論　説

Under the Löschian competition game with Deter and New Entry

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Abstract

This paper applies the Abreu [1] strategy to spatial monopoly and Löschian entry cases to show how crucially it depends on market radius. A credible punishment requires an intriguing combination of production and distance costs.

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1 Introduction

This paper is based upon the second essay in the dissertation of Naito [10]. We attempt to explore another possibility of spatial competition in a monopolistic game.

Several earlier studies have dealt with free entry in the context of spatial equilibrium models. (See Damania, D. [2] [3], Eaton, B. C. and Lispecy [4], Eaton, B.C. and M.H. Wooders [5]) However, the studies by Eaton, B. C. and Lispecy [4], Eaton, B.C. and M.H. Wooders [5] do not use a game theoretical approach, and are based on the work of Hotelling [6]. The original Hotelling model has a two-firm locational choice while Eaton, B. C. and Lipsecy [4], Eaton, B.C. and M.H. Wooders [5] consider multiple-firm models. All these models assume a horizontal infinite line in order to avoid endpoint problems. In contrast, We assume the circumference of a circle to avoid the endpoint problem. The advantage of a circle model is its simple applicability to the taste space. Damania, D. [2] uses this assumption along with a linear demand function in his game theoretic approach, but he assumes [3] a perfectly inelastic demand function. Both models adopt the so-called "trigger strategy" in a collusion game. We point out the flaw in Damania, D. [3]: there is a solution for the collusion price despite the fact that he assumes a perfectly inelastic demand function. In order to avoid this flaw, We assume a linear demand and a "carrot and stick strategy" instead of a "trigger strategy". [1]

This paper is organized as follows. Section 2 sets up a spatial monopoly model with one product and a market with radius γ. Section 3 deals with a imperfect and complete information game, and then following section considers monopolistic game.

2 The Model of a Single Firm.

We consider the conditions that a monopoly market occurs without any strategic behavior in this section. We assume that there is a loop representing the circumference of a circle which has radius γ. Hence, the length of the loop is $2\pi\gamma$. Consumers are uniformly distributed, with density D on the loop. There exists a firm on the loop. Another firm may enter the market. On the loop, each of consumer must travel to the firm to buy the commodity, incurring a cost equal to a positive constant freight rate t times the distance x between the consumer's location and the firm.

For simplicity, we suppose that the individual commodity demand func-

[1] See Abreu [1] for more details.

tion is given by

$$q_x = \frac{a-m-tx}{b} \qquad \forall x \in \left(0, \frac{a-m}{t}\right),$$

where q_x is the quantity of commodity demand at x, m the f.o.b. mill price, t the freight rate, and a and b positive constants. The aggregate spatial demand function can be derived as follows;

$$Q = 2\int_0^{\frac{a-m}{t}} \frac{a-m-tx}{b}dx = \frac{(a-m)^2}{bt} \qquad for\ m \geq a - tx_0 \tag{1}$$

and

$$Q = 2\int_0^{x_0} \frac{a-m-tx}{b}dx = \frac{2(a-m)x_0 - tx_0^2}{b} \qquad for\ m < a - tx_0 \tag{2}$$

where $x_0 = \pi\gamma$ is the length of the semicircle. The former equation presents non-boundary demand function, while the latter equation shows us fixed boundary demand function. The fixed boundary means that the demand is bounded by the size of the circle. If the market size is so small, the analytically farther consumer from the firm is located beyond the half-length of the circle, that is, $\frac{a-m}{t} > x_0$. Since the consumer choose the nearest way to travel to the firm, the consumers located between x_0 and $\frac{a-m}{t}$ go another way.

The firm faces marginal cost of zero. We assume that the firm cannot produce when accumulated profit is zero. Under these conditions, we have the following Propositions.

Now we suppose that an entrant selling the homogeneous product attempts to locate at the opposite side of the existing firm. The consumers travel to the nearer firm to buy the product. Under the conditions, we have Proposition 1.

Proposition 1 *If market size γ is larger than γ_1, a monopolist firm cannot keep out newcomers, where $\gamma_1 = \frac{2a}{t\pi}$.*

Proof of Proposition 1 *According to equation(2), we obtain a profit function as follows;*

$$\pi = \frac{2m(a-m)x_0 - tmx_0^2}{b}$$

$$m^* = \frac{a}{2} - \frac{tx_0}{4}$$

Therefore, at the location of $x^* = \frac{2a}{t}$, mill price is equal to the transportation cost. The residence at x^* purchase a commodities from either an existing firm or a newcomer firm, which is indifferent. Thus, This implies $x^* = \gamma_1\pi$, which, in turn, implies

$$\gamma_1 = \frac{2a}{t\pi}. \qquad \blacksquare$$

According to the proposition 1, the market radius larger than γ_1 shows us that a monopolist may not exist. This existence depends upon strategic behavior of the firm. In the next section we solve the sub-game perfect Nash equilibrium.

