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Estimation of Economic Development Factors of Japan

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1. Introduction

Estimation of economic development factors of Japan in the period 1980-2002 with the usage of production functions is made in the article. To solve the contradictions of traditional modeling, that presupposes dependence of one and the same economic process on the type of production functions, the author shows the equivalence of production functions characteristics in case of technical progress and its absence. The equivalence of characteristics follows the solution of the problem how to choose the type of production function in modeling.

In the course of research new equivalent analogues of linear and power production functions were found, and forecast methods with the usage of production function with variable technical progress were elaborated. The mentioned forecast method lets forecast the effects stipulated by the influence of accounted and unaccounted production factors. Modeling of economic development of Japan with the help of a modified production function let estimate and determine the tendency of influence of technical progress and unaccounted production factors upon the economic growth of Japan.

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2. Analysis of Effectiveness of Japanese Production in 1980-2002

According to traditional modeling, estimation of effectiveness of Japanese production by using production functions (PF) with constant parameters is impossible without solving the problem of their selection. However, due to the equivalence of characteristics of PF there is not any need in such selection, because parameters values of one function let determine the values of the other [2-3].

PF with linear degree of homogeneity are often used in modeling due to the fact that statistical problems do not let estimate the production function's parameters of arbitrary degree of homogeneity. It causes the necessity to elaborate methods how to modify production function with linear degree of homogeneity.

The parameters in traditional PF - linear function

$$Y=A^{*}+a\cdot K+b\cdot L \tag{1}$$

and Cobb-Douglas' power function

$$Y=A\cdot K^{\alpha}\cdot L^{\beta} \tag{2}$$

are assumed to be constant during the analyzed period.

Comparison of parameters of functions (1) and (2) is impossible due to the fact that marginal efficiencies of factors a and b of linear PF are dimensional values and the elasticity of α and β of power PF are relative values, dimensionless ones.

The parameters of functions (1) and (2) will be commensurable if we estimate characteristics of a linear function by formulas:

$$\alpha^{*}=a\cdot\frac{\sum_{i=1}^{n}K_i}{\sum_{i=1}^{n}Y_i}=a\cdot\frac{\bar{K}}{\bar{Y}},\ \beta^{*}=b\cdot\frac{\sum_{i=1}^{n}L_i}{\sum_{i=1}^{n}Y_i}=b\cdot\frac{\bar{L}}{\bar{Y}},\ v^{*}=(\alpha^{*}+\beta^{*}),\ A^{*}=(1-v^{*})\cdot\bar{Y}, \tag{3}$$

$$\bar{Y}=\frac{\sum_{i=1}^{n}Y_i}{n},\ \bar{K}=\frac{\sum_{i=1}^{n}K_i}{n},\ L=\frac{\sum_{i=1}^{n}L_i}{n}.$$

If the degree of function's homogeneity equals one

$$v^{*}=(\alpha^{*}+\beta^{*})=1, \tag{4}$$

$$A^{*}=(1-v^{*})\cdot\bar{Y}=0. \tag{5}$$

Consequently, PF (1) and (2) with regard to (4) and (5) are modified into:

$$Y=a\cdot K+b\cdot L; \tag{6}$$

$$Y=A\cdot K^{\alpha}\cdot L^{1-\alpha}; \tag{7}$$

$$\frac{Y}{L}=a\cdot\frac{K}{L}+b; \tag{8}$$

$$\frac{Y}{L}=A\cdot\left(\frac{K}{L}\right)^{\alpha}. \tag{9}$$

Elasticity coefficients of factors of linear homogeneous functions (6), (8) and (9) are estimated by formulas:

$$\alpha^{*}=a\cdot\frac{K}{Y},\ \beta^{*}=b\cdot\frac{L}{Y}=1-\alpha^{*}. \tag{10}$$

Table 1. Static Production Functions of Japan (1) and (2)

Period	$Y=A^{*}+a\cdot K+b\cdot L$					$Y=A\cdot K^{\alpha}\cdot L^{\beta}$		
	A^{*}	a	b	$\alpha^{*}=a\cdot\frac{K}{Y}$	$\beta^{*}=b\cdot\frac{L}{Y}$	lnA	α	β
1980-1996	-573540.2	0.776	148.077	0.221	2.166	-8.14	0.252	2.081
1980-1997	-599056.4	0.736	153.165	0.209	2.218	-8.46	0.245	2.128
1980-1998	-637964.1	0.648	161.402	0.184	2.317	-8.99	0.228	2.212
1980-2002	-693144.7	0.747	169.525	0.209	2.355	-10.08	0.250	2.309

Table 2. Static Production Functions of Japan (6), (8) and (9) ($v=(\alpha+\beta)=1$)

