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Profitable Mergers in Cournot and Stackelberg Markets: 80 Percent Share Rule Revisited

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Abstract

In this paper, we examine the share rule for profitable mergers in the standard Cournot and Stackelberg models. We show that mergers are unprofitable unless they involve at least 80% of the firms with the same output choice timing in the industry.

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1 Introduction

In a Cournot model, some exogenous mergers reduce the joint profits of the firms that are assumed to collude. Such unexpected result was initially observed by Salant, Switzer, and Reynolds (1983) (hereafter SSR). In the Cournot oligopoly with linear demand and cost functions, SSR (1983) showed that mergers among firms that produce homogeneous goods are unprofitable unless they involve at least 80% of the firms in the industry. The robustness of their conclusion was analyzed in various frameworks. Daughety (1990) investigated the issue of profitable horizontal mergers in a Stackelberg market with homogeneous goods and complemented the conclusion of SSR (1983) in the Cournot case. Recently, in the same framework, Huck, Konrad, and Müller (2001) studied that mergers between two leaders (two followers) are profitable only if there are two leaders (two followers).[1]

In this paper, we examine the share rule for profitable mergers in the Stackelberg market as well as in the Cournot market with homogeneous goods. In spite of the recent development in the studies on mergers, comparison of such share rule in different market structures was not undertaken. Our study extends and complements the analysis of SSR (1983). We show that mergers are unprofitable unless they involve at least 80% of the firms with the same timing to choose output in the industry. Thus, the 80% share rule for profitable mergers in the Cournot case remains valid in the Stackelberg equilibrium in the sense that the mergers should include at least 80% of the leaders or the followers.

The remainder of this paper is organized as follows. In Section 2, we re-present the result of SSR (1983). Section 3 analyzes profitable mergers in the Stackelberg market. Final section

[1] Concerning papers on the profit of mergers in other settings, see also, Deneckere and Davidson (1985), Perry and Porter (1985), Kwoka (1989), and Farrell and Shapiro (1990).

concludes the paper with some remarks.

2 The result of SSR (1983): 80% share rule

We briefly re-present the result of SSR (1983) under Cournot oligopoly. Consider a market for homogeneous product with N firms. Costs are assumed to be linear and the marginal cost is denoted by c. Inverse demand function is given by $p(Q) = a - bQ$ with $Q = \sum_{i=1}^{N} q_i$ denoting total supply and q_i firm i's individual quantity. $a > c$ is assumed. Firm i's profit is $\pi_i(q_i) = (p(Q) - c)q_i$. Under N homogeneous Cournot quantity competition, the Cournot-Nash equilibrium implies that $q_i = \frac{a-c}{b(N+1)}$. This gives a total supply of $Q = \frac{N(a-c)}{b(N+1)}$ and a price minus marginal cost of $p - c = \frac{a-c}{N+1}$. The profit of each firm can be written as

$$\pi^C(N) = \frac{(a-c)^2}{b(N+1)^2}. \tag{1}$$

The firms have an incentive to merge if the profit of the post-merged firm exceeds the sum of the pre-merged firms' profit. Then, the following inequality must be satisfied if the merger of $k+1$ firms is profitable under Cournot oligopoly ($k \geq 0,\ N \geq k+1$).

$$\pi^C(N-k) \geq (k+1)\pi^C(N) \Longleftrightarrow (N+1)^2 \geq (k+1)(N-k+1)^2. \tag{2}$$

From (2), SSR (1983) showed that no merger involving less than 80% of the firms in the industry would be privately profitable.

3 The share rule in the Stackelberg equilibrium

In this section, we consider the two-stage homogenous Stackelberg oligopoly. The cost and demand conditions are the same as those under the Cournot oligopoly. There are $n^L < N$

Stackelberg leaders who independently and simultaneously decide about their individual quantity. The $n^F(= N - n^L)$ Stackelberg followers decide upon their quantity after learning about the total quantity supplied by the n^L leaders. Let q^L be the quantity of the identical typical leader and q^F be the quantity of the identical typical follower. The subgame-perfect Stackelberg equilibrium implies that $(q_L, q_F) = (\frac{a-c}{b(n^L+1)}, \frac{a-c}{b(n^L+1)(n^F+1)})$. This gives the sum of leaders' and followers' quantities denoted by $Q^L = \frac{n^L(a-c)}{b(n^L+1)}$ and $Q^F = \frac{n^F(a-c)}{b(n^L+1)(n^F+1)}$. The total supply is $Q = \frac{(n^Ln^F+n^L+n^F)(a-c)}{b(n^L+1)(n^F+1)}$ and subtracting marginal cost from the price, we have $p - c = \frac{a-c}{(n^L+1)(n^F+1)}$. Then, the profits of a leader and a follower can be written as

$$\pi^L(n^L, n^F) = \frac{(a-c)^2}{b(n^L+1)^2(n^F+1)} \text{ and } \pi^F(n^L, n^F) = \frac{(a-c)^2}{b(n^L+1)^2(n^F+1)^2}. \tag{3}$$

To provide an incentive for $k+1$ leader firms to merge, the following inequality must be satisfied.

$$\pi^L(n^L - k, n^F) \geq (k+1)\pi^L(n^L, n^F) \Longleftrightarrow (n^L+1)^2 \geq (k+1)(n^L - k + 1)^2. \tag{4}$$

A special feature to be noted in (4) is that the inequality does not depend on the number of the followers, n^F.

