards giving the home firm an advantage on output competition under strategic substitutes. The second is to adjust to competitive distortion originated from costs differences. If the costs are identical, the second effect does not appear and, through the subsidy, the output is adjusted at the level of the Stackelberg leader.

As for the firm's profit, $\pi^{Ci} = b(q_i^C)^2$, the relation about the profit size is obtained from the previous relation about the output size. From Proposition 4, it is obtained that $\pi^{Ci}(s_i^{uC}, 0) \geq \pi^{Ci}(s_i^{bSC}, s_j^{bSC})$ if $c_i \geq c_j$, $\pi^{Cj}(s_i^{uC}, 0) < \pi^{Cj}(s_i^{bSC}, s_j^{bSC})$, if $a - 7c_j + 6c_i > 0$, and $\pi^{Cj}(0, s_j^{uC}) > \pi^{Cj}(s_i^{bSC}, s_j^{bSC})$, if $a - 6c_i + 5c_j > 0$. Moreover, it is satisfied that if $a - 5c_i + 4c_j > 0$, then $\pi^{Ci}(0, 0) < \pi^{Ci}(s_i^{bSC}, s_j^{bSC})$, and if $c_i \leq c_j$, then $\pi^{Cj}(0, 0) \geq \pi^{Cj}(s_i^{bSC}, s_j^{bSC})$.

When the firm's cost is higher (lower) than that of the rival, the firm that is subsidized by the leader government prefers the unilateral (resp. bilateral) intervention to the bilateral (resp. unilateral) one. When firms are almost identical, the firm that is subsidized by the follower government always prefers the unilateral intervention to the bilateral one. Comparing nonintervention with the bilateral one, the leader government always prefers the bilateral intervention, while the follower government will prefer the bilateral intervention to nonintervention only if the marginal cost is lower.

Finally, the effect of the subsidy on the welfare is examined. By direct calculation, it is obtained that $G^{Ci}(s_i^{bSC}, s_j^{bSC}) = \frac{(a-3c_i+2c_j)^2}{12b}$ and $G^{Cj}(s_i^{bSC}, s_j^{bSC}) = \frac{(a-4c_j+3c_i)^2}{18b}$.²⁵ Hence $G^{Ci}(s_i^{uC}, 0) > G^{Ci}(0, 0)$ and $G^{Ci}(s_i^{bSC}, s_j^{bSC}) = \frac{(a-3c_i+2c_j)^2}{12b} > G^{Ci}(0, s_j^{uC}) = \frac{(a-3c_i+2c_j)^2}{16b}$. If the costs are almost identical, $G^{Cj}(s_i^{bSC}, s_j^{bSC}) < G^{Cj}(s_i^{uC}, 0)$.²⁶ If the costs are identical, then $G^{Ci}(0, 0) > G^{Ci}(s_i^{bSC}, s_j^{bSC})$ and $G^{Cj}(0, 0) > G^{Cj}(s_i^{bSC}, s_j^{bSC})$.²⁷ Different from Case B-1, when

²⁵ Because $G^{Ci}(s_i^{uC}, 0) - G^{Ci}(s_i^{bSC}, s_j^{bSC}) = \frac{(a-c_j)^2 - 6(c_i-c_j)^2}{24b}$, it is shown that $G^{Ci}(s_i^{uC}, 0) > G^{Ci}(s_i^{bSC}, s_j^{bSC})$,

if $a - c_j > \pm\sqrt{6}(c_i - c_j)$. It is satisfied when the costs are almost identical.

²⁶ $G^{Cj}(s_i^{bSC}, s_j^{bSC}) - G^{Cj}(s_i^{uC}, 0) = -\frac{(a-6c_i+5c_j)^2 - 72(c_i-c_j)^2}{144b} < 0$, if $(a - 6c_i + 5c_j)^2 - 72(c_i - c_j)^2 > 0$.

²⁷ $G^{Ci}(s_i^{bSC}, s_j^{bSC}) - G^{Ci}(0, 0) = -\frac{(a-2c_j+c_i)^2 - 12(c_i-c_j)^2}{36b} < 0$, if $(a - 2c_j + c_i)^2 - 12(c_i - c_j)^2 > 0$.

the costs are almost identical, the leader government chooses to intervene and the follower government chooses not to intervene. The first-mover advantage of government i on the subsidy choice can deter the rival follower government from exercising the subsidy. This result is summarized in the following proposition. See also Table 2.

Proposition 5. (*comparison of the welfare under nonintervention and Case B-2*)

Under the Cournot competition, when firms have almost identical marginal costs, the welfare under the bilateral intervention is smaller than that under nonintervention. In the equilibrium, the result is that only the leader government intervenes. In this case, the prisoner's dilemma of the bilateral intervention for both governments is avoided.

		government j	
		nonintervention	intervention
government i	nonintervention	$G^{Ci}(0, 0), G^{Cj}(0, 0)$	$G^{Ci}(0, s_j^{uC}), G^{Cj}(0, s_j^{uC})$
	intervention	$G^{Ci}(s_i^{uC}, 0), G^{Cj}(s_i^{uC}, 0)$	$G^{Ci}(s_i^{bSC}, s_j^{bSC}), G^{Cj}(s_i^{bSC}, s_j^{bSC})$

If the costs are almost identical, $G^{Ci}(0, 0) < G^{Ci}(s_i^{uC}, 0)$, $G^{Ci}(0, s_j^{uC}) < G^{Ci}(s_i^{bSC}, s_j^{bSC})$,

$G^{Cj}(s_i^{bSC}, s_j^{bSC}) < G^{Cj}(s_i^{uC}, 0)$, $G^{Ci}(0, 0) > G^{Ci}(s_i^{bSC}, s_j^{bSC})$, and $G^{Cj}(0, 0) > G^{Cj}(s_i^{bSC}, s_j^{bSC})$.

		government j	
		nonintervention	intervention
government i	nonintervention	$\frac{(a-2c_i+c_j)^2}{9b}, \frac{(a-2c_j+c_i)^2}{9b}$	$\frac{(a-3c_i+2c_j)^2}{16b}, \frac{(a-2c_j+c_i)^2}{8b}$
	intervention	$\frac{(a-2c_i+c_j)^2}{8b}, \frac{(a-3c_j+2c_i)^2}{16b}$	$\frac{(a-3c_i+2c_j)^2}{12b}, \frac{(a-4c_j+3c_i)^2}{18b}$

Table 2: Welfare under nonintervention, Case A, and Case B-2

C. Unilateral intervention under Stackelberg model

Now, we proceed to examine the unilateral intervention under the Stackelberg model.

$$G^{Cj}(s_i^{bSC}, s_j^{bSC}) - G^{Cj}(0, 0) = -\frac{(a-c_i)^2 - 8(c_i-c_j)^2}{18b} < 0, \text{ if } (a-c_i)^2 - 8(c_i-c_j)^2 > 0.$$

C-1. Unilateral intervention by government i

We examine the case in which government i , whose firm i is the Stackelberg leader, intervenes. As $s_j = 0$, that is, $e_j = c_j$, government i maximizes the following objectives: $\max_{s_i \geq 0} G^{Si}(s_i, 0) = \pi^i(q_i^S(s_i, 0), q_j^S(s_i, 0); s_i, 0) - s_i q_i^S$. The f.o.c. is as follows: $\frac{\partial G^{Si}(s_i, 0)}{\partial s_i} = \frac{d\pi^i(q_i^S(s_i, 0), q_j^S(s_i, 0); s_i, 0)}{ds_i} - q_i - s_i \frac{\partial q_i}{\partial s_i} = \pi_i^i \frac{\partial q_i^S}{\partial s_i} + \pi_j^i \frac{\partial q_j^S}{\partial s_i} + \frac{\partial \pi^i}{\partial s_i} - q_i - s_i \frac{\partial q_i^S}{\partial s_i} \leq 0$.²⁸ By the f.o.c. of the Stackelberg leader, it is satisfied that $\pi_i^i = -\pi_j^i R_j'(q_i) = -\frac{b}{2} q_i$. By substituting $\pi_j^i = -b q_i$ and $\frac{\partial \pi^i}{\partial s_i} = q_i$ into the f.o.c., $(-\frac{b}{2} q_i)(\frac{1}{b}) + (-b q_i)(-\frac{1}{2b}) + q_i - q_i - s_i \frac{1}{b} = -s_i \frac{1}{b} \leq 0$. That is, the subsidy level is zero, $s_i^{uS} = 0$.

In this case, the Stackelberg output levels in the equilibrium are as follows:

$$(q_i^S(s_i^{uS}, 0), q_j^S(s_i^{uS}, 0)) = (q_i^S(0, 0), q_j^S(0, 0)) = \left(\frac{a - 2c_i + c_j}{2b}, \frac{a - 3c_j + 2c_i}{4b} \right). \quad (14)$$

The optimal subsidy moves to the Stackelberg leader-follower position. This result is summarized in the following proposition.