3 Löschian Game

Let's consider spatial game in this section. The monopolist profit may be eroded by new entrants and can be sustained only if the firm deters further entry in the market. If the monopolist can threaten new entrants, she could keep earning the monopolist profit. The profit function for a firm is given by

$$\pi = m \cdot Q = \frac{m(a-m)^2}{bt}.$$

The monopolist price is given by

$$\frac{d\pi}{dm} = \frac{(a-m)\,(a-3m)}{bt} = 0.$$

Therefore, the monopolist price will be $m = \frac{a}{3}$.[2] Likewise, the monopolists profit π, is

$$\pi = \frac{4a^3}{27bt}.$$

This game consists of four stages. In the first stage, Nature gives us the radius of the market. In the second stage, newcomer firm decides to enter the market or not. In the third stage, a newcomer selects its location. In forth stage, the firms simultaneously choose their prices if they play Löschian competition.

This model gives us the following a imperfect, complete information game. The players basically consist of three different players: Nature, an existing monopolist firm, and a newcomer firm, while the locations of households are parametrically fixed. We present the three players as a set of $M = \{N, f_e, f_n\}$. Each player's pure strategy is given by

$$C_N = \{\gamma|\gamma \in (0,\infty)\} \;\; C_e = \{p_e|p_e \in [0,a)\}, \;\; C_n = \{\alpha|\alpha \in \{\alpha_0, \alpha_1, p_n\}\}, \qquad \forall p_n \in [0,a)$$

where α_0 is the action of non entrance to the market, and α_1 is the action of entrance to the market. A strategy profile is $C_N \times C_e \times C_n$. The expected payoff of the players are

[2] The second derivative of profit function equation is $\frac{d^2\pi}{dm^2} = \frac{-4a+6m}{bt}$. The sufficient condition requires $\frac{d^2\pi}{dm^2} < 0$, which is satisfied because $\frac{d^2\pi(\frac{a}{3})}{dm^2} < 0$.

$$\Pi_e \in \left\{ \frac{2m(a-m)x_0 - tmx_0^2}{b}, \frac{4a^3}{27bt} \right\}$$

and

$$\Pi_n \in \max \left\{ \frac{2m(a-m)\hat{x}_0 - tm\hat{x}_0^2}{b}, 0 \right\}.$$

These expected payoffs depend upon the mill price, m, and the market size, γ. Formally, a strategic form for this game is given by Γ, (see Figure 1) where

$$\Gamma = (3, C_N \times C_e \times C_n, \Pi_e, \Pi_n).$$

The sub-game perfect equilibrium of the model is solved by backward induction,with beginning of the final stage. Assume in this section that firms play Löschian competition. Each firm chooses its mill price to maximize its profits given the market border. Profit of each firm are defined as follows;

$$\pi_i = \frac{2m(a-m)x_0 - tmx_0^2}{b} \qquad \forall i = e, n.$$

Once Löschian competition carries out, newcomer enters the market until the profit goes to zero. Thus, each of profit function is given by;

$$\pi_i = \frac{2m(a-m)x_0 - tmx_0^2}{b} = 0 \qquad \forall i = e, n.$$

Then, we have;

$$x_0 = \frac{2(a-m)}{t}.$$

Therefore, n firms produce a commodity in this market, i.e.,

$$n = \frac{\gamma \pi t}{2(a-m)}.$$

Hence, the sub-game perfect equilibrium are;

$$\{Enter, the\ \ midpoint\ \ between\ \ borders, m = \frac{2a - tx_0}{4}\}$$

and

$$\{Not\ \ enter, the\ \ midpoint\ \ between\ \ borders, \frac{a}{3}\}.$$

Since the midpoint between borders is strictly dominated strategy. In the next section we examine the case in an aggressive strategy i.e., it is called, a “carrot and stick strategy” rather than “trigger strategy”. In this carrot and stick strategy, an existing firm sets a punishing price from a monopoly price, as soon as a newcomer enters.