Period	$\frac{Y}{L}=a\cdot\frac{K}{L}+b$			$Y=a\cdot K+b\cdot L$			$\frac{Y}{L}=A\cdot\left(\frac{K}{L}\right)^{\alpha}$	
	a	b	$\alpha^{*}=a\cdot\frac{K}{Y}$	a	b	$\alpha^{*}=a\cdot\frac{K}{Y}$	lnA	α
1980-1996	2.463	20.461	0.700	2.459	20.625	0.699	2.159	0.697
1980-1997	2.536	19.226	0.721	2.537	19.304	0.721	2.109	0.715
1980-1998	2.603	18.166	0.738	2.604	18.239	0.739	2.063	0.731
1980-2002	2.900	13.450	0.813	2.895	13.646	0.811	1.864	0.803

The comparison of factor coefficients of elasticity of functions (1) and (2) as well as (6), (8) and (9) using the formulas (3) and (10) testifies to their proximity. For example, parameters α^*=0.209 и β^*=2.355 of PF (1) of Japan in 1980-2002 are in proximity with the analogous parameters α=0.250 and β=2.309 of the function (2) (table 1). The proximity of characteristics of linear and power PF intensifies when we deal with linear homogeneous functions (8) and (9): α^*=0.813; α= 0.803 (table 2).

The difference in characteristics of linear and power PF is stipulated by different hypothesis regarding the constancy of the functions' parameters. Thus, if in linear function (1) the marginal efficiencies of factors a and b are assumed to be constant, in power function (2) the elasticity coefficients α and β are assumed to be constant.

In fact, all characteristics of power PF are variable as a result of technical progress' and unaccounted factors' influence:

$$Y_t = A_t \cdot K_t^{\alpha(t)} \cdot L_t^{\beta(t)}. \tag{11}$$

Conclusion on the equivalence of production functions' characteristics with variable parameters results from the divergence of the corresponding parameters of static PF caused by the difference in the hypotheses on the constancy of these functions' parameters. Really, if PF with variable parameters describe one and the same economic process, the corresponding characteristics of the functions must coincide irrespective of production function's type.

The equivalence of production functions' characteristics with variable parameters on the basis of the estimates of variable parameters A_t, $\alpha_t = \alpha(t)$, $\beta_t = \beta(t)$ of power function (11) lets estimate variable parameters of a linear function

$$Y_t = A_t^* + a_t \cdot K_t + b_t \cdot L_t \tag{12}$$

by formulas:

$$a_t = \alpha_t \cdot \frac{Y_t}{K_t},\ b_t = \beta_t \cdot \frac{Y_t}{L_t},\ A_t^* = (1 - v_t) \cdot Y_t,\ v_t = (\alpha_t + \beta_t). \tag{13}$$

For PF (11) and (12) the elasticity of substitution $\sigma^{(t)}$ and the marginal rate of substitution h coincide and are variable.

Thus, the hypothesis on the variability of parameters lets overcome the limitation of traditional PF with constant parameters whose elasticities of substitution are not variable.

The process of function (11) estimation consists of transition into PF:

$$Y_t=A_0 \cdot K_t^{\alpha(0)} \cdot L_t^{\beta(0)} \cdot e^{\Theta(t)}, \tag{14}$$

$$e^{\Theta_t} = \frac{A_t \cdot K_t^{\alpha_t} \cdot L_t^{\beta_t}}{A_0 \cdot K_t^{\alpha_0} \cdot L_t^{\beta_0}}. \tag{15}$$

Parameters A_0, α_0, β_0 are estimated by least-squares method (MLS) from the equation which we get by exclusion of the value of unaccounted factors' influence Θ_t from the equations in absolute and relative values:

$$ln(Y_t^{(0)})=lnA_0+\alpha_0 \cdot ln(K_t^{(0)})+\beta_0 \cdot ln\ (L_t^{(0)})+\varepsilon_t; \tag{16}$$

$$Y_t^{(0)}=Y_t \cdot e^{-y(1,t)},\ K_t^{(0)}=K_t \cdot e^{-k(1,t)},\ L_t^{(0)}=L_t \cdot e^{-l(1,t)}; \tag{17}$$

$$y(1,t) = \sum_{i=1}^{t} \frac{\Delta Y_i}{Y_{i-1}},\quad k(1,t) = \sum_{i=1}^{t} \frac{\Delta K_i}{K_{i-1}},\quad l(1,t) = \sum_{i=1}^{t} \frac{\Delta L_i}{L_{i-1}}. \tag{18}$$

One of the drawbacks of the least-squares method is the instability of the estimations of the production functions' parameters accounted by this method to insignificant changes of the dynamic series' length. This drawback is overcome in the method of estimation PF (14), as due to the exclusion of unaccounted factors' influence Θ_t from the equations in absolute and relative values, the number of estimated parameters lessens and their stability increases.

In traditional modeling when selecting PF, special attention is paid to the selection of the period during which there will not be any drastic changes in technical development. If this assumption is not valid, it is often impossible to build a PF for all the period.

Modified PF (14) lets overcome the limitation of traditional modeling and build a PF for all the heterogeneous period which has both the periods of growth and decline. It is stipulated by the fact that usual least-squares method presupposes the usage of basic data *Y*, *K* and *L* without reducing them to common base year; and the method of estimation PF (14) presupposes the usage of modified variables $Y_t^{(0)}$, $K_t^{(0)}$ and $L_t^{(0)}$ which are reduced to the common base year $t=0$.

