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A Note on Profitable Mergers in a Hierarchical Stackelberg Model

Kojun Hamada*

Abstract

In this paper, we attempt to examine whether or not mergers are profitable in a hierarchical Stackelberg model under quantity competition. Under general demand function, a significant result cannot be obtained with regard to whether or not mergers are profitable. Instead, we show that mergers are always profitable, regardless of any curvature of demand under a specified demand structure. This result supports the intuition that mergers are always profitable in a hierarchical Stackelberg model.

JEL Classification: D43; L13**Keywords:** hierarchical Stackelberg model, profitable merger

* Faculty of Economics, Niigata University

Address: 8050 Ikarashi 2-no-cho, Nishi-ku, Niigata City, 950-2181 Japan

Tel. and fax: +81-25-262-6538

E-mail address: khamada@econ.niigata-u.ac.jp

1 Introduction

This paper attempts to examine whether or not mergers are profitable in a hierarchical Stackelberg model under quantity competition. Under the hierarchical Stackelberg model (henceforth HSM), multiple firms choose outputs sequentially under Stackelberg quantity competition. In a representative paper on the analysis of the HSM, Boyer and Moreaux (1986) show that competitive equilibrium can be seen as the limit of some hierarchical game in which the rights (or information) of the players can be strictly ordered. Vives (1988) reexamine the relationship between potential competition, industry structure, and welfare in a market, subject to a sequential entry threat where firms can make quantity commitments and have access to a constant returns technology. It is shown that, provided the entry is not blocked, the incumbent(s) will either prevent entry or allow all the potential entrants in. Anderson and Engers (1992) compare an n -firm Cournot model with an n -hierarchical Stackelberg model and they showed that the Stackelberg equilibrium price is lower, total surplus is higher, and total profits are lower. However, the existing literature has paid scant attention to the profitability of mergers in a HSM. In this paper, we investigate in more details the profitability of mergers in the HSM.

As a well-known result, Salant, Switzer, and Reynolds (1983) show that mergers are not profitable if the market share of the merged firm does not exceed over 80% under Cournot quantity competition. Daugherty (1990) and Huck, Konrad, and Müller (2001) independently present the similar extensive results under the two-stage Stackelberg quantity competition. They showed that any merger between a leader and a follower is profitable. We extend the existing analyses on profitable mergers to an n -hierarchical Stackelberg model and examine the profit of mergers under the HSM.

We focus on the analysis of an n -hierarchical Stackelberg equilibrium, applying the same setting as in the HSM in Anderson and Engers (1992) and attempt to examine whether or not mergers are profitable. It is simply shown that when the demand curve is linear, any merger is profitable in a HSM. Likewise, we may anticipate that any merger is profitable in a HSM, even if demand structure is in a more general form. So far, however, it has not yet been clarified that any merger is profitable in a HSM. Therefore, we attempt to examine whether or not any merger is profitable in a HSM. Unfortunately, a significant result cannot be obtained under

general demand function, with regard to whether or not any merger is profitable, although we would expect this result to be satisfied as a conjecture. Instead, we focus on analyzing a specified form of demand function, which takes the curvature of the more general demand function into consideration, including that the functions are linear, concave and convex.

We show that any merger is profitable, regardless of any curvature of the demand curve under this specified demand function. This result supports the intuition that any merger is profitable in a hierarchical Stackelberg model, although the result is accompanied with some loss of generality on the functional form of demand.

The remainder of the paper is organized as follows. Section 2 describes the model and derives firms' outputs and profits in the equilibrium. Section 3 examines whether or not mergers are profitable under general demand structure. Section 4 specifies the form of demand function and presents the main results. Section 5 contains concluding remarks.

2 The Model

In Section 2, we describe a situation where n firms choose output sequentially in a HSM. The Stackelberg equilibrium is the subgame perfect outcome that arises when firms choose their outputs sequentially according to some exogenously determined order of moves. We shall assume that firms that supply perfect substitutes are identical, except for the order of moves.

The output level of firm $i = 1, 2, \dots, n$ is denoted by q_i . The total output and the price of homogeneous goods are denoted by $Q \equiv \sum_{i=1}^n q_i$ and P , respectively. The market demand function is denoted by $Q = Q(P)$. The demand function is a strict monotone decreasing function of P and it is assumed to be more than twice differentiable for all $P > 0$. That is, $Q'(P) < 0$ is satisfied for all $P > 0$. Thus, the inverse demand function exists and it is given by $P = P(Q)$, $P'(Q) < 0$. $P(Q)$ is more than twice differentiable for all $Q > 0$. The demand specification that we consider in Section 2 is a general one. Later on, in Section 4, we specify the demand function. Therefore, the inverse demand function is allowed to be convex, $P''(Q) > 0$, or concave, $P''(Q) < 0$, and it is linear when $P''(Q) = 0$.

It is assumed that the second-order conditions for profit maximization for all firms are satisfied. In particular, $2P' + P''q_i < 0, \forall q_i$ is assumed. It is presumed that the output levels

for all firms in the equilibrium are the interior solution. Furthermore, in order to satisfy the condition that the best reply functions of all firms are strictly downward-sloping and as a result, the equilibrium is unique and stable, it is assumed that the usual downward-sloping marginal revenue condition is satisfied: as $MR_i(q_i; \widehat{Q}_{-i}) \equiv P + P'q_i$ where $\widehat{Q}_{-i} \equiv \sum_{j \neq i} q_j$, $\frac{\partial MR_i}{\partial \widehat{Q}_{-i}} = P' + P''q_i < 0 \forall q_i$.

For simplification of analysis, it is assumed that the marginal cost is constant and it is denoted by c . Firm i 's profit is denoted by $\pi_i(q_i) \equiv [P(q_i + \widehat{Q}_{-i}) - c]q_i$, where $\widehat{Q}_{-i} \equiv \sum_{j \neq i} q_j$.