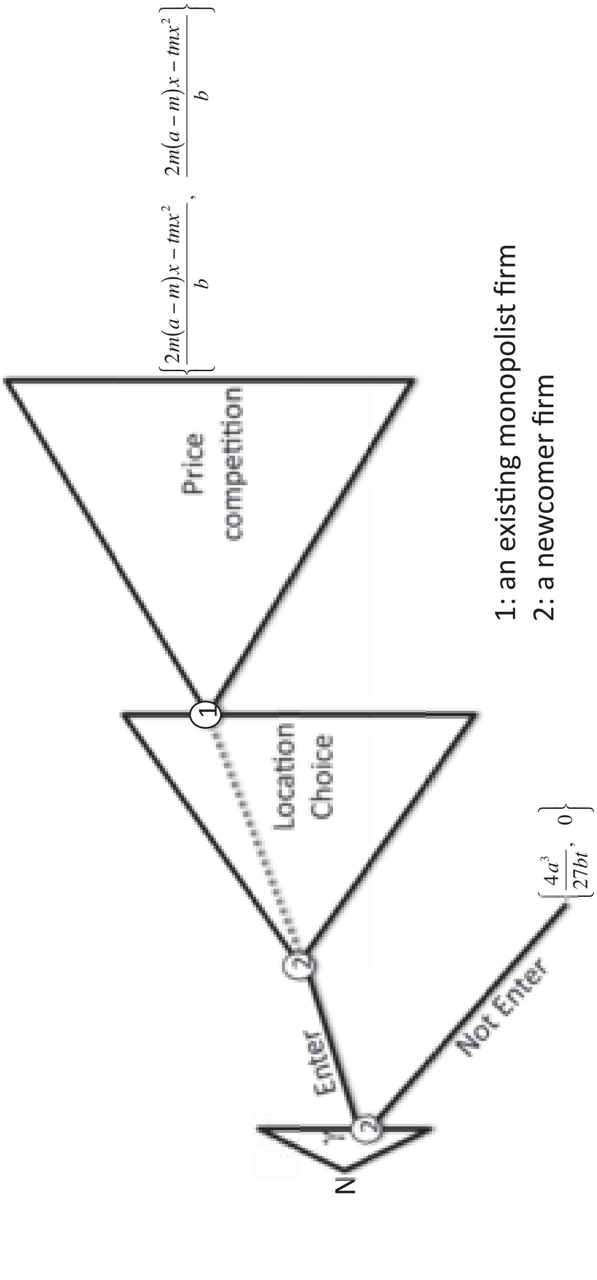

Figure 1: Löschian game

4 Monopolistic Game

We assume imperfect complete information and then will modify this assumption later as perfect incomplete information. The central issue in this game is credibility. The credible threats can influence current behavior. We apply an Abreu [1] strategy to, which is based on the idea that the most effective way to deter a firm from entering is to threaten to administer the strongest credible punishment. In the first period, an existing firm produces $\frac{(a-m)^2}{bt}$ with mill price $\frac{a}{3}$. In the t^{th} period, a monopolist firm produces $\frac{(a-m)^2}{bt}$ and charges a mill price of $\frac{a}{3}$ if a newcomer firm does not enter the market in period $t-1$, and otherwise charges a zero mill price. This price policy is a punishment phase. Once an entering firm chooses α_1 strategy, the punishment phase begins.

Let us evaluate the accumulated profit function of player 1. In the second stage, the existing firm has $\frac{4a^3}{27bt} + \delta\Pi^e(x^*)$, where δ is the discount factor and x^* is $\frac{a}{2t}$. If player 2 does not enter this industry, the accumulated profit function in the third stage will be $(1+\delta)(\frac{4a^3}{27bt}) + \delta^2\Pi^e(x^*)$. As long as this accumulated profit plus $\delta^3\Pi^e(x^*, m^*)$ is positive, player 2 will not enter, where m^* is the punishment mill price. That is, in the t^{th} stage, the accumulated profit will be

$$\frac{1-\delta^{t-1}}{1-\delta}(\frac{4a^3}{27bt}) + \delta^{t-1}\Pi^e(x^*) + \delta^t\Pi^e(x^*, m^*). \tag{3}$$

Once this accumulated profit goes negative, player 2 will enter. Then, each player faces spatial competition. The accumulated profit function(3) is a function of δ.

In this game, we have a unique Perfect Nash equilibrium under the following conditions:

1. Suppose $\gamma = 0$, then, no firm exists in this industry. Thus, the payoff vector is given by $\{0, 0\}$.

2. Suppose $\gamma \in (0, \gamma_1)$. Then, the incumbent firm charges $\frac{a}{2}$ while the potential entrant does not enter. Since the proposition 1 said that the market radius, which is smaller than γ_1, assures us of existence at least a firm in this market, the payoff vector is given by
$$\{\frac{1}{1-\delta}(\frac{4a^3}{27bt}), 0\}.$$

3. Suppose
$$\gamma \in [\gamma_1, \infty).$$
If
$$\frac{1-\delta^{t-1}}{1-\delta}(\frac{4a^3}{27bt}) + \delta^{t-1}\pi^e(x^*) > 0$$

at the t^{th} stage, the payoff vector is

$$\{\frac{1}{1-\delta}(\frac{4a^3}{27bt}),0\}.$$

If

$$\frac{1-\delta^{t-1}}{1-\delta}(\frac{4a^3}{27bt})+\delta^{t-1}\pi^e(x^*)\leq 0$$

at tth stage, the payoff vector becomes

$$\{\frac{49}{512tb},\frac{49}{512tb}\}.$$

In this three-player game we can obtain unique sub-game perfect Nash equilibrium. Most of the strategies in this game are dominated.

5 Conclusion

The stability of this monopolistic equilibrium depends upon the size of the market radius. As accumulated profit shrinks to a sufficiently small number, a firm cannot punish a newcomer and thus the monopoly equilibrium breaks down. This outcome shows that monopoly price is only feasible and sustainable when a monopolistic firm can earn a sufficiently large profit in the first stage, otherwise spatial competition equilibrium is Nash.

As an extension of our model, we plan on the following for future writing. First we will consider simultaneous price determination as a game which is imperfect and either complete or incomplete. In this connection, we use sequential equilibrium analysis as it has a stronger equilibrium than does Bayesian equilibrium. [3] Second, we could consider a quasi-general equilibrium model which includes a labor market.

[3] See Myerson [9]

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