Proposition 6. *Under the Stackelberg competition in which firm i is the leader, the unilateral intervention by the government of the leader firm i has no effect. That is, there is no subsidy.*

This proposition is just another corollary of Proposition 3 in Brander and Spencer (1985, p.89). The optimal subsidy has the profit-shifting effect and moves the Cournot competition to the Stackelberg leader-follower position. Under the Stackelberg competition, the government of the leader firm i has nothing to do. Without subsidy, the profit of firm i is $\pi^{Si}(s_i^{uS}, 0) = \pi^{Si}(0, 0) = \frac{b}{2}(q_i^S)^2$ and $\pi^{Sj}(s_i^{uS}, 0) = \pi^{Sj}(0, 0) = b(q_j^S)^2$. The welfare is $G^{Si}(s_i^{uS}, 0) = G^{Si}(0, 0) = \pi^{Si}(0, 0)$ and $G^{Sj}(s_i^{uS}, 0) = \pi^{Sj}(0, 0)$.

C-2. Unilateral intervention by government j

²⁸The s.o.c. is satisfied because $\frac{\partial^2 G^{Si}(s_i, s_j)}{\partial s_i^2} = \pi_{ii}^i \frac{\partial q_i^S}{\partial s_i} + \pi_{ji}^i \frac{\partial q_j^S}{\partial s_i} - \frac{\partial q_i^S}{\partial s_i} = (-2b)\frac{1}{b} + (-b)(-\frac{1}{2b}) - \frac{1}{b} = -(\frac{3}{2} + \frac{1}{b}) < 0$.

We examine the case in which government j , whose firm j is the Stackelberg follower, intervenes. As $s_i = 0$, $e_i = c_i$, government j maximizes the following objectives: $\max_{s_j \geq 0} G^{Sj}(0, s_j) = \pi^j(q_j^S(0, s_j), q_i^S(0, s_j); 0, s_j) - s_j q_j^S$. The f.o.c. of government j is as follows: $\frac{\partial G^{Sj}(0, s_j)}{\partial s_j} = \frac{d\pi^j(q_j^S(0, s_j), q_i^S(0, s_j); 0, s_j)}{ds_j} - q_j - s_j \frac{\partial q_j^S}{\partial s_j} = \pi_j^j \frac{\partial q_j^S}{\partial s_j} + \pi_i^j \frac{\partial q_i^S}{\partial s_j} + \frac{\partial \pi^j}{\partial s_j} - q_j - s_j \frac{\partial q_j^S}{\partial s_j} = 0$.²⁹ By the f.o.c. of the Stackelberg follower, $\pi_j^j = 0$ is satisfied. By substituting $\pi_i^j = -bq_j$ and $\frac{\partial \pi^j}{\partial s_j} = q_j$ into the f.o.c., $(-bq_j)(-\frac{1}{2b}) + q_j - q_j - s_j \frac{3}{4b} = 0 \Leftrightarrow s_j = \frac{2b}{3} q_j = \frac{a-3(c_j-s_j)+2c_i}{6}$.

$$s_j^{uS} = \frac{a - 3c_j + 2c_i}{3}. \quad (15)$$

In this case, the Stackelberg output levels in the equilibrium are as follows:

$$(q_i^S(0, s_j^{uS}), q_j^S(0, s_j^{uS})) = \left(\frac{a - 4c_i + 3c_j}{3b}, \frac{a - 3c_j + 2c_i}{2b} \right). \quad (16)$$

This output level is equivalent to that in Case B-2. The following proposition is obtained.

Proposition 7. (*comparison of the output levels under Case B-2 and Case C-2*)

Under the Stackelberg competition in which firm i is the leader, the unilateral intervention by the government of the follower firm j yields the same result on output as the bilateral sequential intervention under Cournot competition in which government j is the first-mover. That is,

$$(q_i^S(0, s_j^{uS}), q_j^S(0, s_j^{uS})) = (q_j^C(s_i^{bSC}, s_j^{bSC}), q_i^C(s_i^{bSC}, s_j^{bSC})).$$

This proposition implies that the subsidy of the government works as if it changes the competition mode from Stackelberg to Cournot. The optimal subsidy improves the Stackelberg-follower position compared to the Cournot one. Even if firm j is the Stackelberg follower under quantity competition, the optimal subsidy by government j makes the disadvantage of the follower reduce until it disappears.

By substituting (16) into $\pi^{Si}(s_i, s_j) = \frac{b}{2}(q_i^S)^2$ and $\pi^{Sj}(s_i, s_j) = b(q_j^S)^2$, the profits of the

²⁹The s.o.c. is satisfied because $\frac{\partial^2 G^{Sj}(s_i, s_j)}{\partial s_j^2} = \pi_{jj}^j \frac{\partial q_j^S}{\partial s_j} + \pi_{ij}^j \frac{\partial q_i^S}{\partial s_j} - \frac{\partial q_j^S}{\partial s_j} = (-2b)\frac{3}{4b} + (-b)(-\frac{1}{2b}) - \frac{3}{4b} = -(1 + \frac{3}{4b}) < 0$.

firms are obtained: $\pi^{Si}(0, s_j^{uS}) = \frac{(a-4c_i+3c_j)^2}{18b}$ and $\pi^{Sj}(0, s_j^{uS}) = \frac{(a-3c_j+2c_i)^2}{4b}$. It is shown that $\pi^{Si}(s_i^{uS}, 0) = \frac{(a-2c_i+c_j)^2}{8b} > \pi^{Si}(0, s_j^{uS})$ and $\pi^{Sj}(s_i^{uS}, 0) = \frac{(a-3c_j+2c_i)^2}{16b} < \pi^{Sj}(0, s_j^{uS})$.³⁰

Finally, the welfare is examined. It is obtained that $G^{Si}(0, s_j^{uS}) = \pi^{Si}(0, s_j^{uS}) = \frac{(a-4c_i+3c_j)^2}{18b} < G^{Si}(s_i^{uS}, 0) = \pi^{Si}(s_i^{uS}, 0) = \frac{(a-2c_i+c_j)^2}{8b}$. Also it is obtained that $G^{Sj}(0, s_j^{uS}) = b(q_j^S)^2 - s_j^{uS}q_j^S = \frac{(a-3c_j+2c_i)^2}{12b} > G^{Sj}(s_i^{uS}, 0) = \pi^{Sj}(s_i^{uS}, 0) = \frac{(a-3c_j+2c_i)^2}{16b}$.

For simplification of analysis, it is assumed that $c \equiv c_i = c_j$ throughout the following analysis.

Under this identical assumption, the following proposition is obtained:

Proposition 8. (*comparison of the profit and the welfare levels under Case C-1 and Case C-2*)
Consider the Stackelberg competition in which firm i is the leader. Suppose that the costs are identical. In the case in which the government of the follower firm j intervenes unilaterally, the profit of the follower firm j (the leader firm i) is larger (resp. smaller) than the profit of the leader firm i (resp. the follower firm j) in the case in which the government of the leader firm i intervenes unilaterally. That is,

$$\begin{aligned} \pi^{Si}(s_i^{uS}, 0) &= \frac{(a-c)^2}{8b}, & \pi^{Si}(0, s_j^{uS}) &= \frac{(a-c)^2}{18b}, \\ \pi^{Sj}(s_i^{uS}, 0) &= \frac{(a-c)^2}{16b}, & \pi^{Sj}(0, s_j^{uS}) &= \frac{(a-c)^2}{4b}. \end{aligned}$$

When government j intervenes unilaterally, the welfare of government j (i) is smaller than that of government i (resp. j) when government i intervenes unilaterally. That is,

$$\begin{aligned} G^{Si}(s_i^{uS}, 0) &= \frac{(a-c)^2}{8b}, & G^{Si}(0, s_j^{uS}) &= \frac{(a-c)^2}{18b}, \\ G^{Sj}(s_i^{uS}, 0) &= \frac{(a-c)^2}{16b}, & G^{Sj}(0, s_j^{uS}) &= \frac{(a-c)^2}{12b}. \end{aligned}$$

Note that this proposition also holds when firms have almost identical cost. When the costs are almost identical, although it looks at first glance that the subsidized leader firm may enjoy higher profit than that of the subsidized follower, the previous proposition shows that such

³⁰Because $\pi^{Si}(0, s_j^{uS}) = (G^{Si}(0, s_j^{uS}) = \frac{b}{2}(q_i^S(0, s_j^{uS}))^2$ and $\pi^{Si}(s_i^{uS}, 0) = (G^{Si}(s_i^{uS}, 0) = \frac{b}{2}(q_i^S(s_i^{uS}, 0))^2$, it is satisfied that $\pi^{Si}(0, s_j^{uS}) < \pi^{Si}(s_i^{uS}, 0)$, if and only if $q_i^S(0, s_j^{uS}) < q_i^S(s_i^{uS}, 0)$. It is shown that $q_i^S(s_i^{uS}, 0) - q_i^S(0, s_j^{uS}) = \frac{a-3c_j+2c_i}{6b} > 0$.

a preconceived idea is incorrect. This implication is derived from the fact that government i does not subsidize at all, because the advantage of the Stackelberg leader had already been acquired by firm i in Case C-1. On the other hand, this intuition is correct when the welfare is considered. Even if the government makes the follower firm recover the first-mover advantage through intervention, it takes an extra cost to subsidize this firm.