The solution concept under the HSM is the subgame perfect equilibrium, which can be obtained by backward induction. In the following subsection, we derive outputs and profits of firms in the equilibrium by solving the profit maximization problems from firm n to firm 1 in backward sequence.

2.1 The Final Firm

First, we solve the profit maximization problem of the final firm (firm n) in a HSM. Firm n 's profit is denoted by $\pi_n(q_n; Q_{-n}) \equiv [P(q_n + Q_{-n}) - c]q_n$; $Q_{-n} \equiv \sum_{i=1}^{n-1} q_i = q_1 + \dots + q_{n-1}$.¹ The f.o.c. is as follows:

$$\frac{\partial \pi_n(q_n; Q_{-n})}{\partial q_n} = P(Q) + P'(Q)q_n - c = 0. \quad (1)$$

By assumption, the s.o.c. is satisfied.

From (1), the best reply function of firm n is obtained. It is denoted by $q_n \equiv r_n(Q_{-n})$. Substituting $q_n = r_n(Q_{-n})$ into (1), the following identity is satisfied:

$$r_n(Q_{-n}) = -\frac{P(r_n(Q_{-n}) + Q_{-n}) - c}{P'(r_n(Q_{-n}) + Q_{-n})}. \quad (2)$$

Totally differentiating (2) with regard to Q_{-n} , we can obtain the slope of the best reply function of firm n as follows:

$$r'_n = -\frac{P' + P''q_n}{2P' + P''q_n} < 0. \quad (3)$$

By assumption, the best reply function of firm n is strictly downward-sloping. Also it is shown that $1 + r'_n > 0$ by assumption, which guarantees the uniqueness and the stability of the equilibrium. From (3), if $P'' = 0$, then $r'_n = -\frac{1}{2}$.

¹ Note that $Q_{-i} \equiv \sum_{j=1}^{i-1} q_j$ is different from $\widehat{Q}_{-i} \equiv \sum_{j \neq i} q_j$. It is satisfied that $\widehat{Q}_{-i} = Q_{-i} + \sum_{j=i+1}^n q_j$. The reaction functions of firm i depends on Q_{-i} .

2.2 The Penultimate Firm

Next, we solve the profit maximization problem of the penultimate firm (firm $n - 1$) in a HSM. This problem is described as follows:

$$\begin{aligned} \max_{q_{n-1}} \pi_{n-1}(q_{n-1}; q_n, Q_{-(n-1)}) &= [P(q_{n-1} + q_n + Q_{-(n-1)}) - c]q_{n-1}, \\ \text{subject to } q_n &= r_n(Q_{-n}), \end{aligned} \quad (4)$$

where $Q_{-(n-1)} = \sum_{i=1}^{n-2} q_i = q_1 + \cdots + q_{n-2}$. The f.o.c. is as follows:

$$\frac{\partial \pi_{n-1}(q_{n-1}; r_n(Q_{-n}), Q_{-(n-1)})}{\partial q_{n-1}} = P + P' \times (1 + r'_n)q_{n-1} - c = 0. \quad (5)$$

By assumption, the s.o.c. of this problem is satisfied and the marginal revenue of firm $n - 1$ is downward-sloping. The best reply function of firm $n - 1$ can be obtained by (5). It is denoted by $q_{n-1} = r_{n-1}(Q_{-(n-1)})$. Substituting $q_{n-1} = r_{n-1}(Q_{-(n-1)})$ into (5), the following identity is satisfied:

$$r_{n-1}(Q_{-(n-1)}) = -\frac{P(r_{n-1}(Q_{-(n-1)}) + q_n + Q_{-(n-1)}) - c}{P'(r_{n-1}(Q_{-(n-1)}) + q_n + Q_{-(n-1)})(1 + r'_n)}, \quad (6)$$

where $q_n = r_n(Q_{-n}) = r_n(q_{n-1} + Q_{-(n-1)}) = r_n(r_{n-1}(Q_{-(n-1)}) + Q_{-(n-1)})$. Thus it is satisfied that $\frac{\partial q_n}{\partial Q_{-(n-1)}} = r'_n(1 + r'_{n-1})$ is satisfied.

Totally differentiating (6) with regard to $Q_{-(n-1)}$, we obtain the following equation:

$$r'_{n-1} = -\frac{(P' + P'' \times (1 + r'_n)r_{n-1})(1 + r'_n) + P'r''_nr_{n-1}}{(2P' + P'' \times (1 + r'_n)r_{n-1})(1 + r'_n) + P'r''_nr_{n-1}}. \quad (7)$$

Comparing (1) with (5), we obtain the following equation:

$$q_n = (1 + r'_n)q_{n-1} = \frac{P'}{2P' + P''q_n}q_{n-1}. \quad (8)$$

By (3) and (8), it is satisfied that $q_n = (1 + r'_n)q_{n-1} < q_{n-1}$.

In advance, we assume that $r'_{n-1} < 0$. If $P'' = 0$, then $r'_{n-1} = -\frac{1}{2}$.

2.3 The Antepenultimate Firm

We proceed to solve the profit maximization problem of the antepenultimate firm (firm $n - 2$). This problem is described as follows:

$$\begin{aligned} \max_{q_{n-2}} \pi_{n-2}(q_{n-2}; \{q_n, q_{n-1}\}, Q_{-(n-2)}) &= [P(q_{n-2} + q_{n-1} + q_n + Q_{-(n-2)}) - c]q_{n-2}, \\ \text{s.t. } q_n &= r_n(Q_{-n}) \text{ and } q_{n-1} = r_{n-1}(Q_{-(n-1)}). \end{aligned} \quad (9)$$

The f.o.c. is as follows:

$$\frac{\partial \pi_{n-2}(q_{n-2}; \{q_n, q_{n-1}\}, Q_{-(n-2)})}{\partial q_{n-2}} = P + P' \times (1 + r'_n)(1 + r'_{n-1})q_{n-2} - c = 0. \quad (10)$$

By assumption, the s.o.c. of this problem is satisfied and the marginal revenue of firm $n-2$ is downward-sloping. The best reply function of firm $n-2$, $q_{n-2} = r_{n-2}(Q_{-(n-2)})$, can be obtained from (10), and the slope is assumed to be downward-sloping.