Thus, the transition from the basic data *Y*, *K* and *L* to the modified $Y_t^{(0)}$, $K_t^{(0)}$ and $L_t^{(0)}$ lets reduce the basic data to the common base year $t=0$ and build PF for heterogeneous periods which have both the periods of growth and decline.

The results of estimation of modified PF (14) and the traditional dynamic Tinbergen's PF

$$Y^{*}=A\cdot K^{\alpha}\cdot L^{\beta}\cdot e^{\lambda\cdot t} \qquad (19)$$

are given in tables 3-4.

Table 3. Dynamic Production Functions of Japan (14) and (19)

Period	$Y_t=A_0\cdot K_t^{\alpha(0)}\cdot L_t^{\beta(0)}\cdot e^{\Theta(t)}$			$Y=A\cdot K^{\alpha}\cdot L^{\beta}\cdot e^{\lambda\cdot t}$			
	lnA_0	α_0	β_0	lnA	α	β	λ
1980-1996	-10.174	0.280	2.278	3.719	0.308	0.631	0.015
1980-1997	-10.251	0.280	2.287	3.021	0.309	0.711	0.013
1980-1998	-9.911	0.281	2.247	1.768	0.307	0.858	0.012
1980-2002	-8.108	0.286	2.031	2.436	0.315	0.772	0.012

Table 4. Dynamic Homogeneous PF of Japan (14) and (19) *($v=\alpha_0+\beta_0=1$)*

Period	$Y_t=A_0\cdot K_t^{\alpha(0)}\cdot L_t^{1-\alpha(0)}\cdot e^{\Theta(t)}$		$\frac{Y_t}{L_t}=A\cdot\left(\frac{K_t}{L_t}\right)^{\alpha_0}\cdot e^{\lambda\cdot t}$		
	lnA_0	α_0	lnA	α_0	λ
1980-1996	3.072	0.348	3.208	0.303	0.014
1980-1997	3.069	0.349	3.188	0.311	0.014
1980-1998	3.071	0.348	3.147	0.326	0.013
1980-2002	3.077	0.346	3.128	0.333	0.013

The accuracy of economic analysis and forecasting depends on the right consideration of technical progress' impact on economic growth. The criterion to the adequacy of description of real economic processes by Tinbergen's dynamic function is the proximity of the dynamics of values Θ_t^* of function (14) and $\lambda\cdot t$ Tibergen's function: the closer their dynamics is, the more accurately traditional dynamic function describes actual process.

In 1980-2002 the Japanese production efficiency underwent a stable growth tendency. The highest growth rate of production efficiency was in 1980-s, their decline was since early 1990s. Strong decline in the Japanese production efficiency took place after the financial crisis in 1997, when the growth rates of production efficiency have declined almost to the zero level. Thus, by 1990 production efficiency has grown up to 124.6% of the efficiency level in 1980; in 1997 the production efficiency has reached only 112.4% relatively the level in 1990:

$$\frac{e^{\Theta(1990,*)}}{e^{\Theta(1980,*)}} = \frac{1.246}{1.000} = 1.246; \quad \frac{e^{\Theta(1997,*)}}{e^{\Theta(1990,*)}} = \frac{1.400}{1.246} = 1.124; \quad \frac{e^{\Theta(1998,*)}}{e^{\Theta(1990,*)}} = 1.126.$$

By 2000 and later on the growth rate of Japanese production efficiency has increased. Thus, by 2002 the production efficiency has reached 105.1 % relatively the level in 1997:

$$\frac{e^{\Theta(2002,*)}}{e^{\Theta(1997,*)}} = \frac{1.472}{1.400} = 1.051.$$

Consequently, for Japan in 1980-2002 the hypothesis of constant economic growth within the framework of Tinbergen's dynamic PF was valid regardless certain decline in production efficiency as a result of the financial crisis.

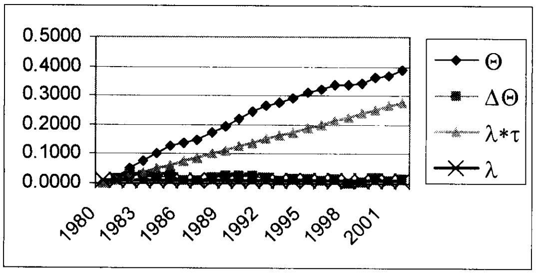

Chart 1. The Dynamics of values Θ_t^*, $\Delta\Theta_t$, λ and $\lambda{\cdot}t$ of Japan in 1980-2002.

It is proved by the proximity of values' dynamics Θ_t^* of function (14) and $\lambda{\cdot}t$ Tinbergen's function (chart 1). The exceeding of coefficients β_0 values over α_0 of the given functions testifies to the fact that change of factor L influences the economic growth more than the change of factor K (table 4).

Thus the increase in the manpower to 1% within the framework of function (14) results in GDP growth in average up to 0.654%, and the increase to 1% of fixed capital leads to GDP growth only up to 0.346%. Thus, within the framework of dynamic PF of

Japan the influence of skilled manpower changes on economic growth is bigger than changes in fixed capital influence.