D. Bilateral intervention under Stackelberg model

We analyze the bilateral intervention under Stackelberg model.

D-1. Simultaneous decision of subsidy

We consider the simultaneous decision of subsidy (s_i, s_j) , similar to the simultaneous quantity choice in the Cournot model. Government i , whose firm i is leader, maximizes the following objective: given s_j , $\max_{s_i \geq 0} G^{Si}(s_i, s_j) = \pi^i(q_i^S(s_i, s_j), q_j^S(s_i, s_j); s_i, s_j) - s_i q_i^S$. The f.o.c. is as follows: $\frac{\partial G^{Si}(s_i, s_j)}{\partial s_i} = \frac{d\pi^i(q_i^S(s_i, s_j), q_j^S(s_i, s_j); s_i, s_j)}{ds_i} - q_i - s_i \frac{\partial q_i}{\partial s_i} = \pi_i^i \frac{\partial q_i^S}{\partial s_i} + \pi_j^i \frac{\partial q_j^S}{\partial s_i} + \frac{\partial \pi^i}{\partial s_i} - q_i - s_i \frac{\partial q_i^S}{\partial s_i} = 0$. By the f.o.c. of the Stackelberg leader, it is satisfied that $\pi_i^i = -\pi_j^i R_j'(q_i) = -\frac{b}{2} q_i$. By substituting $\pi_j^i = -bq_i$ and $\frac{\partial \pi^i}{\partial s_i} = q_i$ into the f.o.c., $(-\frac{b}{2} q_i) \frac{1}{b} + (-bq_i)(-\frac{1}{2b}) + q_i - q_i - s_i \frac{1}{b} = -s_i \frac{1}{b} \leq 0 \Rightarrow s_i^{bCS} = 0$. The reaction function is $s_i^{bCS} = R_i(s_j^{bCS}) = 0$.

Government j , whose firm j is the follower, maximizes the following objective: given s_i , $\max_{s_j \geq 0} G^{Sj}(s_i, s_j) = \pi^j(q_j^S(s_i, s_j), q_i^S(s_i, s_j); s_i, s_j) - s_j q_j^S$. The f.o.c. is as follows: $\frac{\partial G^{Sj}(s_i, s_j)}{\partial s_j} = \frac{d\pi^j(q_j^S(s_i, s_j), q_i^S(s_i, s_j); s_i, s_j)}{ds_j} - q_j - s_j \frac{\partial q_j}{\partial s_j} = \pi_j^j \frac{\partial q_j^S}{\partial s_j} + \pi_i^j \frac{\partial q_i^S}{\partial s_j} + \frac{\partial \pi^j}{\partial s_j} - q_j - s_j \frac{\partial q_j^S}{\partial s_j} = 0$. From the f.o.c. of the Stackelberg follower, it is satisfied that $\pi_j^j = 0$. By substituting $\pi_i^j = -bq_j$ and $\frac{\partial \pi^j}{\partial s_j} = q_j$ into the f.o.c., $(-bq_j)(-\frac{1}{2b}) + q_j - q_j - s_j \frac{3}{4b} = 0 \Leftrightarrow s_j = \frac{2b}{3} q_j = \frac{a-3(c_j-s_j)+2(c_i-s_i)}{6}$. The reaction function is $s_j^{bCS} = R_j(s_i^{bCS}) = \frac{-2s_i + a - 3c_j + 2c_i}{3}$.

In order to work out the simultaneous decision of the subsidy levels by both governments,

the intersection of the reaction functions is reduced as follows:

$$s_i^{bCS} = 0, s_j^{bCS} = \frac{a - 3c_j + 2c_i}{3}. \quad (17)$$

Note that $s_i^{bCS} = s_i^{uS} = 0$ and $s_j^{bCS} = s_j^{uS} = \frac{a - 3c_j + 2c_i}{3}$.

By substituting s_i into the outputs, the optimal Stackelberg output levels are obtained as follows:

$$(q_i^S(s_i^{bCS}, s_j^{bCS}), q_j^S(s_i^{bCS}, s_j^{bCS})) = \left(\frac{a - 4c_i + 3c_j}{3b}, \frac{a - 3c_j + 2c_i}{2b} \right). \quad (18)$$

Hence $q_i^S(s_i^{bCS}, s_j^{bCS}) = q_i^S(0, s_j^{uS})$ and $q_j^S(s_i^{bCS}, s_j^{bCS}) = q_j^S(0, s_j^{uS})$.

In this case, as a result of the simultaneous decision of subsidy levels, differently from the Cournot model, under the Stackelberg model, the subsidy policy of the government to the leader firm is nullified. In Case D-1, the Stackelberg leader and the follower have the same behavior as in Case C-2.

Proposition 9. (*equivalence between Case D-1 and Case C-2*)

Consider the Stackelberg competition in which firm i is the leader. The result in the equilibrium is the same under Case D-1 and under Case C-2. The government whose firm is the leader does not subsidize its firm at all.

Also the profit and the welfare are the same as those under Case C-2. That is, the profits are $\pi^{Si}(s_i^{bCS}, s_j^{bCS}) (= \pi^{Si}(0, s_j^{uS})) = \frac{(a - 4c_i + 3c_j)^2}{18b}$ and $\pi^{Sj}(s_i^{bCS}, s_j^{bCS}) (= \pi^{Sj}(0, s_j^{uS})) = \frac{(a - 3c_j + 2c_i)^2}{4b}$.

The welfares are $G^{Si}(s_i^{bCS}, s_j^{bCS}) = \pi^{Si}(s_i^{bCS}, s_j^{bCS}) = \frac{(a - 4c_i + 3c_j)^2}{18b}$ and $G^{Sj}(s_i^{bCS}, s_j^{bCS}) = G^{Sj}(0, s_j^{uS}) = \frac{(a - 3c_j + 2c_i)^2}{12b}$.

D-2. Sequential decision of subsidy ($s_i \rightarrow s_j$)

Then, we examine the sequential decision of subsidy ($s_i \rightarrow s_j$). In this case, the government i of the Stackelberg leader i moves first and then the government j of the follower j , after observing s_i , decides the subsidy level, $s_j = R_j(s_i)$.

Government j maximizes the following objective: given s_i , $\max_{s_j \geq 0} G^{Sj}(s_i, s_j) = \pi^j(q_j^S(s_i, s_j), q_i^S(s_i, s_j); s_i, s_j) - s_j q_j^S$. The f.o.c. is as follows: $\frac{\partial G^{Sj}(s_i, s_j)}{\partial s_j} = \frac{d\pi^j(q_j^S(s_i, s_j), q_i^S(s_i, s_j); s_i, s_j)}{ds_j} - q_j - s_j \frac{\partial q_j^S}{\partial s_j} = \pi_j^j \frac{\partial q_j^S}{\partial s_j} + \pi_i^j \frac{\partial q_i^S}{\partial s_j} + \frac{\partial \pi^j}{\partial s_j} - q_j - s_j \frac{\partial q_j^S}{\partial s_j} = 0$. From the f.o.c. of the Stackelberg follower, it is satisfied that $\pi_j^j = 0$. By substituting $\pi_i^j = -bq_j$ and $\frac{\partial \pi^j}{\partial s_j} = q_j$ into the f.o.c., $(-bq_j)(-\frac{1}{2b}) + q_j - q_j - s_j \frac{3}{4b} = 0 \Leftrightarrow s_j = \frac{2b}{3}q_j = \frac{a-3(c_j-s_j)+2(c_i-s_i)}{6}$. The reaction function is $s_j^{bSiS} = R_j(s_i^{bSiS}) = \frac{-2s_i+a-3c_j+2c_i}{3}$. The slope is $R_j'(s_i^{bSiS}) = -\frac{2}{3}$. This maximization problem is the same procedure taken by the follower government in Case D-1.