Substituting $q_{n-2} = r_{n-2}(Q_{-(n-2)})$ into (10), the following identity is satisfied:

$$r_{n-2}(Q_{-(n-2)}) = -\frac{P(r_{n-2}(Q_{-(n-2)}) + q_n + q_{n-1} + Q_{-(n-2)}) - c}{P'(r_{n-2}(Q_{-(n-2)}) + q_n + q_{n-1} + Q_{-(n-2)})(1 + r'_n)(1 + r'_{n-1})}, \quad (11)$$

where $q_n = r_n(r_{n-2}(Q_{-(n-2)}) + r_{n-1}(r_{n-2}(Q_{-(n-2)}) + Q_{-(n-2)}) + Q_{-(n-2)})$ and $q_{n-1} = r_{n-1}(r_{n-2}(Q_{-(n-2)}) + Q_{-(n-2)}) + Q_{-(n-2)}$. Note that $\frac{\partial q_n}{\partial Q_{-(n-2)}} = r'_n(1 + r'_{n-1})(1 + r'_{n-2})$ and $\frac{\partial q_{n-1}}{\partial Q_{-(n-2)}} = r'_{n-1}(1 + r'_{n-2})$.

Comparing (5) with (10), we obtain the following equation:

$$q_{n-1} = (1 + r'_{n-1})q_{n-2}. \quad (12)$$

By (12) and $r'_{n-1} < 0$, $q_{n-1} = (1 + r'_{n-1})q_{n-2} < q_{n-2}$ is satisfied.

2.4 The Intermediate Firm

By inducing backward in turn, we can deal with the profit maximization problem of the intermediate firm (firm i). This problem is as follows:

$$\begin{aligned} \max_{q_i} \pi_i(q_i; \{q_n, q_{n-1}, \dots, q_{i+1}\}, Q_{-i}) &= [P(q_i + \sum_{j=i+1}^n q_j + Q_{-i}) - c]q_i, \\ \text{s.t. } q_n &= r_n(Q_{-n}), q_{n-1} = r_{n-1}(Q_{-(n-1)}), \dots, q_{i+1} = r_{i+1}(Q_{-(i+1)}). \end{aligned} \quad (13)$$

The f.o.c. is as follows:

$$P + P' \prod_{j=i+1}^n (1 + r'_j)q_i - c = 0. \quad (14)$$

By assumption, the s.o.c. is satisfied and the marginal revenue of firm i is downward-sloping. The best reply function of firm i , $q_i = r_i(Q_{-i})$, can be obtained from (14), and the slope is assumed to be downward-sloping, $r'_i < 0$.²

² When $i = n$, the f.o.c. can be calculated as the product term in (14) is unity, $\prod_{j=i+1}^n (1 + r'_j) = 1$.

Substituting $q_i = r_i(Q_{-i})$ into (14), the following identity is satisfied:

$$r_i(Q_{-i}) \equiv -\frac{P - c}{P' \prod_{j=i+1}^n (1 + r'_j)}. \quad (15)$$

Note that $\frac{\partial q_{i+m}}{\partial Q_{-i}} = r'_{i+m} \prod_{j=i+m+1}^n (1 + r'_j)$.

2.5 Comparison of Outputs and Profits in the Equilibrium

We compare the output levels and firms' profits in the equilibrium under the HSM. As we assume $-1 < r'_i < 0 \forall i$, the interior solution for all firms is guaranteed in the subgame perfect equilibrium. The assumption, $1 + r'_i > 0$, implies that the self-effect of the output on the profit function exceeds over the cross-effect and guarantees the uniqueness and the stability of equilibrium.

First, the following proposition can be derived with regard to the firms' outputs in the HSM.

Proposition 1. *The output of the firm moving fast is always greater than that of the firms moving late, under general demand function. That is,*

$$q_1 > q_2 > \cdots > q_i > q_{i+1} > \cdots > q_{n-1} > q_n. \quad (16)$$

Proof. The result is immediately derived by comparing the two adjoining first-order conditions between firm i and $i + 1$. The margins of firm i and $i + 1$ are by f.o.c. as follows:

$$P - c = -P' \prod_{j=i+1}^n (1 + r'_j) q_i = -P' \prod_{j=i+2}^n (1 + r'_j) q_{i+1}. \quad (17)$$

By (17), the following equation is satisfied:

$$(1 + r'_{i+1}) q_i = q_{i+1} \quad \forall i. \quad (18)$$

By (18) and $r'_{i+1} < 0$, $q_i > (1 + r'_{i+1}) q_i = q_{i+1}$ is satisfied. \square

Second, the following proposition can be obtained with regard to the firms' profits in the HSM.