3. ESTIMATION OF INTENSIVE FACTORS' CONTRIBUTION TO GDP GROWTH OF JAPAN

Parameters of function (14) corresponding to parameters of a base year t=0 of function (11) A_0, α_0, β_0, meet the ratio:

$$\alpha_0=a_0\cdot\frac{K_0}{Y_0},\ \beta_0=b_0\cdot\frac{L_0}{Y_0}. \tag{20}$$

To estimate the contribution of extensive and intensive factors we use the formula, which we got from the equation in growth rates of PF (14):

$$1=a_0\cdot\frac{K_t-K_0}{Y_t-Y_0}+b_0\cdot\frac{L_t-L_0}{Y_t-Y_0}+\frac{a_t^{(0)}-a_0}{Y_t-Y_0}\cdot K_0+\frac{b_t^{(0)}-b_0}{Y_t-Y_0}\cdot L_0+$$

$$+\frac{a_t^{(0)}-a_0}{Y_t-Y_0}\cdot(K_t-K_0)+\frac{b_t^{(0)}-b_0}{Y_t-Y_0}\cdot(L_t-L_0)+\frac{\sum_{i=1}^{t}\Delta\Theta_i\cdot Y_{i-1}}{Y_t-Y_0}, \tag{21}$$

$$K_{t,0}=K_0+\sum_{i=1}^{t}f_{i-1}^{(0)}\cdot\Delta K_i=K_{(t-1),0}+f_{t-1}^{(0)}\cdot\Delta K_t, \tag{22}$$

$$L_{t,0}=L_0+\sum_{i=1}^{t}p_{i-1}^{(0)}\cdot\Delta L_i=L_{(t-1),0}+p_{t-1}^{(0)}\cdot\Delta L_t, \tag{23}$$

$$f_t^{(0)}=\frac{Y_t}{K_t}\Big/\frac{Y_0}{K_0},\quad p_t^{(0)}=\frac{Y_t}{L_t}\Big/\frac{Y_0}{L_0}, \tag{24}$$

$$a_t^{(0)}=a_0\cdot\frac{K_{t,0}}{K_t},\ b_t^{(0)}=b_0\cdot\frac{L_{t,0}}{L_t}. \tag{25}$$

In formula (21) the first two items E_K and E_L show the contribution of factors K and L in output increase as a result of their extensive growth; the third and the fourth items $I_K(a)$ and $I_L(b)$ show the contribution of factors K and L in output increase as a result of change in their efficiency. The fifth and the sixth items $I_K(a,K)$ and $I_L(b,L)$ are indivisible remainders which were got due to the change in the values of factors and their efficiencies. Last item $I_{(unaccounted)}$ (which we denote as $I_{(un)}$) shows the contribution of unaccounted factors.

When we indicate as I_K and I_L the sum of $I_K(a)$ and $I_K(a,K)$ as well as $I_L(b)$ and $I_L(b,L)$, we get:

$$I_K = \frac{a_t^{(0)} - a_0}{Y_t - Y_0} \cdot K_0 + \frac{a_t^{(0)} - a_0}{Y_t - Y_0} \cdot (K_t - K_0) = I_K(a) + I_K(a,K), \qquad (26)$$

$$I_L = \frac{b_t^{(0)} - b_0}{Y_t - Y_0} \cdot L_0 + \frac{b_t^{(0)} - b_0}{Y_t - Y_0} \cdot (L_t - L_0) = I_L(b) + I_L(b,L), \qquad (27)$$

$$I_{(un)} = \frac{\sum_{i=1}^{t} \Delta \Theta_i \cdot Y_{i-1}}{Y_t - Y_0} = 1 - (E_K + E_L + I_K + I_L). \qquad (28)$$

Indices I_K and I_L characterize intensive contribution of factors K and L in output increase which is stipulated by both qualitative changes of K and L and their growth.

Consequently, if first two items determine E_t – the contribution of extensive factors, all the other items I_K, I_L, $I_{(un)}$ determine the contribution of all intensive factors:

$$E_t = E_K + E_L = a_0 \cdot \frac{K_t - K_0}{Y_t - Y_0} + b_0 \cdot \frac{L_t - L_0}{Y_t - Y_0}; \qquad (29)$$

$$I_t = I_K + I_L + I_{(un)} = 1 - E_t. \qquad (30)$$

Let's estimate the contribution of factors in GDP growth of Japan (table 5) according to data of function (14) for period 1980-2002 (table 4).