The leader government induces this reaction function, R_j , and solves the following maximization problem: $\max_{s_i \geq 0} G^{Si}(s_i, R_j(s_i)) = \pi^i(q_i^S(s_i, R_j(s_i)), q_j^S(s_i, R_j(s_i)); s_i, R_j(s_i)) - s_i q_i^S(s_i, R_j(s_i))$. The f.o.c. is as follows: $\frac{dG^{Si}(s_i, R_j(s_i))}{ds_i} = \frac{d\pi^i(q_i^S(s_i, R_j(s_i)), q_j^S(s_i, R_j(s_i)); s_i, R_j(s_i))}{ds_i} - q_i^S(s_i, R_j(s_i)) - s_i(\frac{\partial q_i^S}{\partial s_i} + \frac{\partial q_i^S}{\partial s_j} R_j'(s_i)) = \pi_i^i(\frac{\partial q_i^S}{\partial s_i} + \frac{\partial q_i^S}{\partial s_j} R_j'(s_i)) + \pi_j^i(\frac{\partial q_j^S}{\partial s_i} + \frac{\partial q_j^S}{\partial s_j} R_j'(s_i)) + (\frac{\partial \pi^i}{\partial s_i} + \frac{\partial \pi^i}{\partial s_j} R_j'(s_i)) - q_i - s_i(\frac{\partial q_i^S}{\partial s_i} + \frac{\partial q_i^S}{\partial s_j} R_j'(s_i)) = 0$. Similar to Case C, it is satisfied that $\pi_i^i = -\pi_j^i R_j'(q_i) = -\frac{b}{2}q_i$. By substituting $\pi_j^i = -bq_i$, $\frac{\partial \pi^i}{\partial s_i} = q_i$, and $\frac{\partial \pi^i}{\partial s_j} = 0$ into the f.o.c., $-\frac{b}{2}q_i(\frac{\partial q_i^S}{\partial s_i} + \frac{\partial q_i^S}{\partial s_j} R_j'(s_i)) - bq_i(\frac{\partial q_j^S}{\partial s_i} + \frac{\partial q_j^S}{\partial s_j} R_j'(s_i)) - s_i(\frac{\partial q_i^S}{\partial s_i} + \frac{\partial q_i^S}{\partial s_j} R_j'(s_i)) = 0 \Leftrightarrow s_i = \frac{b}{4}q_i = \frac{a-2(c_i-s_i)+(c_j-R_j(s_i))}{8}$.

Under the sequential decision, the optimal subsidy levels are as follows:

$$s_i^{bSiS} = \frac{a-4c_i+3c_j}{8}, \quad s_j^{bSiS} = R_j(s_i^{bSiS}) = \frac{a-5c_j+4c_i}{4}. \quad (19)$$

Note that $s_j^{bSiS} > s_i^{bSiS}$ if the costs are almost identical.³¹ In this case, the subsidy to the follower is larger than that to the leader.

By substituting s_i into the output, the optimal Stackelberg output levels are obtained as follows:

$$(q_i^S(s_i^{bSiS}, s_j^{bSiS}), s_j^{bSiS}, q_j^S(s_i^{bSiS}, s_j^{bSiS})) = \left(\frac{a-4c_i+3c_j}{2b}, \frac{3(a-5c_j+4c_i)}{8b} \right). \quad (20)$$

³¹ It is satisfied that $s_j^{bSiS} > s_i^{bSiS}$, if $a-13c_j+12c_i = a-3c_j+2c_i+10(c_i-c_j) > 0$, because $s_j^{bSiS} - s_i^{bSiS} = \frac{a-13c_j+12c_i}{8}$.

Note that $q_i^S(s_i^{bSiS}, s_j^{bSiS}) > q_j^S(s_i^{bSiS}, s_j^{bSiS})$, if the costs are almost identical.³² In this case, the output of the leader is larger than that of the follower. In particular, when costs are identical, $q_i^S(s_i^{bSiS}, s_j^{bSiS}) = \frac{a-c}{2b} > q_j^S(s_i^{bSiS}, s_j^{bSiS}) = \frac{3(a-c)}{8b}$.

From $(\pi^{Si}, \pi^{Sj}) = (\frac{b}{2}(q_i^S)^2, b(q_j^S)^2)$, the profits are calculated as follows: $\pi^{Si}(s_i^{bSiS}, s_j^{bSiS}) = \frac{(a-4c_i+3c_j)^2}{8b}$ and $\pi^{Sj}(s_i^{bSiS}, s_j^{bSiS}) = \frac{9(a-5c_j+4c_i)^2}{64b}$. When the costs are identical, it is worth noting that $\pi^{Si}(s_i^{bSiS}, s_j^{bSiS}) = \frac{(a-c)^2}{8b} < \pi^{Sj}(s_i^{bSiS}, s_j^{bSiS}) = \frac{9(a-c)^2}{64b}$. In other words, the profit of the follower is larger than that of the leader.

From $G^{Si} = \pi^{Si} - s_i q_i$, the welfares are calculated as follows: $G^{Si}(s_i^{bSiS}, s_j^{bSiS}) = \frac{(a-4c_i+3c_j)^2}{16b}$ and $G^{Sj}(s_i^{bSiS}, s_j^{bSiS}) = \frac{3(a-5c_j+4c_i)^2}{64b}$. When the costs are identical, then $G^{Si}(s_i^{bSiS}, s_j^{bSiS}) = \frac{(a-c)^2}{16b} > G^{Sj}(s_i^{bSiS}, s_j^{bSiS}) = \frac{3(a-c)^2}{64b}$. With regard to the welfare, the welfare of the leader government is larger than that of the follower. This result is summarized in the following proposition.

Proposition 10. (*profit and welfare levels under Case D-2*)

Consider the Stackelberg competition in which firm i is the leader. In the symmetric equilibrium under Case D-2, the profit of the leader is less than that of the follower. The welfare of the leader government i is larger than that of the follower government j.

Although the leader produces more than the follower, the government of the leader subsidizes less than that of the follower. As a result, the profit of the leader is less than that of the follower and the welfare of the first-mover government is larger than that of the second-mover government. This result provides a new viewpoint considering the subsidy policy determined sequentially.

Furthermore, by comparing the welfare under nonintervention with that under Case D-2, it is obtained that $G^{Si}(0,0) = \frac{(a-2c_i+c_j)^2}{8b} > G^{Si}(s_i^{bSiS}, s_j^{bSiS}) = \frac{(a-4c_i+3c_j)^2}{16b}$ and $G^{Sj}(0,0) =$

³² $q_i^S(s_i^{bSiS}, s_j^{bSiS}) - q_j^S(s_i^{bSiS}, s_j^{bSiS}) = \frac{a-c_i-27(c_i-c_j)}{8b} > 0$, if $a - 28c_i + 27c_j > 0$.

$\frac{(a-3c_j+2c_i)^2}{16b} > G^{Sj}(s_i^{bSiS}, s_j^{bSiS}) = \frac{3(a-5c_j+4c_i)^2}{64b}$, if the costs are almost identical.³³ Thus, the following proposition is derived.

Proposition 11. (*comparison of the welfare under nonintervention and Case D-2*)

Under the Stackelberg competition in which firm i is the leader, when the costs are almost identical, the welfares of the governments under the bilateral intervention in Case D-2 are smaller than under nonintervention. That is, the bilateral intervention falls into the prisoner's dilemma.

D-3. Sequential decision of subsidy ($s_j \rightarrow s_i$)

Finally, we examine the sequential decision of subsidy ($s_j \rightarrow s_i$). This case is the adverse case of Case D-2 with regard to the timing of decision-making by governments. In this case, the government j of the Stackelberg follower firm j moves first and then the government i of the leader firm, after observing s_j , decides the subsidy $s_i = R_i(s_j)$.

The follower government i maximizes the following objective: given s_j , $\max_{s_i \geq 0} G^{Si}(s_i, s_j) = \pi^i(q_i^S(s_i, s_j), q_j^S(s_i, s_j); s_i, s_j) - s_i q_i^S$. The f.o.c. is as follows: $\frac{\partial G^{Si}(s_i, s_j)}{\partial s_i} = \frac{d\pi^i(q_i^S(s_i, s_j), q_j^S(s_i, s_j); s_i, s_j)}{ds_i} - q_i - s_i \frac{\partial q_i^S}{\partial s_i} = 0$. From the f.o.c. of Stackelberg leader, it is satisfied that $\pi_j^i = -\pi_j^i R_j'(q_i) = -\frac{b}{2} q_i$. By substituting $\pi_j^i = -bq_i$ and $\frac{\partial \pi^i}{\partial s_i} = q_i$ into the f.o.c., $(-\frac{b}{2} q_i) \frac{1}{b} + (-bq_i)(-\frac{1}{2b}) + q_i - q_i - s_i \frac{1}{b} = -\frac{s_i}{b} \leq 0 \Rightarrow s_i = 0$. The reaction function is $s_i^{bSjS} = R_i(s_j^{bSjS}) = 0$. This is the same procedure taken by the leader government i in Case D-1.