Proposition 2. *The profit of the firm moving fast is always greater than that of the firms moving late, under general demand function. That is,*

$$\pi_1 > \pi_2 > \cdots > \pi_i > \pi_{i+1} > \cdots > \pi_{n-1} > \pi_n. \quad (19)$$

Proof. The result is immediately derived from Proposition 1. By the first-order conditions of firms i and $i + 1$, (17), the profits of firm i and $i + 1$ are rewritten as follows:

$$\pi_i = (P - c)q_i = -P' \prod_{j=i+1}^n (1 + r'_j)q_i^2, \quad (20)$$

$$\pi_{i+1} = (P - c)q_{i+1} = -P' \prod_{j=i+2}^n (1 + r'_j)q_{i+1}^2. \quad (21)$$

By (20) and (21), $\pi_i > \pi_{i+1}$ if and only if $(1 + r'_{i+1})q_i^2 > q_{i+1}^2$. This inequality holds because if $(1 + r'_{i+1})q_i = q_{i+1}$, then $(1 + r'_{i+1})q_i^2 > (1 + r'_{i+1})^2 q_i^2 = q_{i+1}^2$ is satisfied by (17). \square

By Propositions 1 and 2, it is shown that the faster the order of move of the firm is, the greater the output and the profit of the firm are, regardless of demand structure. As a well-known fact, when the demand function is linear, $P'' = 0$, $r'_i = -\frac{1}{2}$ holds. Therefore, $q_{i+1} = \frac{1}{2}q_i$ and $\pi_{i+1} = \frac{1}{2}\pi_i$ hold.

3 Merger Profitability

In this section, we examine whether or not a merger is profitable under the HSM. We denote the output of firm i and the total output under the n -HSM by $q_i(n)$ and $Q(n) = \sum_{i=1}^n q_i(n)$, respectively.

Comparing the total output of the n -hierarchy with that of the $n - 1$ -hierarchy, the following proposition can be derived.

Proposition 3. (i) *The monopoly output is always less than the total output of any n -hierarchy and it is always greater than a final firm's output of any n -hierarchy. That is, $Q(1) < Q(n)$ and $q_1(1) > q_n(n) \forall n \geq 2$.*

(ii) *If and only if the total output of the n -hierarchy is always lower than that of the $(n + 1)$ -hierarchy, the output of the final firm is always greater under the n -hierarchy than under the $(n + 1)$ -hierarchy. That is, $Q(n) < Q(n + 1)$ if and only if $q_n(n) > q_{n+1}(n + 1)$.*

Proof. Rewriting the f.o.c. of firm i , (17), with regard to the numbers of hierarchies, n , the following equation is obtained:

$$P(Q(n)) + P'(Q(n)) \prod_{j=i+1}^n (1 + r'_j(n))q_i(n) - c = 0. \quad (22)$$

(i) $Q(1)$ and $Q(n) \forall n \geq 2$ satisfy the following equations. Note that when $n = 1$, $Q(1) \equiv q_1(1)$ is the monopoly output.

$$P(Q(1)) + P'(Q(1))Q(1) - c = 0, \quad (23)$$

$$P(Q(n)) + P'(Q(n))q_n(n) - c = 0. \quad (24)$$

By (24), it is satisfied that $P(Q(n)) + P'(Q(n))Q(n) - c < 0$; $Q(n) \equiv \sum_{i=1}^n q_i(n) > q_n(n)$. As $2P' + P''q_i < 0 \forall q_i$ is satisfied by the s.o.c., $Q(1) < Q(n) \forall n \geq 2$ is immediately satisfied. By the assumption that the marginal revenue is downward-sloping, $\frac{\partial MR(q_i)}{\partial Q_{-i}} \equiv \frac{\partial P(Q) + P'(Q)q_i}{\partial Q_{-i}} = P'(Q) + P''(Q)q_i < 0 \forall q_i$, $P(q_1(1)) + P'(q_1(1))q_1(1) - c = 0 > P(Q(n)) + P'(Q(n))q_1(1) - c$ is satisfied by (23). In order to satisfy (24), $q_1(1) > q_n(n)$ must be satisfied.

(ii) From (22), the outputs of the final firm n in a n -hierarchy and that of firm $n + 1$ in a $(n + 1)$ -hierarchy satisfy the following equations respectively.

$$P(Q(n)) + P'(Q(n))q_n(n) - c = 0, \quad (25)$$

$$P(Q(n + 1)) + P'(Q(n + 1))q_{n+1}(n + 1) - c = 0. \quad (26)$$

First, we show that if $Q(n) < Q(n + 1)$, then $q_n(n) > q_{n+1}(n + 1)$. Under the downward-sloping marginal revenue, $P'(Q) + P''(Q)q_i < 0 \forall q_i$, if $Q(n) < Q(n + 1)$, $P(Q(n)) + P'(Q(n))q_n(n) - c = 0 > P(Q(n + 1)) + P'(Q(n + 1))q_n(n) - c$ is satisfied from (25) by the assumption, $P'(Q) + P''(Q)q_i < 0 \forall q_i$. In order to satisfy (26), $q_n(n) > q_{n+1}(n + 1)$ must be satisfied by the same logic as part (i). Second, we show that if $q_n(n) > q_{n+1}(n + 1)$, then $Q(n) < Q(n + 1)$. By (25) and $P' < 0$, $P(Q(n)) + P'(Q(n))q_n(n) - c = 0 < P(Q(n)) + P'(Q(n))q_{n+1}(n + 1) - c$ is satisfied, if $q_n(n) > q_{n+1}(n + 1)$. In order to satisfy (26), $Q(n) < Q(n + 1)$ must be satisfied under the assumption, $P'(Q) + P''(Q)q_i < 0 \forall q_i$. Thus, $Q(n) < Q(n + 1) \Leftrightarrow q_n(n) > q_{n+1}(n + 1)$. \square

Part (i) in Proposition 3 implies that the monopoly output is less than the total output under multi-stage Stackelberg quantity competition and it is greater than that of the Stackelberg follower. Part (ii) implies that if and only if the output of the final firm of n -HSM is less than that of $(n + 1)$ -HSM, the total output of n -hierarchical Stackelberg model exceeds that of $(n + 1)$ -HSM.

Note that Proposition 3 does not imply that $Q(n) < Q(n + 1)$ is satisfied. Whether or not $Q(n) < Q(n + 1)$ is satisfied depends on the details of demand structure and it is indeterminate

under general demand structure. Therefore, we cannot compare $Q(n)$ with $Q(n+1)$ without further presumption.