Table 5. Contribution of E_t Extensive and I_t Intensive Factors in GDP Growth of Japan (%)

PERIOD	E_t		I_K		I_L		$I_{(un)}$	E_t	I_K	I_L	I_t=100-E_t
	E_K	E_L	$I_K(a)$	$I_K(a,K)$	$I_L(b)$	$I_L(b,L)$					
1980-1981	22.5	18.1	0.0	0.0	0.0	0.0	59.3	40.7	0.0	0.0	59.3
1980-1982	7.1	20.9	0.0	0.0	0.2	0.0	71.8	28.0	0.0	0.2	72.0
1980-1983	-7.3	31.1	-0.6	0.0	0.7	0.0	76.0	23.8	-0.6	0.8	76.2
1980-1984	2.8	25.1	0.3	0.0	0.6	0.0	71.1	27.9	0.3	0.7	72.1
1980-1985	17.8	19.5	1.6	0.1	0.6	0.0	60.3	37.3	1.8	0.6	62.7
1980-1986	24.8	18.8	1.9	0.3	0.8	0.0	53.4	43.6	2.2	0.8	56.4
1980-1987	32.1	18.1	1.9	0.4	1.0	0.1	46.3	50.2	2.4	1.0	49.8
1980-1988	40.6	17.1	1.4	0.6	1.3	0.1	38.9	57.7	2.0	1.4	42.3
1980-1989	43.6	17.5	0.8	0.4	1.7	0.2	35.7	61.2	1.2	1.9	38.8
1980-1990	45.6	17.9	0.3	0.2	2.1	0.3	33.7	63.5	0.4	2.4	36.5
1980-1991	43.8	18.9	0.1	0.1	2.6	0.4	34.2	62.7	0.2	3.0	37.3
1980-1992	40.0	19.8	0.2	0.1	2.9	0.5	36.4	59.9	0.4	3.4	40.1
1980-1993	36.8	20.0	0.3	0.2	3.0	0.5	39.2	56.8	0.5	3.5	43.2
1980-1994	34.2	19.5	0.3	0.2	2.9	0.5	42.4	53.7	0.5	3.4	46.3
1980-1995	33.1	18.6	0.3	0.2	2.8	0.5	44.6	51.7	0.5	3.2	48.3
1980-1996	35.7	17.5	0.3	0.2	2.7	0.5	43.0	53.3	0.6	3.2	46.7
1980-1997	34.9	18.0	0.3	0.2	3.0	0.5	43.1	52.9	0.5	3.5	47.1
1980-1998	32.5	17.7	0.3	0.2	2.8	0.5	46.0	50.2	0.6	3.3	49.8
1980-1999	31.7	16.7	0.3	0.2	2.5	0.4	48.1	48.4	0.5	2.9	51.6
1980-2000	31.7	15.4	0.3	0.2	2.2	0.4	49.8	47.1	0.6	2.6	52.9
1980-2001	30.5	14.6	0.3	0.2	2.0	0.3	52.0	45.2	0.5	2.3	54.8
1980-2002	25.9	13.4	0.2	0.1	1.5	0.2	58.7	39.3	0.3	1.7	60.7

Maximum contribution of intensive factors I_t into the analyzed period of time was in early 1980s. It was to a considerable extent stipulated by the influence of technical progress and unaccounted factors $I_{(un)}$. Thus, intensive contribution I_t into GDP growth in 1980-1983 which amounted to 76.2% was 76.0% stipulated by the influence of technical progress and unaccounted factors, 0.8% by the increase in efficiency of factor L and -0.6% by the decrease of factor K efficiency.

Later on, intensive contribution of factors I_t in GDP output at first reached the lowest level of 36.5% in 1980-1990 and then underwent a stable growth tendency and reached 60.7% in 1980-2002. This level was 58.7% stipulated by the influence of technical progress and unaccounted factors $I_{(un)}$ and respectively 0.3% and 1.7% by the growth in efficiency of accounted factors K and L (table 5). Financial crisis 1997 resulted in insignificant changes of intensive factors' I_t contribution.

The excess of intensive factors' I_t contribution in the last years of the analyzed period over extensive factors E_t testifies to predominantly intensive type of Japanese economy development.

4. Methods of dynamic production function's transformation

Characteristics of PF coincide in the periods of lack and existence of technical progress. As it follows from the equation in growth rates, power PF (14) transforms into linear PF with variable influence of technical progress

$$Y_t = A_0 + a_0 \cdot K_{t,0} + b_0 \cdot L_{t,0} + \sum_{i=1}^{t} Y_{i-1} \cdot \Delta\Theta_i, \tag{31}$$

$$A_0 = Y_0 - a_0 \cdot K_0 - b_0 \cdot L_0, \tag{32}$$

and $K_{t,0}$ and $L_{t,0}$ are estimated by formulas (22) and (23).

Tinbergen's dynamic PF (19) is a special case of PF (14) with variable influence of technical progress when $\Delta\Theta_t$ are constant for all the analyzed period and equal λ:

$$\Delta\Theta_1 = \Delta\Theta_2 = \ldots = \Delta\Theta_t = \lambda. \tag{33}$$

In this case total value Θ_t will be:

$$\Theta_t = \Delta\Theta_1 + \Delta\Theta_2 + \ldots + \Delta\Theta_t = \lambda \cdot t. \tag{34}$$

Thus, power PF (14) transforms into function (19) if condition (34) is fulfilled.

On the other hand, linear PF (31) with account of PF (31) modifies into:

$$Y_t = A_0 + a_0 \cdot K_{t,0} + b_0 \cdot L_{t,0} + \lambda \cdot \sum_{i=1}^{t} Y_{i-1}. \tag{35}$$

Consequently, Tinbergen's dynamic PF (19) is transformed into dynamic linear PF (35) with constant economic growth as a result of technical progress.