The leader government j induces this reaction and solves the following maximization problem substituting $s_i^{bSjS} = R_i(s_j^{bSjS}) = 0$: $\max_{s_j \geq 0} G^{Sj}(0, s_j) = \pi^j(q_j^S(0, s_j), q_i^S(0, s_j); 0, s_j) - s_j q_j^S(0, s_j)$. The f.o.c. is as follows: $\frac{dG^{Sj}(0, s_j)}{ds_j} = \frac{d\pi^j(q_j^S(0, s_j), q_i^S(0, s_j); 0, s_j)}{ds_j} - q_j^S(0, s_j) - s_j \frac{\partial q_j^S}{\partial s_j} = \pi_j^j \frac{\partial q_j^S}{\partial s_j} + \pi_j^j \frac{\partial q_i^S}{\partial s_j} + \frac{\partial \pi^j}{\partial s_j} - q_j - s_j \frac{\partial q_j^S}{\partial s_j} = 0$. Similar to Case C-2, it is satisfied that $\pi_j^j = 0$. By

³³ $G^{Si}(0, 0) - G^{Si}(s_i^{bSiS}, s_j^{bSiS}) = \frac{(a-c_j)^2 - 8(c_i-c_j)^2}{16b}$ and $G^{Sj}(0, 0) - G^{Sj}(s_i^{bSiS}, s_j^{bSiS}) = \frac{(a-4c_i+3c_j)^2 - 48(c_i-c_j)^2}{64b}$.

substituting $\pi_i^j = -bq_j$ and $\frac{\partial \pi_i^j}{\partial s_j} = q_j$ into the f.o.c., $-bq_j \frac{\partial q_i}{\partial s_j} - s_j \frac{\partial q_j}{\partial s_j} = 0 \Leftrightarrow s_j = \frac{2b}{3}q_j = \frac{a-3(c_j-s_j)+2c_i}{6} \Leftrightarrow s_j = \frac{a-3c_j+2c_i}{3}$.

Under the sequential decision, the optimal subsidy levels are as follows:

$$s_i^{bSjS} = 0, s_j^{bSjS} = \frac{a-3c_j+2c_i}{3}. \quad (21)$$

By substituting s_i into the output, the optimal Stackelberg output levels are as follows:

$$(q_i^S(s_i^{bSjS}, s_j^{bSjS}), q_j^S(s_i^{bSjS}, s_j^{bSjS})) = \left(\frac{a-4c_i+3c_j}{3b}, \frac{a-3c_j+2c_i}{2b} \right). \quad (22)$$

The equilibrium in this case is the same as that in Case D-1 (and also Case C-2).

Proposition 12. (equivalence between Case D-3 and Case D-1 (Case C-2))

Consider the Stackelberg competition in which firm i is the leader. The result in the equilibrium under Case D-3 is the same as that under Case D-1 (and Case C-2). The government whose firm is the leader does not subsidize its firm at all.

In this case, the follower government of the country where there is the Stackelberg leader firm has nothing to do, by the same reason as Case D-1 and Case C-2. Also the profit and the welfare are the same as those under Case D-1 and Case C-2. That is, the profits are $\pi^{Si}(s_i^{bSjS}, s_j^{bSjS}) = \frac{(a-4c_i+3c_j)^2}{18b}$ and $\pi^{Sj}(s_i^{bSjS}, s_j^{bSjS}) = \frac{(a-3c_j+2c_i)^2}{4b}$. The welfares are $G^{Si}(s_i^{bSjS}, s_j^{bSjS}) = \frac{(a-4c_i+3c_j)^2}{18b}$ and $G^{Sj}(s_i^{bSjS}, s_j^{bSjS}) = \frac{(a-3c_j+2c_i)^2}{12b}$.

We compare the profit and the welfare in Case D-2 with those in Case D-3. When the costs are identical, the following inequality is satisfied: $\pi^{Si}(s_i^{bSjS}, s_j^{bSjS}) < \pi^{Si}(s_i^{bSiS}, s_j^{bSiS}) < \pi^{Sj}(s_i^{bSiS}, s_j^{bSiS}) < \pi^{Sj}(s_i^{bSjS}, s_j^{bSjS})$.³⁴ And it is also satisfied that $G^{Si}(s_i^{bSjS}, s_j^{bSjS}) < G^{Si}(s_i^{bSiS}, s_j^{bSiS}) < G^{Sj}(s_i^{bSiS}, s_j^{bSiS}) < G^{Sj}(s_i^{bSjS}, s_j^{bSjS})$.³⁵ For the government, the first-mover advantage still exists.

³⁴By direct calculation, it is obtained that $\frac{(a-c)^2}{18b} < \frac{(a-c)^2}{8b} < \frac{9(a-c)^2}{64b} < \frac{(a-c)^2}{4b}$.

³⁵By direct calculation, it is obtained that $\frac{(a-c)^2}{18b} < \frac{3(a-c)^2}{64b} < \frac{(a-c)^2}{16b} < \frac{(a-c)^2}{12b}$.

E. Wholly sequential decision

As the remaining possible combination, we examine the wholly sequential decision ($s_i \rightarrow q_i \rightarrow s_j \rightarrow q_j$). First, we consider the bilateral intervention of sequential decision-making: $s_i \rightarrow q_i \rightarrow s_j \rightarrow q_j$.³⁶ The equilibrium can be solved by backward induction.

In the subgame at the fourth stage, the f.o.c. for the profit maximization of firm j , given (s_i, q_i, s_j) , is as follows: $\pi_j^j = (a - b(q_i + q_j) - e_j) - bq_j = 0$. The reaction function is $q_j = R_j^s(q_i, s_j) = \frac{a - bq_i - e_j}{2b}$; $\frac{\partial R_j^s(q_i, s_j)}{\partial s_j} = \frac{1}{2b}$. Note that this reaction function does not depend on s_i .

In the subgame at the third stage, the subsidy decision by government j is determined by maximizing $G^j(s_i, q_i, s_j, q_j) \equiv \pi_j^j(q_j, q_i; s_i, s_j) - s_j q_j$. Government j maximizes the following objective: given (s_i, q_i) , $\max_{s_j \geq 0} G^j(s_i, q_i, s_j, q_j), s.t. q_j = R_j^s(q_i, s_j) = \frac{a - bq_i - e_j}{2b}$, that is, $\max_{s_j \geq 0} G^j(s_i, q_i, s_j, R_j^s(q_i, s_j))$. The f.o.c. for government j is $\frac{dG^j(s_i, q_i, s_j, R_j^s(q_i, s_j))}{ds_j} = \frac{d\pi_j^j(R_j^s(q_i, s_j), q_i; s_i, s_j)}{ds_j} - q_j - s_j \frac{\partial q_j}{\partial s_j} = 0$, if the solution is interior ($s_j \geq 0$). If $\frac{dG^j(s_i, q_i, s_j, R_j^s(q_i, s_j))}{ds_j} < 0$, the solution is corner, $s_j = 0$. The f.o.c. is rewritten as follows: $\frac{\partial G^j(s_i, q_i, s_j, R_j^s(q_i, s_j))}{\partial s_j} = \frac{d\pi_j^j(R_j^s(q_i, s_j), q_i; s_i, s_j)}{ds_j} - q_j - s_j \frac{\partial q_j}{\partial s_j} = \pi_j^j \frac{\partial R_j^s(q_i, s_j)}{\partial s_j} + \frac{\partial \pi_j^j}{\partial s_j} - q_j - s_j \frac{\partial R_j^s(q_i, s_j)}{\partial s_j} = 0$. By substituting $\pi_j^j = 0$, the f.o.c. of firm j , and $\frac{\partial \pi_j^j}{\partial s_j} = q_j$ into the f.o.c. of government j , we obtain $0 \times \frac{1}{2b} + q_j - q_j - s_j \frac{1}{2b} = -s_j \frac{1}{2b} \leq 0 \Rightarrow s_j^{bs} = 0$. As a result, the subsidy level is zero, regardless of any q_i . The reaction function is $s_j^{bs} = R_j^s(q_i) = 0$. Thus $q_j = R_j^s(q_i, 0) = \frac{a - bq_i - c_j}{2b}$.

In the subgame at the second stage, the output choice by firm i is solved as follows: given s_i , inducing $s_j^{bs} = 0$ and $q_j = R_j^s(q_i, 0) = \frac{a - bq_i - c_j}{2b}$, firm i maximizes the profit function $\pi^i(q_i, R_j^s(q_i, 0); s_i, 0)$. That is, the maximizing problem is $\max_{q_i} \pi^i(q_i, R_j^s(q_i, 0); s_i, 0)$, noting that $R_j^{s'}(q_i) = -\frac{1}{2}$. The f.o.c. is $\pi_i^i + \pi_j^j R_j^{s'}(q_i) = ((a - b(q_i + R_j^s(q_i, 0)) - e_i) - bq_i) - bq_i(-\frac{1}{2}) = 0$. This result is the same as for the Stackelberg equilibrium with no subsidy: $q_i = R_i^s(s_i) = \frac{a - 2e_i + c_j}{2b} =$

³⁶Note that if $s_i = 0$, the unilateral intervention of sequential decision-making is analyzed: $q_i \rightarrow s_j \rightarrow q_j$. If $s_j = 0$, This is Case C-1.