Assuming $Q(n) < Q(n+1) \forall n$ in this section, we proceed to examine the total profits of firms. If $Q(n) < Q(n+1) \forall n$, the price in the equilibrium is always higher under n -HSM than that under $(n+1)$ -HSM by $P' < 0$, that is, $P(Q(n)) > P(Q(n+1)) \forall n$. We denote firm i 's profit and the total profits under the n -HSM by $\pi_i(n)$ and $\Pi(n) \equiv \sum_{i=1}^n \pi_i(n)$, respectively. Comparing the total profits under the n -HSM with those under the $(n+1)$ -HSM, the following proposition can be derived.

Proposition 4. *Suppose that $Q(n) < Q(n+1) \forall n$.*

(i) *The total profit of the n -hierarchy is always greater than that of the $(n+1)$ -hierarchy.*

That is, $\Pi(n) > \Pi(n+1) \forall n$.

(ii) *The profit of the final firm of the n -hierarchy is always greater than that of the $(n+1)$ -hierarchy. That is, $\pi_n(n) > \pi_{n+1}(n+1) \forall n$.*

Proof. (i) Since $\Pi(n) \equiv \sum_{i=1}^n \pi_i(n) = (P(Q(n)) - c)Q(n)$, $\arg \max_n \Pi(n) = (P(Q(n)) - c)Q(n) = 1$, because $Q(1)$ is the monopoly output. That is, the f.o.c., $P(Q(1)) + P'(Q(1))Q(1) - c = 0$, is satisfied. Since the s.o.c. $2P' + P''Q < 0$, is satisfied by assumption, the larger the total output Q grows, the smaller the total profit Π decreases. Therefore, if $Q(n) < Q(n+1)$, then $\Pi(Q(n)) > \Pi(Q(n+1))$ is satisfied.

(ii) if $Q(n) < Q(n+1)$, then $P(Q(n)) > P(Q(n+1))$. From part (ii) in Proposition 3, $q_n(n) > q_{n+1}(n+1)$ is also satisfied. Since the output of the final firm is $\pi_n(n) \equiv (P(Q(n)) - c)q_n(n)$, $\pi_n(n) > \pi_{n+1}(n+1)$ immediately follows. \square

Part (i) in Proposition 4 implies that if the total output is larger under n -HSM than that under $(n+1)$ -HSM, the total profits is also larger under n -HSM than that under $(n+1)$ -HSM, even if the equilibrium price decreases, $P(Q(n)) > P(Q(n+1))$. The reason is because the increase of total output involves more intensified competition under the HSM and ends in the decline in the whole industrial profit. See Figure 1 and Figure 2.

By rewriting $\Pi(Q(n)) > \Pi(Q(n+1))$, the following equation is obtained:

If $Q(n) < Q(n+1) \forall n$, then

$$\pi_1(1) > \pi_1(2) + \pi_2(2) > \pi_1(3) + \pi_2(3) + \pi_3(3) > \pi_1(4) + \pi_2(4) + \pi_3(4) + \pi_4(4) > \dots \dots \dots \quad (27)$$

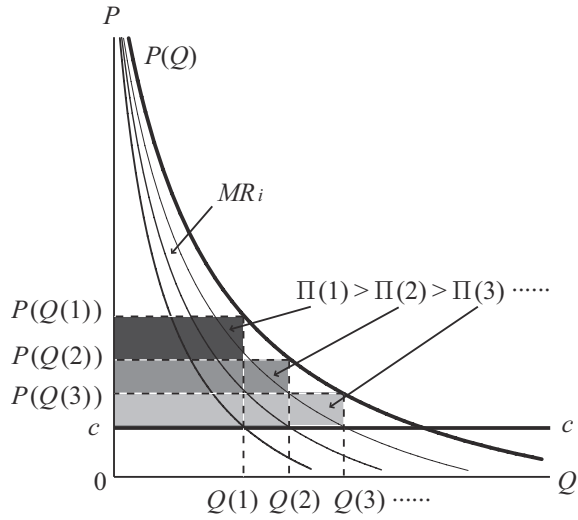


Figure 1: Total outputs and total profits under the demand curve

If $Q(n) < Q(n+1)$, then $P(Q(n)) > P(Q(n+1))$ and $\Pi(n) > \Pi(n+1)$.

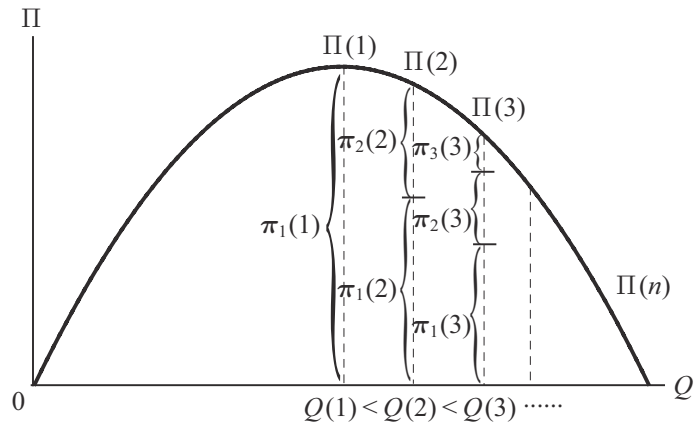


Figure 2: Total profit function with regard to total output

If $Q(n) < Q(n+1)$, then $\Pi(n) > \Pi(n+1)$ and $\pi_n(n) > \pi_{n+1}(n+1)$.