If there are no qualitative changes in the production, we get

$$\Delta\Theta_1 = \Delta\Theta_2 = \ldots = \Delta\Theta_t = 0, \tag{36}$$

and values $K_{t,0} = K_t$ and $L_{t,0} = L_t$ because with invariable capital productivity ratio and productivity of labor values $f_{t-1}^{(0)}$ and $p_{t-1}^{(0)}$ equal one:

$$f_{i-1}^{(0)} = p_{i-1}^{(0)} = 1,\ i = 1, 2, \ldots, t.$$

Consequently, in the lack of qualitative changes in the production, dynamic PF (19) and (35) are transformed into static PF (1) and (2).

To compare characteristics of dynamic PF (19) and (35) we need to calculate factor elasticity of the output by formulas (20) using estimations a_0 and b_0 of dynamic linear PF.

Table 6. Dynamic Linear (35) and Power (19) Production Functions of Japan

Period	(35) $Y_t=A_0+a_0\cdot K_{t,0}+b_0\cdot L_{t,0}+\lambda\cdot\sum_{i=1}^{t} Y_{i-1}$						(19) $Y=A\cdot K^{\alpha}\cdot L^{\beta}\cdot e^{\lambda\cdot t}$			
	A_0	a_0	b_0	λ	$\alpha_0=a_0\cdot\frac{K_0}{Y_0}$	$\beta_0=b_0\cdot\frac{L_0}{Y_0}$	lnA	α	β	λ
1980- 1996	25922.99	1.14	34.56	0.014	0.321	0.611	3.72	0.308	0.631	0.015
1980-1997	4777.03	1.14	38.44	0.013	0.321	0.680	3.02	0.309	0.711	0.013
1980-1998	-38733.3	1.13	46.49	0.011	0.319	0.822	1.77	0.307	0.858	0.012
1980-2002	-973.86	1.17	39.08	0.012	0.330	0.691	2.44	0.315	0.772	0.012

Due to the fact that hypothesis of a constant economic growth as a result of technical progress is valid for Japan in 1980-2002, parameters ($\alpha_0=0.330$, $\beta_0=0.691$, $\lambda=0.012$) of a linear function (35) are close to the analogous parameters ($\alpha_0=0.315$, $\beta_0=0.772$, $\lambda=0.012$) of power function (19) (table 6).

The proximity of characteristics of dynamic PF (35) and (19) as well as for static PF increases in case of dynamic linear homogeneous PF:

$$\frac{Y_t}{L_t}=A\cdot\left(\frac{K_t}{L_t}\right)^{\alpha(0)}\cdot e^{\lambda\cdot t}, \qquad (37)$$

$$Y_t=a_0\cdot K_{t,0}+b_0\cdot L_{t,0}+\lambda\cdot\sum_{i=1}^{t} Y_{i-1}. \qquad (38)$$

Table 7. Dynamic Homogeneous PF of Japan (37) and (38) *($v=\alpha_0+\beta_0=1$)*

Period	$Y_t=a_0 \cdot K_{t,0}+b_0 \cdot L_{t,0}+\lambda \cdot \sum_{i=1}^{t} Y_{i-1}$				$\frac{Y_t}{L_t}=A \cdot \left(\frac{K_t}{L_t}\right)^{\alpha(0)} \cdot e^{\lambda \cdot t}$		
	a_0	b_0	λ	$\alpha_0=a_0 \cdot \frac{K_0}{Y_0}$	lnA	α_0	λ
1980-1996	1.110	39.680	0.013	0.313	3.208	0.303	0.014
1980-1997	1.132	39.391	0.013	0.320	3.188	0.311	0.014
1980-1998	1.185	38.654	0.012	0.334	3.147	0.326	0.013
1980-2002	1.173	38.866	0.012	0.331	3.128	0.333	0.013

Thus, parameters of functions (37) and (38) in 1980-2002 were equal (table 7): $\alpha_0=0.333$, $\lambda=0.013$; $\alpha_0=0.331$, $\lambda=0.012$.

Characteristics of dynamic linear PF (35) and (38) will be equivalent if the hypotheses of constant economic growth as a result of technical progress are valid. The slightest violation of the hypothesis increases the difference of estimations of these functions' parameters.

If we insert real indices K_t and L_t, instead of corrected indices $K_{t,0}$ and $L_{t,0}$ into functions (35) and (38) we will get:

$$Y_t=A_0+a_0 \cdot K_t+b_0 \cdot L_t+\lambda \cdot \sum_{i=1}^{t} Y_{i-1}, \qquad (39)$$

$$Y_t=a_0 \cdot K_t+b_0 \cdot L_t+\lambda \cdot \sum_{i=1}^{t} Y_{i-1}, \qquad (40)$$

Recall, that $K_{t,0}=K_t$ and $L_{t,0}=L_t$ only if the capital ratio and productivity of labor are invariable in the analyzed period, that is $f_t^{(0)}=p_t^{(0)}=1$.