$q_i^S(s_i, 0)$, $q_j = R_j(q_i, 0) = \frac{a-3c_j+2e_i}{4b} = q_j^S(s_i, 0)$, and also. The f.o.c. is $\pi_i^i + \pi_j^i R_j^{SI}(q_i) = ((a - b(q_i + R_j^S(q_i, 0)) - c_i) - bq_i) - bq_i(-\frac{1}{2}) = 0$.

In the subgame at the first stage, the subsidy decision by government i is determined by the same procedure as in Case C-1. That is, there is no subsidy, $s_i = 0$.

$$(s_i^{bs}, s_j^{bs}) = (0, 0). \quad (23)$$

$$(q_i^{bs}, q_j^{bs}) = \left(\frac{a - 2c_i + c_j}{2b}, \frac{a - 3c_j + 2c_i}{4b} \right). \quad (24)$$

It is satisfied that $(q_i^{bs}, q_j^{bs}) = (q_i^S(0, 0), q_j^S(0, 0))$. The profit and welfare levels are as follows:

$$\pi^{Si}(0, 0) = G^{Si}(0, 0) = \frac{(a-2c_i+c_j)^2}{8b} \text{ and } \pi^{Sj}(0, 0) = G^{Sj}(0, 0) = \frac{(a-3c_j+2c_i)^2}{16b}.$$

4 Comparison

In this section, we compare the different structures with regard to the timing of decision-making on the subsidy by governments and the output by the firms. In particular, we compare the simultaneous decision with the sequential one in the equilibrium which is derived from the previous section. Before proceeding to the analysis, it is convenient to digest the equilibrium outcome under the different timing of decision-making in a table. See Table 3. To visualize the output levels in the equilibrium, refer to Figure 2 and 3.³⁷

³⁷For simplification, we illustrate only the case in which the costs are identical.

	subsidy	output	price
1. no-subsidy Cournot	nothing	$(\frac{a-2c_i+c_j}{3b}, \frac{a-2c_j+c_i}{3b})$	$\frac{a+c_i+c_j}{3}$
2. no-subsidy Stackelberg	nothing	$(\frac{a-2c_i+c_j}{2b}, \frac{a-3c_j+2c_i}{4b})$	$\frac{a+2c_i+c_j}{4}$
A. unilateral Cournot	$(\frac{a-2c_i+c_j}{4}, 0)$	$(\frac{a-2c_i+c_j}{2b}, \frac{a-3c_j+2c_i}{4b})$	$\frac{a+2c_i+c_j}{4}$
B. bilateral Cournot			
B-1. simultaneous	$(\frac{a-3c_i+2c_j}{5}, \frac{a-3c_j+2c_i}{5})$	$(\frac{2(a-3c_i+2c_j)}{5b}, \frac{2(a-3c_j+2c_i)}{5b})$	$\frac{a+2c_i+2c_j}{5}$
B-2. sequential	$(\frac{a-3c_i+2c_j}{3}, \frac{a-4c_j+3c_i}{6})$	$(\frac{a-3c_i+2c_j}{2b}, \frac{a-4c_j+3c_i}{3b})$	$\frac{a+3c_i+2c_j}{6}$
C. unilateral Stackelberg			
C-1. government i	$(0, 0)$	$(\frac{a-2c_i+c_j}{2b}, \frac{a-3c_j+2c_i}{4b})$	$\frac{a+2c_i+c_j}{4}$
C-2. government j	$(0, \frac{a-3c_j+2c_i}{3})$	$(\frac{a-4c_j+3c_i}{3b}, \frac{a-3c_j+2c_i}{2b})$	$\frac{a+2c_i+3c_j}{6}$
D. bilateral Stackelberg			
D-1. simultaneous	$(0, \frac{a-3c_j+2c_i}{3})$	$(\frac{a-4c_i+3c_j}{3b}, \frac{a-3c_j+2c_i}{2b})$	$\frac{a+2c_i+3c_j}{6}$
D-2. sequential gov. i	$(\frac{a-4c_i+3c_j}{8}, \frac{a-5c_j+4c_i}{4})$	$(\frac{a-4c_i+3c_j}{2b}, \frac{3(a-5c_j+4c_i)}{8b})$	$\frac{a+4c_i+3c_j}{8}$
D-3. sequential gov. j	$(0, \frac{a-3c_j+2c_i}{3})$	$(\frac{a-4c_i+3c_j}{3b}, \frac{a-3c_j+2c_i}{2b})$	$\frac{a+2c_i+3c_j}{6}$
E. wholly sequential	$(0, 0)$	$(\frac{a-2c_i+c_j}{2b}, \frac{a-3c_j+2c_i}{4b})$	$\frac{a+2c_i+c_j}{4}$

	profit	welfare
1. no-subsidy Cournot	$(\frac{(a-2c_i+c_j)^2}{9b}, \frac{(a-2c_j+c_i)^2}{9b})$	$(\frac{(a-2c_i+c_j)^2}{9b}, \frac{(a-2c_j+c_i)^2}{9b})$
2. no-subsidy Stackelberg	$(\frac{(a-2c_i+c_j)^2}{8b}, \frac{(a-3c_j+2c_i)^2}{16b})$	$(\frac{(a-2c_i+c_j)^2}{8b}, \frac{(a-3c_j+2c_i)^2}{16b})$
A. unilateral Cournot	$(\frac{(a-2c_i+c_j)^2}{4b}, \frac{(a-3c_j+2c_i)^2}{16b})$	$(\frac{(a-2c_i+c_j)^2}{8b}, \frac{(a-3c_j+2c_i)^2}{16b})$
B. bilateral Cournot		
B-1. simultaneous	$(\frac{4(a-3c_i+2c_j)^2}{25b}, \frac{4(a-3c_j+2c_i)^2}{25b})$	$(\frac{2(a-3c_i+2c_j)^2}{25b}, \frac{2(a-3c_j+2c_i)^2}{25b})$
B-2. sequential	$(\frac{(a-3c_i+2c_j)^2}{4b}, \frac{(a-4c_j+3c_i)^2}{9b})$	$(\frac{(a-3c_i+2c_j)^2}{12b}, \frac{(a-4c_j+3c_i)^2}{18b})$
C. unilateral Stackelberg		
C-1. government i	$(\frac{(a-2c_i+c_j)^2}{8b}, \frac{(a-3c_j+2c_i)^2}{16b})$	$(\frac{(a-2c_i+c_j)^2}{8b}, \frac{(a-3c_j+2c_i)^2}{16b})$
C-2. government j	$(\frac{(a-4c_i+3c_j)^2}{18b}, \frac{(a-3c_j+2c_i)^2}{4b})$	$(\frac{(a-4c_i+3c_j)^2}{18b}, \frac{(a-3c_j+2c_i)^2}{12b})$
D. bilateral Stackelberg		
D-1. simultaneous	$(\frac{(a-4c_i+3c_j)^2}{18b}, \frac{(a-3c_j+2c_i)^2}{4b})$	$(\frac{(a-4c_i+3c_j)^2}{18b}, \frac{(a-3c_j+2c_i)^2}{12b})$
D-2. sequential gov. i	$(\frac{(a-4c_i+3c_j)^2}{8b}, \frac{9(a-5c_j+4c_i)^2}{64b})$	$(\frac{(a-4c_i+3c_j)^2}{16b}, \frac{3(a-5c_j+4c_i)^2}{64b})$
D-3. sequential gov. j	$(\frac{(a-4c_i+3c_j)^2}{18b}, \frac{(a-3c_j+2c_i)^2}{4b})$	$(\frac{(a-4c_i+3c_j)^2}{18b}, \frac{(a-3c_j+2c_i)^2}{12b})$
E. wholly sequential	$(\frac{(a-2c_i+c_j)^2}{8b}, \frac{(a-3c_j+2c_i)^2}{16b})$	$(\frac{(a-2c_i+c_j)^2}{8b}, \frac{(a-3c_j+2c_i)^2}{16b})$