Furthermore, part (ii) in Proposition 4 implies that the final firm acquires less profit, as the number of hierarchies is larger. The reason is that as total output increases, the price becomes reduced and the residual demand of the final firm under HSM also becomes smaller. By rewriting $\pi_n(n) > \pi_{n+1}(n + 1)$, the following equation is obtained:

$$\text{If } Q(n) < Q(n + 1) \forall n, \text{ then } \pi_1(1) > \pi_2(2) > \pi_3(3) > \pi_4(4) > \dots \dots \dots \quad (28)$$

However, it is worth noting that Proposition 4 does not mention any other relationships among firms' profits. For example, we cannot say that $\pi_i(n) > \pi_i(n + 1) \forall i < n$, even if $Q(n) < Q(n + 1)$ from Proposition 4.

Now we proceed to examine whether or not a merger is profitable. Suppose that a merged firm produces in firm i 's order of move after the merger. If the profit of the merged firm is greater than the sum of profits of the two adjoining firms before the merger, it is better for two firms to choose to merge each other. In other words, if the inequality, $\pi_i(n - 1) > \pi_i(n) + \pi_{i+1}(n)$, is satisfied, mergers by adjoining firms are always profitable regardless of the form of demand function. See Figure 3.

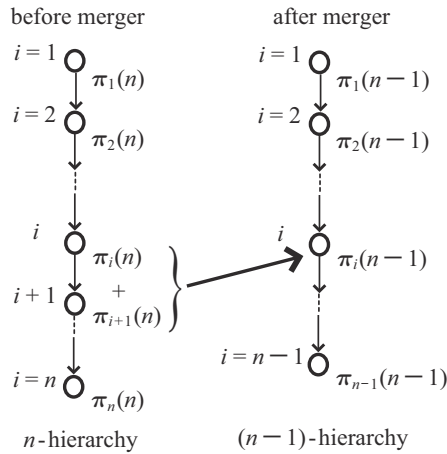


Figure 3: Firm's profit before and after merger

If $\pi_i(n - 1) > \pi_i(n) + \pi_{i+1}(n)$, the adjoining merger is profitable.

In order to compare $\pi_i(n - 1)$ with $\pi_i(n) + \pi_{i+1}(n)$, applying (17) and (18), we obtain the

following equations:

$$\begin{aligned}\pi_i(n) + \pi_{i+1}(n) &= -P'(Q(n))\left\{\prod_{j=i+1}^n (1 + r'_j(n))q_i^2(n) + \prod_{j=i+2}^n (1 + r'_j(n))q_{i+1}^2(n)\right\} \\ &= -P'(Q(n))\prod_{j=i+1}^n (1 + r'_j(n))(2 + r'_{i+1}(n))q_i^2(n).\end{aligned}\quad (29)$$

$$\pi_i(n-1) = -P'(Q(n-1))\prod_{j=i+1}^{n-1} (1 + r'_j(n-1))q_i^2(n-1).\quad (30)$$

By using the above two equations, (29) and (30), and adapting the result of Proposition 4, can we compare $\pi_i(n-1)$ with $\pi_i(n) + \pi_{i+1}(n)$? Unfortunately, we cannot make any comparison between the sum of profits of two adjoining firms before merger and firm's profit after merger under general demand structure, without further detailed assumptions. The reason why this comparison cannot be made is because the sizes of slopes of best reply functions, $r'_i(n)$, and also the output sizes of firm i in the n -hierarchy, $q_i(n)$, cannot be determined explicitly under the general demand function. As a result, any explicit result on whether or not a merger is profitable cannot be obtained.

At a first glance, this comparison seems to be easy to calculate and such result that mergers are profitable seems to be satisfied. However, it is necessary to solve a very complicated calculation and it is too hard to present the calculating result in practice. If $Q(n) < Q(n+1) \forall n$, it seems to be satisfied that $\pi_i(n-1) > \pi_i(n) + \pi_{i+1}(n)$ with regard to any firms' profits, such that that is concluded in Proposition 4. In reality, although only $\pi_1(1) > \pi_1(2) + \pi_2(2)$ is necessarily satisfied, any other relationships, for example, $\pi_1(2) > \pi_1(3) + \pi_2(3)$ and $\pi_2(2) > \pi_2(3) + \pi_3(3)$, are not necessarily satisfied such that that is shown in Figure 2. As a result, whether or not a merger is profitable, $\pi_i(n-1) > \pi_i(n) + \pi_{i+1}(n)$, cannot be shown under general demand structure.

In this section, we could not make a comparison and present a fruitful result on whether or not a merger is profitable in a HSM in the end, despite our best endeavors. In the following section, we specify the functional form of demand and answer the question on whether or not any merger is certainly profitable regardless of the curvature of demand function explicitly.

4 Specification of the Demand Function

In this section, we specify the functional form of demand, although this specification of function is accompanied with some loss of generality, and present the main result that any merger is profitable in a HSM.

Suppose that the demand function is given by $Q(P) = 1 - P^\alpha, \alpha > 0$.³ The inverse demand function is denoted by $P(Q) = (1 - Q)^{\frac{1}{\alpha}}$. The demand curve is always downward-sloping and includes to be convex ($0 < \alpha < 1$) or concave ($\alpha > 1$). If $\alpha = 1$, the demand function is linear, $P = 1 - Q$. It is assumed that the constant marginal cost is normalized as zero, $c = 0$, for analytical simplification. See Figure 4 with regard to the form of the demand functions.

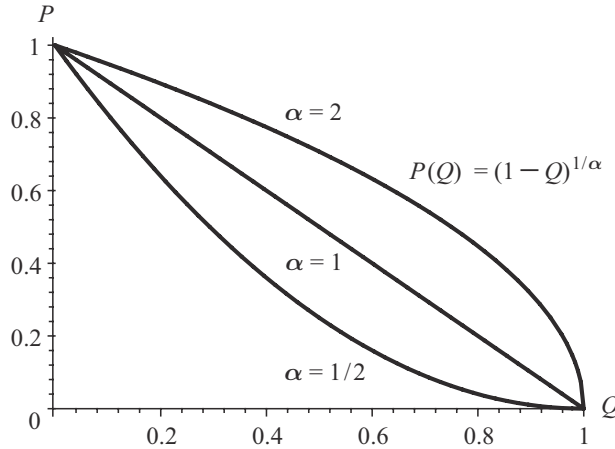


Figure 4: Specified demand function form: $P(Q) = (1 - Q)^{\frac{1}{\alpha}}$

When $\alpha = \frac{1}{2}, 1$, and 2 , the demand curve is convex, linear, and concave respectively.