Table 8. Dynamic PF of Japan *(39)* and *(19)* for case $K_{t,0}=K_t$, $L_{t,0}=L_t$

Period	$Y_t=A_0+a_0 \cdot K_t+b_0 \cdot L_t+\lambda \cdot \sum_{i=1}^{t} Y_{i-1}$						$Y=A \cdot K^{\alpha} \cdot L^{\beta} \cdot e^{\lambda \cdot t}$			
	A_0	a_0	b_0	λ	$\alpha_0=a_0 \cdot \frac{K_0}{Y_0}$	$\beta_0=b_0 \cdot \frac{L_0}{Y_0}$	lnA	α	β	λ
1980-1996	-42850.4	1.091	47.536	0.014	0.308	0.840	3.719	0.308	0.631	0.015
1980-1997	-67212.9	1.082	52.098	0.013	0.305	0.921	3.021	0.309	0.711	0.013
1980-1998	-116353.7	1.067	61.208	0.011	0.301	1.082	1.768	0.307	0.858	0.012
1980-2002	-77369.0	1.104	53.661	0.012	0.312	0.949	2.436	0.315	0.772	0.012

Table 9. Dynamic Homogeneous PF of Japan (40) and (37) for case $K_{t,0}=K_t$, $L_{t,0}=L_t$, $v=\alpha_0+\beta_0=1$

Period	$Y_t=a_0 \cdot K_t+b_0 \cdot L_t+\lambda \cdot \sum_{i=1}^{t} Y_{i-1}$				$\frac{Y_t}{L_t}=A \cdot \left(\frac{K_t}{L_t}\right)^{\alpha(0)} \cdot e^{\lambda \cdot t}$		
	a_0	b_0	λ	$\alpha_0=a_0 \cdot \frac{K_0}{Y_0}$	lnA	α_0	λ
1980-1996	1.134	39.166	0.015	0.320	3.208	0.303	0.014
1980-1997	1.157	38.881	0.014	0.327	3.188	0.311	0.014
1980-1998	1.221	37.995	0.013	0.345	3.147	0.326	0.013
1980-2002	1.276	37.254	0.013	0.360	3.128	0.333	0.013

Characteristics of linear dynamic PF (39) and (40) will be close to estimations of parameters of power dynamic PF (19) and (37) only under the clear growth tendency e^{Θ_t} of production efficiency (table 8-9). In this connection, insignificant changes of hypothesis of constant economic growth as a result of technical progress lead to greater difference in parameters of PF (39) and (40) than in case with PF (35) and (38).

5. Method of forecasting of Accounted Factors' Influence

PF (14) can be used not only in retrospective modeling but in forecasting as well. When forecasting, let's proceed from the following assumptions:

1) Parameters A_0, α_0, β_0 of PF (14) are found by retrospective modeling of the least squares method from equation (16);

2) Let's assume factors K_{t+1} and L_{t+1} to be known for the forecasting period;

3) In forecasting year $(t+1)$ let's assume the influence of unaccounted factors $\Theta_{t+1}{}^{**(unaccounted)}$ (which we denote as $\Theta_{t+1}{}^{**(un)}$) upon the economic growth to be invariable and equal $\Theta_{t+1}{}^{*(unaccounted)}$ (which we denote as $\Theta_{t+1}{}^{*(un)}$) the value of unaccounted factors' influence in retrospective year t

$$\Theta_{t+1}{}^{**(un)}=\Theta_t{}^{*(un)},\ \Delta\Theta_{t+1}{}^{**(un)}=0; \tag{41}$$

4) The efficiency of output $e^{\Theta^{**}_{t+1}}$ in forecasting year $(t+1)$ depends on the production efficiency $e^{\Theta^{*}_{t}}$ in retrospective year t and $e^{\Delta\Theta(t+1,acc)}$ the effect of joint influence upon the year $(t+1)$ only of accounted factors K_{t+1} and L_{t+1}:

$$e^{\Theta(t+1,**)}= e^{\Theta(t,*)+\Delta\Theta(t+1,acc)}; \tag{42}$$

5) Values $Y_{t+1}{}^{(accounted)}$ (which we denote as $Y_{t+1}{}^{(acc)}$) and $\Delta\Theta_{t+1}{}^{(acc)}$ at the moment $(t+1)$ are unknown.

Forecasting is possible either with the help of variable growth rates of output $\frac{\Delta Y_{t+1}}{Y_t}$ or with the help of $e^{\Delta\Theta(t+1,acc)}$ variable effects of joint influence upon the growth output of accounted factors K and L. Forecast method by $\frac{\Delta Y_{t+1}}{Y_t}$ gives analogous forecast results like the second method only in case of constant growth of factors K and L in the forecasting period. However the first method does not consider changes and forecast the process adequately, when factors K and L in the forecasting period not only increase but decrease as well. Thus, let's consider the forecast method with the help of $e^{\Delta\Theta(t+1,acc)}$ variable effects of influence on accounted factors' output growth.