Table 3: Equilibrium results

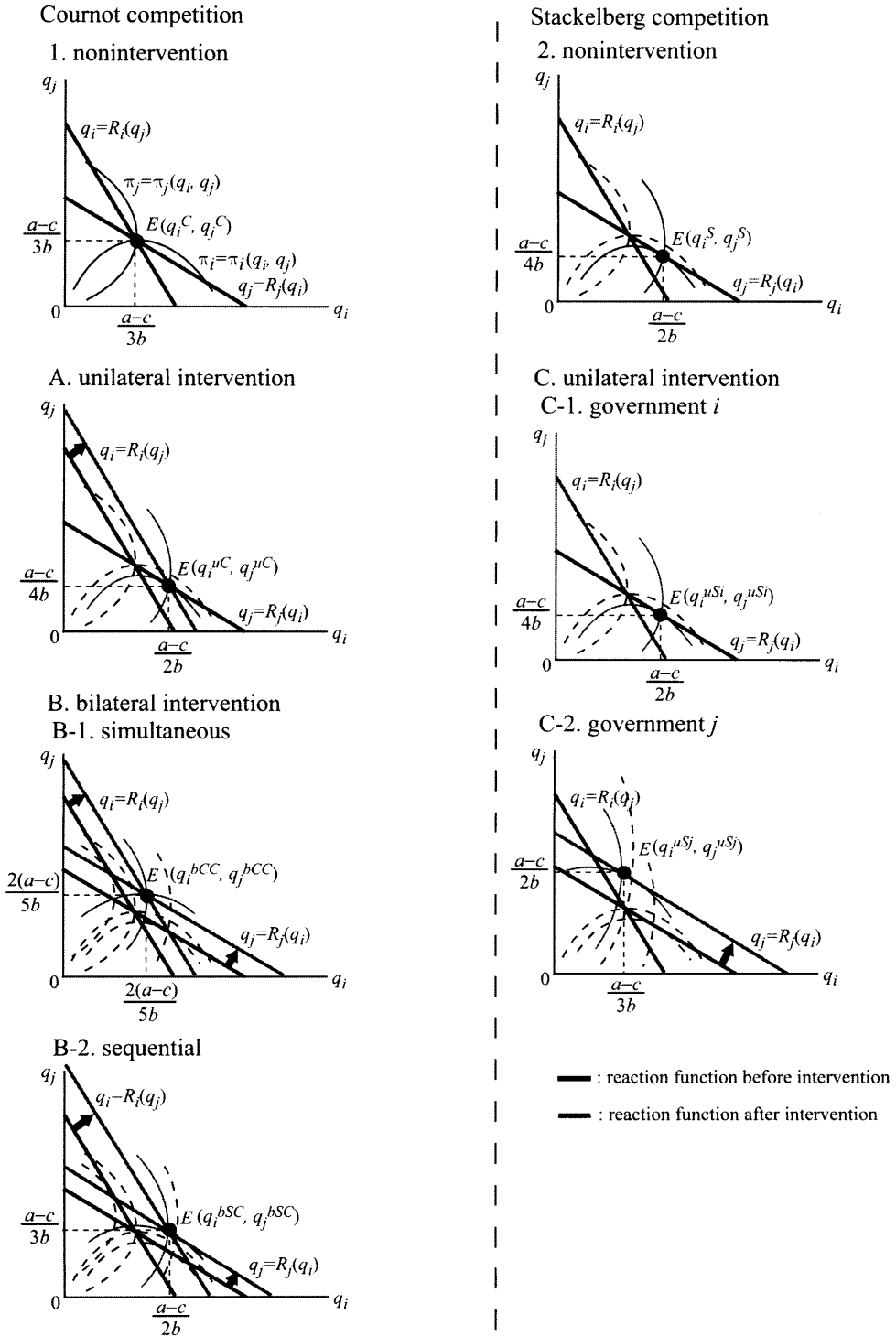
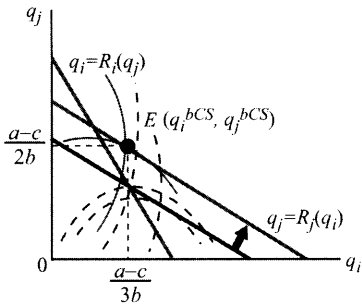


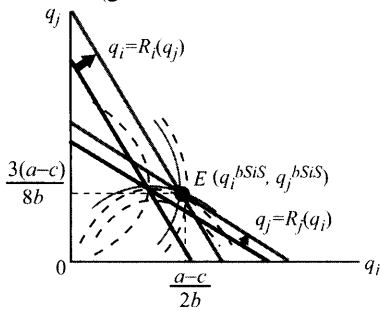
Figure 2: Reaction functions and output levels in the equilibrium

Stackelberg competition (continued)

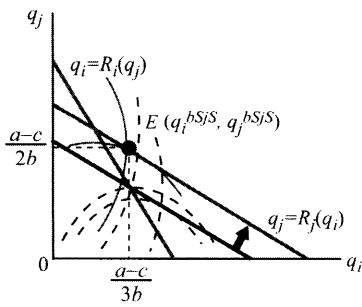
D-1. simultaneous decision-making



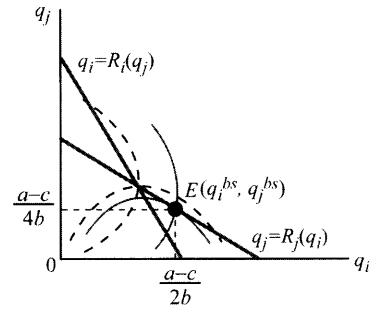
D-2. sequential decision-making (government *i* moves first.)



D-3. sequential decision-making (government *j* moves first.)



E. Wholly sequential decision-making



— : reaction function before intervention
 — : reaction function after intervention

Figure 3: Reaction functions and output levels in the equilibrium (continued)

From Table 3, we can examine how the different structures about the timing of decision-making by firms and governments affects the efficiency of the subsidy policy. In the following comparison, for simplification, the argument is limited to the situation in which the costs are identical. Although this paper does not present an exhaustive comparison in a comprehensive way, several noticeable results are shown in the following propositions.³⁸

Proposition 13. (*profit levels between Cournot and Stackelberg competition under the unilateral intervention*)

Consider the unilateral intervention of the government. When the firm faces the Cournot competition, the profit is larger than when it competes as the Stackelberg leader and it is equal to that when it competes as the Stackelberg follower. That is,

$$\pi^{Ci}(s_i^{uC}, 0) = \pi^{Sj}(0, s_j^{uS}) = \frac{(a-c)^2}{4b} > \pi^{Si}(s_i^{uS}, 0) = \frac{(a-c)^2}{8b}.$$

This proposition implies that although it looks at first glance that the Stackelberg leader has more advantage than under Cournot competition, this impression is not correct. If the unilateral intervention of the government (and nonintervention of the rival government) is necessarily guaranteed, the firm prefers to face the Cournot competition rather than become the Stackelberg leader. The reason is that under the Stackelberg competition, the firm receives no subsidy, but under the Cournot competition, by being subsidized, the firm can raise its profit. Whereas, the proposition also implies that the subsidized firm can recover the competitive position from the Stackelberg follower to the Cournot, being able to compete on equal terms with the rival firm.

Then, we compare the profits between two competitive forms under the bilateral intervention.

³⁸Although we do not compare the welfare in the third country, it is worth noting that the third country's welfare is reduced by the size of total quantity, Q , because this welfare consists of only the consumer's surplus, $CS(Q) = \frac{1}{2}Q^2$. From the price level in Table 3, we can compare the third country's welfare level directly. Likewise, the sum of the welfare of two exporting countries and the world welfare are immediately derived by tedious calculation, although these comparisons are omitted.

First, a comparison between Case B-1 and Case D-1 is made.

Proposition 14. (*profit levels between Cournot and Stackelberg competition under the bilateral simultaneous intervention*)

Consider the bilateral simultaneous intervention of the governments. When the firm faces the Cournot competition, its profit is larger than when it competes as a Stackelberg leader. On the other hand, the profit under the Cournot competition is smaller than when it competes as a Stackelberg follower. That is,

$$\pi^{Si}(s_i^{bCS}, s_j^{bCS}) = \frac{(a-c)^2}{18b} < \pi^{Ci}(s_i^{bCC}, s_j^{bCC}) = \frac{4(a-c)^2}{25b} < \pi^{Sj}(s_i^{bCS}, s_j^{bCS}) = \frac{(a-c)^2}{4b}.$$

This proposition is the extended version of **Proposition 13** to the bilateral intervention. Although it looks at first glance that the Stackelberg leader has more advantage than under Cournot competition, the firm prefers to compete in the Cournot way rather than become the Stackelberg leader and moreover prefers to become the follower. The reason is similar to that of **Proposition 13**. Under the bilateral simultaneous intervention, the government of the Stackelberg leader does not have any influence to change the market structure. When both governments under the Cournot competition subsidize their firms, their subsidies influence market structure and raise the firms' profits. Whereas, the Stackelberg follower is fully supported by its government and improves its competitive position vastly. It can compete on more advantageous terms than the rival firm in Case D-1.