Firm i 's output in the equilibrium can be determined as the solution to $\max_{q_i} P(Q_{-i} + q_i + \sum_{j=i+1}^n q_j)q_i; Q_{-i} \equiv \sum_{j=1}^{i-1} q_j, i = 1, 2, \dots, n$.

When $i = n, q_n = r_n(Q_{-n}) = \frac{\alpha(1-Q_{-n})}{1+\alpha}$ is obtained by solving the f.o.c. of the final firm, (1). Inducing backward, when $i = n - 1, q_{n-1} = r_{n-1}(Q_{-(n-1)}) = \frac{\alpha(1-Q_{-(n-1)})}{1+\alpha}$ is obtained by

³ By a change of units, we can always reduce a demand function of the form $Q = a - bP^\alpha, a > 0, b > 0$ to the form of $Q = 1 - P^\alpha$, so that our demand specification is a generalization of linear demand.

solving the f.o.c. of the penultimate firm, (5).⁴ Likewise, by calculating the f.o.c. of firm i , (14), the best reply function of firm i is obtained as follows:

$$q_i = r_i(Q_{-i}) = \frac{\alpha(1 - Q_{-i})}{1 + \alpha}; \quad Q_{-i} \equiv \sum_{j=1}^{i-1} q_j. \quad (31)$$

Q_{-i} is the output produced by all preceding firms.

Solving (31) yields

$$q_i = \frac{\alpha}{(1 + \alpha)^i}, \quad i = 1, \dots, n. \quad (32)$$

so that $Q = \sum_{i=1}^n q_i = 1 - (1 + \alpha)^{-n}$ by the sum of finite geometric progression. The Stackelberg equilibrium price is

$$P(Q) = (1 + \alpha)^{-\frac{n}{\alpha}}. \quad (33)$$

Combining (33) with (32) yields firm i 's profit as

$$\pi_i(n) = P(Q) \times q_i = \alpha(1 + \alpha)^{-\frac{n}{\alpha} - i}, \quad i = 1, \dots, n. \quad (34)$$

For the linear demand case $\alpha = 1$, the equilibrium price is 2^{-n} and output combination, $(q_1(n), q_2(n), \dots, q_i(n), \dots, q_n(n))$, are $(1/2, 1/4, \dots, 2^{-i}, \dots, 2^{-n})$. Note that $q_{i+1}(n) = \frac{1}{2}q_i(n)$. Thus, under linear demand, each firm earns half the profit of its immediate predecessor, that is, $\pi_{i+1}(n) = \frac{1}{2}\pi_i(n)$.

4.1 All Mergers Are Profitable

We examine whether or not mergers between firms are profitable under the above specification of demand curve in the HSM. Suppose that when firm i and j , $i < j$ merge in the n -hierarchy, the merged firm produces in firm i 's order of move after merger in the $(n - 1)$ -hierarchy.

We compare the sum of firms' profits before merger with the profit of the merged firm after merger. As the profits of firm i and j before merger are respectively $\pi_i(n) = \alpha(1 + \alpha)^{-\frac{n}{\alpha} - i}$ and $\pi_j(n) = \alpha(1 + \alpha)^{-\frac{n}{\alpha} - j}$, the joint profit of the pre-merged firms is $\pi_i(n) + \pi_j(n) = \alpha(1 + \alpha)^{-\frac{n}{\alpha} - i} + \alpha(1 + \alpha)^{-\frac{n}{\alpha} - j} = \alpha(1 + \alpha)^{-\frac{n}{\alpha}}((1 + \alpha)^{-i} + (1 + \alpha)^{-j}) = \alpha(1 + \alpha)^{-\frac{n}{\alpha}}(1 + \alpha)^{-j}((1 + \alpha)^{-i+j} + 1)$.

⁴ Substituting $1 + r'_n = \frac{1}{1 + \alpha}$ into (5) and arranging it, $P + P'(1 + r'_n)q_{n-1} = (1 - Q)^{\frac{1}{\alpha}} - \frac{1}{\alpha}(1 - Q)^{\frac{1}{\alpha} - 1}(1 + r'_n)q_{n-1} = 0 \Leftrightarrow \frac{1}{\alpha}(1 - Q)^{-1} \frac{1}{1 + \alpha} q_{n-1} = 1$ is obtained. Since $1 - Q = 1 - q_n - q_{n-1} - Q_{-(n-1)} = \frac{1 - q_{n-1} - Q_{-(n-1)}}{1 + \alpha}$ is obtained by substituting $q_n = \frac{\alpha(1 - Q - n)}{1 + \alpha}$, substituting $(1 - Q)^{-1}$ into the above equation, we obtain $q_{n-1} = r_{n-1}(Q_{-(n-1)}) = \frac{\alpha(1 - Q_{-(n-1)})}{1 + \alpha}$.

As the number of firms decreases by mergers, the post-merged firm corresponds to firm i in the $(n - 1)$ -hierarchy. The profit of the post-merged firm is $\pi_i(n - 1) = \alpha(1 + \alpha)^{-\frac{n-1}{\alpha}-i}$.

We obtain the following proposition by tedious calculation.