Likewise, to provide an incentive for $k+1$ follower firms to merge, the following inequality must be satisfied.

$$\pi^F(n^L, n^F - k) \geq (k+1)\pi^F(n^L, n^F) \Longleftrightarrow (n^F+1)^2 \geq (k+1)(n^F - k + 1)^2. \tag{5}$$

Note that (5) does not depend on the number of the leaders, n^L.[2]

It is important to note that the above three equations (2), (4), and (5) represent the same

[2]The share rule for profitable mergers between a leader and a follower is not examined in this paper since it was already analyzed in Huck et al. (2001). In this case, the result of profitable merger is attributed to the decrease in the number of followers after the merger.

condition except the variables on the number of the firms N, n^L, and n^F. Thus, we obtain the following proposition.

Proposition 1. *Suppose that firms face the two-stage Stackelberg quantity competition. Then, (i) the Stackelberg leaders have an incentive to merge if the market share of the merged firm with respect to the leaders exceeds 80%. Likewise, (ii) the Stackelberg followers have an incentive to merge if the market share of the merged firm with respect to the followers exceeds 80%.*

Proof. Rewriting (2) ((4),(5)), we have the necessary and sufficient condition under which the merger is profitable, i.e., $k+1 \leq n \leq k+\sqrt{k+1}; n = \{N, n^L, n^F\}$. Let us denote by $\alpha \equiv \frac{k+1}{n}$ the merged firm's share with respect to the number of firms with the same timing. Thus, the share for the profitable merger is given as follows:

$$\frac{k+1}{k+\sqrt{k+1}} \leq \alpha \leq 1. \tag{6}$$

The least share for a profitable merger is $\alpha(k) \equiv \frac{k+1}{k+\sqrt{k+1}}$. This first and second derivatives are $\alpha'(k) = \frac{1/2\sqrt{k+1}-1}{(k+\sqrt{k+1})^2}$ and $\alpha''(k) = \frac{-3k(k+1)^{-1/2}+7}{4(k+\sqrt{k+1})^3}$. Solving the first-order condition on $\alpha(k)$ with respect to k, $\alpha'(\widehat{k}) = 0$, we obtain $\widehat{k} = 3$. Because the second-order condition, $\alpha''(k) > 0$, is satisfied under any k in the neighborhood of $\widehat{k}$, $\widehat{k}$ is a local minumum value. Moreover, $\alpha(k)$ is strictly decreasing when $0 \leq k < \widehat{k}$ $(\alpha'(k) < 0)$ and it is strictly increasing when $k > \widehat{k}$ $(\alpha'(k) > 0$ respectively). Thus, $\widehat{k} = 3$ is the unique global minimum value. The minimum of the share is $\alpha(\widehat{k}) = \frac{\widehat{k}+1}{\widehat{k}+\sqrt{\widehat{k}+1}} = 0.8$. □

The minimum of the least share for a profitable merger is 80%. When there exist $N = 5$ firms in the industry and $\widehat{k}+1 = 4$ firms merge, the profitable merger occurs under the least market share. The least share function, $\alpha(k), k \geq 0$ is graphed out by Figure 1.

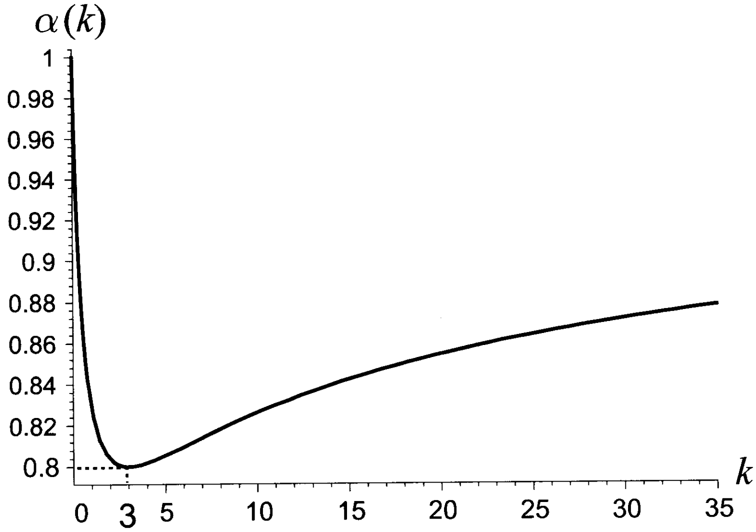

Figure 1: the least share for the profitable merger ($\alpha(k)$)

4 Concluding remarks

In the Stackelberg market as well as in the Cournot market, we showed that mergers are unprofitable unless they involve at least 80% of the firms with the same output choice timing in the industry. Note that the share rule for profitable mergers in the Stackelberg case is less than 80% if it is calculated with respect to the total number of the firms (both the leaders and the followers) in the industry. This implies that a merger between firms producing homogeneous goods is likely to be profitable in the Stackelberg market. The total output is reduced and so is the welfare by mergers in both the Stackelberg and the Cournot cases.

This result can be explained as follows. Let us consider the merger of the followers. In the

Stackelberg market, the followers decide their quantities after having already looked at the sum of the quantity of the leaders. The merger between the followers takes place if the profit of the post-merged firm exceeds the sum of the pre-merged firm under the residual demand after the leaders' output choice. This decision of the second-stage Stackelberg follower is the same as that of the Cournot case except that the demand considered is the residual one. Next, considering the merger of the leaders, the leaders decide on the merger, taking into consideration the best response on the effect that the leaders' quantities choice has on the quantities of the followers. The similar logic explains the 80% share rule among the leaders in the first-stage under the reduced demand which incorporates the effect of the best response. From this logic, we can conjecture that the 80% share rule will hold in the n-stage Stackelberg market.

We discussed the case in which the firms in the industry are identical. One of the remaining issues is the analysis of the share rule for profitable mergers when there exist asymmetric firms. The case of profitable mergers between the asymmetric firms should be examined using more general frameworks.

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