This proposition suggests the following important assertion on trade policy: When governments can intervene in its domestic firm with a certain policy instrument in advance, the difference of the competitive mode between firms, such as Cournot or Stackelberg competition, does not necessarily influence the firm's advantage on the competition. In other words, even if a firm is the Stackelberg follower in the third-market, the subsidization by the government can recover the competitive advantage to some extent.

Finally, a comparison between Case B-2 and Case D-2 (and also Case D-3) is made.

Proposition 15. (*profit levels between Cournot and Stackelberg competition under the bilateral sequential intervention*)

Consider the bilateral sequential intervention of the governments. Suppose that government i moves first.

(i) *When firm i faces the Cournot competition, its profit is larger than when it competes as the Stackelberg leader. On the other hand, the profit of firm j under the Cournot competition is smaller than when it competes as the Stackelberg follower. That is, $\pi^{Ci}(s_i^{bSC}, s_j^{bSC}) = \frac{(a-c)^2}{4b} > \pi^{Si}(s_i^{bSiS}, s_j^{bSiS}) = \frac{(a-c)^2}{8b}$ and $\pi^{Cj}(s_i^{bSC}, s_j^{bSC}) = \frac{(a-c)^2}{9b} < \pi^{Sj}(s_i^{bSiS}, s_j^{bSiS}) = \frac{9(a-c)^2}{64b}$.*

(ii) *When firm i faces the Cournot competition, its profit is equivalent to when it competes as the Stackelberg follower. And also, the profit of firm j under the Cournot competition is larger than when it competes as the Stackelberg leader. That is, $\pi^{Ci}(s_i^{bSC}, s_j^{bSC}) = \pi^{Sj}(s_i^{bSjS}, s_j^{bSjS}) = \frac{(a-c)^2}{4b}$ and $\pi^{Cj}(s_i^{bSC}, s_j^{bSC}) = \frac{(a-c)^2}{9b} > \pi^{Si}(s_i^{bSjS}, s_j^{bSjS}) = \frac{(a-c)^2}{18b}$.³⁹*

Part (i) in this proposition states as follows: Under the bilateral sequential intervention, such as the simultaneous intervention, the firm prefers to face Cournot competition rather than become the Stackelberg leader. Moreover, the firm prefers to become the Stackelberg follower rather than face Cournot competition. At first glance, it seems that the change of decision structure from Case B-2 to Case D-2 gives more advantage about output choice to the firm that becomes the Stackelberg leader. However, this proposition implies that this shift of decision structure does not bring any advantage, but on the contrary, decreases the profit of the leader firm. On the other hand, the firm that becomes the Stackelberg follower acquires more profit than under the previous Cournot competition.

³⁹Note that the index of i is being interchanged that of j in Case D-3, because government i moves first and firm i is the Stackelberg follower.

Under the bilateral sequential intervention, the Stackelberg leader needs less subsidy than if this firm was engaged in Cournot competition, that is, $s_i^{bSC} = \frac{a-c}{3} > s_i^{bSiS} = \frac{a-c}{8}$, because the firm has already enjoyed the first-mover advantage. Whereas, the Stackelberg follower is supported by its government with great care and improves the competitive position, that is, $s_j^{bSC} = \frac{a-c}{6} < s_j^{bSiS} = \frac{a-c}{4}$. As a result of the asymmetric subsidy policy, it occurs that when both governments subsidize the firm under the Cournot competition, the profit of the leader (the follower) is less (resp. more) than the profit under the Stackelberg competition. The sequence of intervention by the governments affects the size of the firm's profit and the efficiency of the trade policy significantly.

Part (ii) of this proposition states as follows: Under the bilateral sequential intervention, the firm's profit is the same whether it behaves as the Stackelberg follower or faces Cournot competition. Moreover, the firm prefers to face Cournot competition rather than become the Stackelberg leader in Case D-3. It implies that when government i moves first, even if the competition form shifts from Cournot to Stackelberg and firm i becomes the follower, its profit does not change. The profit of firm j that becomes the leader in place of firm i becomes less than under the previous Cournot competition. At first glance, it seems that the change of decision structure from Case B-2 to Case D-3 gives the disadvantage on output choice to the firm that becomes the Stackelberg follower. However, this shift of decision structure does not affect the firm's profit.

It is desirable for both firms to shift the competitive mode from the Stackelberg competition under which the government of the Stackelberg follower first decides the subsidy, to the Cournot one under bilateral sequential intervention. This shift of competitive mode avoids the excessive subsidy allocation by governments and saves the subsidy that does not have quite effect on the advantage of the domestic firm in the market.

These propositions insist that the difference in timing of policy implementation and announcement by the government affects the firm's profit and the resulting welfare. And also they insist that the government should exercise the trade policy taking into consideration the competitive mode between firms.

5 Concluding remarks

This paper analyzed the relationship between the different timing of decision-making by exporting firms and their subsidizing governments and its impacts on the export subsidy. Two main results are as follows: First, when governments decide simultaneously the export subsidies in advance under the Stackelberg competition, the original Stackelberg leader firm in the output competition produces as if it was the follower. Different from the Cournot model, under the Stackelberg model, the subsidy policy by the government that can subsidize the leader firm is nullified. Second, under the sequential-move game in which the government that can subsidize the leader firm decides the subsidy level in advance, the profit of the leader is less than that of the follower, although the first-mover advantage is maintained. We conclude that the timing of decision-making affects the effectiveness on the export subsidy policy significantly.

Although the paper mainly focuses on the theoretical aspect, the results in this paper are applicable to make some proper suggestions to the actual strategic trade policies. For example, in the realistic context of the international exporting competition, suppose that there is the leader firm that is the predecessor and lies in the dominant position in the exporting market. When the successor entries and the Stackelberg competition is made, the predecessor government of the leader firm may anticipate the successor's government in deciding the trade policy. In this situation, how does the government implement the strategic subsidy policy? **Proposition 15** in Section 4 suggests that the predecessor government should dare to make its firm acquire less

profit than the rival firm with less subsidy than the successor government and should save the subsidy in order to attain more welfare. Moreover, by **Proposition 15**, even if the government can choose the timing of subsidy policy, it should defend the position as first-mover policy maker and maintain the first-mover advantage. On the other hand, **Proposition 15** also suggests that in order to bring more profit to the successor firm, its government should announce the subsidy policy faster than the rival government of the leader and take the first-mover advantage if possible. This may present one of the reasons that the trade war may be triggered.

Further extension in this paper can be considered. The previous analysis is the linear demand and linear cost model. First, the more general model in which the demand and cost functions have the general forms can be analyzed, although the basic logic remains unchanged as the linear case. It is thought that we can generalize the previous analysis easily and directly. Moreover, we limit the argument to the homogeneous goods. Second, the extension toward product differentiation should be analyzed, although the basic results argued in the previous analysis have remained unchanged.

Thirdly, the extension to strategic complements should be considered as an extension of Eaton and Grossman (1986). They argued the generalization of the analysis by Brander and Spencer (1935). However, they dealt only with the symmetric conjectural variations like Cournot, Bertrand, and consistent conjectures. That is, they dealt only with the simultaneous-move game on output choice. We deal with the asymmetric conjecture such as the Stackelberg leader-follower competition. By using similar conjectural variations to theirs, the generalized analysis can be applied. This issue may bring another problem, because under price competition (or strategic complements in general), the optimal trade policy is the adoption of exporting tariff. This result may be extended to strategic complements and other conjectural variables, although it should be noticed that most of the conjectural variables do not have any economic justification.

Finally, endogeneity of the timing of exporting policy-maker is one important topic. As for the endogenous timing, Ohkawa, Okamura, and Tawada (2002) tackled this issue under the Cournot oligopoly. We believe that the result in this paper is easily applicable to deal with this problem.

As a possibility of further extensions of our paper that investigates the strategic subsidy policy in the sequential-move game, other arguments on the strategic trade policy can be analyzed, taking into consideration the timing of decision-making. For example, sequential timing on other variables such as export tariff, quota, and investment choice of FDI should be investigated. Also the comparative statics of the parameter may be able to be considered, such as externality, spillover, and environmental diseconomy. Moreover, we may be able to deal with the occurrence of sequential-move game due to informational asymmetry.

Acknowledgement

For helpful comments and suggestions, I am grateful to Takao Ohkawa, Yasuhiro Takarada, Makoto Tawada and Takeshi Yamazaki, and seminar participants at the ETSG Meeting 2005 in Dublin (University College Dublin), and the 64th Annual Meeting of the Japan Society of International Economics (Ritsumeikan University). The research for this paper is supported by Grant-in-Aid for Scientific Research (KAKENHI 16730095) from JSPS and MEXT of the Japanese Government. The usual disclaimer applies.

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