Proposition 5. *All mergers in a HSM are profitable, independent of the curvature of the demand curve. That is,*

$$\pi_i(n - 1) > \pi_i(n) + \pi_j(n). \quad (35)$$

Proof. $\pi_i(n) + \pi_j(n) < \pi_i(n - 1) \Leftrightarrow \alpha(1 + \alpha)^{-\frac{n}{\alpha}}((1 + \alpha)^{-i} + (1 + \alpha)^{-j}) < \alpha(1 + \alpha)^{-\frac{n-1}{\alpha}-i} \Leftrightarrow (1 + \alpha)^{-i} + (1 + \alpha)^{-j} < (1 + \alpha)^{\frac{1}{\alpha}-i} \Leftrightarrow 1 < (1 + \alpha)^{\frac{1}{\alpha}} - (1 + \alpha)^{i-j}$. We prove that $1 < (1 + \alpha)^{\frac{1}{\alpha}} - (1 + \alpha)^{i-j}$. When $j = i + 1$, $(1 + \alpha)^{i-j}$, $i < j$ is the maximum. Thus, it is sufficient to prove that $1 < (1 + \alpha)^{\frac{1}{\alpha}} - (1 + \alpha)^{-1}$, because $(1 + \alpha)^{-1} > (1 + \alpha)^{i-j}$, $i < j \neq i + 1$.

The following equivalence is satisfied: $1 < (1 + \alpha)^{\frac{1}{\alpha}} - (1 + \alpha)^{-1} \Leftrightarrow \frac{2+\alpha}{1+\alpha} < (1 + \alpha)^{\frac{1}{\alpha}} \Leftrightarrow 2 + \alpha < (1 + \alpha)^{\frac{1}{\alpha}+1} \Leftrightarrow \ln(2 + \alpha) < \frac{1+\alpha}{\alpha} \ln(1 + \alpha)$.

Let us define $\beta \equiv \frac{1}{\alpha} > 0$ ($\frac{1}{\beta} = \alpha$). We aim to prove that the inequality, $\ln(2 + \alpha) < \frac{1+\alpha}{\alpha} \ln(1 + \alpha) \Leftrightarrow \ln(2 + \frac{1}{\beta}) < (1 + \beta) \ln(1 + \frac{1}{\beta})$, is satisfied. It is defined that $f(\beta) \equiv (1 + \beta) \ln(1 + \frac{1}{\beta}) - \ln(2 + \frac{1}{\beta})$. If $f(\beta) > 0$ is satisfied, then $1 < (1 + \alpha)^{\frac{1}{\alpha}} - (1 + \alpha)^{-1}$ is satisfied.

Since $f(\beta) = \ln \frac{(1+\beta)^{1+\beta}}{\beta^\beta(1+2\beta)} = (1 + \beta) \ln(1 + \beta) - \beta \ln \beta - \ln(1 + 2\beta)$, the limit as β approaches 0 is $\lim_{\beta \rightarrow 0} f(\beta) = 1 \cdot 0 - \lim_{\beta \rightarrow 0} (\beta \ln \beta) - 0 = 0$, because $\lim_{\beta \rightarrow 0} (\beta \ln \beta) = \lim_{\beta \rightarrow 0} (\ln \beta^\beta) = \ln 0^0 = \ln 1 = 0$. Note that β approaches 0 if and only if α approaches $+\infty$. The limit as β approaches $+\infty$ is $\lim_{\beta \rightarrow +\infty} f(\beta) = \lim_{\beta \rightarrow +\infty} \ln(\frac{1+\beta}{\beta})^\beta - \ln(1 + \frac{1}{\beta}) = \ln(\exp) - \ln 2 = 1 - \ln 2 > 0$. Note that β approaches $+\infty$ if and only if α approaches 0.

By tedious calculation, $f'(\beta) = \ln(1 + \beta) - \ln \beta - \frac{2}{1+2\beta}$ and $f''(\beta) = -\frac{1}{\beta(1+\beta)(1+2\beta)^2} < 0$ are satisfied. As $\lim_{\beta \rightarrow 0} f'(\beta) = +\infty$ and $\lim_{\beta \rightarrow +\infty} f'(\beta) = 0$, when $\beta > 0$, it is always satisfied that $f'(\beta) > 0$. Therefore, $f(\beta) > 0$ and $1 < (1 + \alpha)^{\frac{1}{\alpha}} - (1 + \alpha)^{-1}$ are satisfied. As a result, $\pi_i(n - 1) > \pi_i(n) + \pi_j(n)$. \square

Proposition 5 implies that any merger in a HSM is profitable, regardless of the curvature of the demand curve. Although the demand function is specified, the result is applied to broad classes of demand functions including linear demand. Thus, we show that although the intuition that mergers are profitable in the HSM requires some conditions with loss of generality, this intuition is true under broad classes of demand functions.

5 Concluding Remarks

This paper attempted to show that mergers always yield profits in a HSM, regardless of forms of demand function. Under the general demand function, general results on whether or not mergers are profitable has not been obtained, although such result that any merger is profitable has been commonly recognized under the linear demand function.

The reason why mergers are profitable under the linear demand in the HSM is as follows: In the linear Stackelberg model, each firm behaves as the monopolist that takes over the residual demand from the firms whose orders of move are faster. Therefore, the output of each firm is independent of the number of successor firms in the HSM. As merger decreases the number of firms and increases the price, the profit of a merged firm necessarily exceeds the sum of the profits of the pre-merged firms. Unfortunately, a significant result could not be obtained under general demand function, with regard to whether or not any merger is profitable, although we would expect this result to be satisfied. Instead, we showed that any merger is profitable, regardless of any curvature of demand under a specified demand structure. This result supports the intuition that any merger is profitable in a HSM, although the result is accompanied with some loss of generality on the functional form. Moreover, our result contrasts with that of Cournot competition that is primarily shown by Salant, Switzer, and Reynolds (1983), where merger is often not profitable. In the HSM, merger always yields profits.

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