When the listed assumptions are proved, PF (14) in forecasting year $(t+1)$ will be modified into:

$$Y_{t+1}{}^{(acc)}=A_0\cdot K_{t+1}{}^{\alpha(0)}\cdot L_{t+1}{}^{\beta(0)}\cdot e^{\Theta(t,*)}\cdot e^{\Delta\Theta(t+1,acc)}. \tag{43}$$

Thus, index $Y_{t+1}^{(acc)}$ characterizes the forecast quantum of output in year $(t+1)$, which is stipulated by the influence of unaccounted factors K and L.

In PF (43) as was mentioned above unknown values are $Y_{t+1}^{(acc)}$ and $\Delta\Theta_{t+1}^{(acc)}$, and known values are A_0, α_0, β_0, K_{t+1}, L_{t+1}, $e^{\Theta_t^*}$.

If we insert $Y_{t+1}^{(acc)}$ in (43) from equation in growth rates and assume that e^x approximately equals to sum (k+1) of exponential series' item

$$e^x \approx 1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots + \frac{x^k}{k!}, \tag{44}$$

we get:

$$\frac{x^k}{k!} + \ldots + \frac{x^2}{2!} + \left(\frac{D_{t+1} - 1}{D_{t+1}}\right) \cdot x - \left(W_{t+1} - 1\right) = 0; \tag{45}$$

$$x = \Delta\Theta_{t+1}^{(acc)}; \tag{46}$$

$$W_{t+1} = \frac{S_{t+1}}{D_{t+1}}; \tag{47}$$

$$S_{t+1} = 1 + \alpha_0 \cdot \frac{\Delta K_{t+1}}{K_t} + \beta_0 \cdot \frac{\Delta L_{t+1}}{L_t}; \tag{48}$$

$$D_{t+1} = \frac{A_0 \cdot K_{t+1}^{\alpha(0)} \cdot L_{t+1}^{\beta(0)} \cdot e^{\Theta(t,*)}}{Y_t}. \tag{49}$$

Thus, to find the forecast value $\Delta\Theta_{t+1}^{(acc)}$ it is necessary to solve the equation (45) regarding $x = \Delta\Theta_{t+1}^{(acc)}$.

To guarantee high accuracy of calculation it is enough to take six items from exponential series e^x:

$$\frac{x^5}{5!} + \frac{x^4}{4!} + \frac{x^3}{3!} + \frac{x^2}{2!} + \left(\frac{D_{t+1} - 1}{D_{t+1}}\right) \cdot x - \left(W_{t+1} - 1\right) = 0; \tag{50}$$

Value $\Delta\Theta_{t+1}^{(acc)}$ depends on the dynamics of accounted factors K_{t+1} and L_{t+1}. Thus, when we increase the expense of factors K_{t+1} and L_{t+1} in year $(t+1)$ regarding the expense of factors in the previous year t, we get a positive value, and when we decrease the expense of the factors, value $\Delta\Theta_{t+1}^{(acc)}$ becomes negative.

In iteration calculations it is necessary to distinguish values Θ_i and Y_i for retrospective $(i \leq t)$ and perspective $(i > t)$ periods. Let's denote these values as Θ_i^* and Y_i^* for retrospective period and $\Theta_{t+1}^{(acc)}$ and $Y_{t+1}^{(acc)}$ for perspective period.

In retrospective period (i≤t) value Θ_i^* which characterizes the influence $\Theta_i^{(un)}$ of unaccounted factors and $\Theta_i^{(acc)}$ joint influence on output growth of accounted factors K and L is estimated by formula

$$\Theta_i^* = \Theta_i^{(acc)} + \Theta_i^{(un)} = y(1,i) - (\alpha_0 \cdot k(1,i) + \beta_0 \cdot l(1,i)), \tag{51}$$

and Y_i^* is estimated by formula

$$Y_i^* = A_0 \cdot K_i^{\alpha(0)} \cdot L_i^{\beta(0)} \cdot e^{\Theta(i,*)}. \tag{52}$$

Economic development of Japan is characterized by a positive influence of accounted factors $\Theta_t^{(acc)}$. Thus, in 2002 actual level of production $Y_{2002}^{(actual)} = Y_{2002}^{(act)} = 532961.9$ exceeded potential level of production $Y_{2002}^{(acc)} = 458859.0$ (billions of 1995 yen) stipulated by the influence of accounted factors only. Consequently, due to positive influence of new unaccounted factors actual GDP level of Japan in 2002 has increased 1.161 times regarding the potential production volume.

Basing on the data of Japan in 1980-1996, let's illustrate the calculation of forecast GDP values with the help of modified PF (14) for period 1997-2002. To count the value $\Theta_t^{**(un)}$ of unaccounted factors' influence in perspective period 1997-2002 we can estimate the regression using the data $\Theta_t^{*(un)}$ of the whole retrospective period of 1980-1996 or its certain part.

Basing on the data of $\Theta_t^{*(un)}$ period we got a regression:

$$\Theta_t^{**(un)} = -33.066 + 0.017 \cdot t, \; t \in [1990;1995]. \tag{53}$$

Joint influence of accounted and unaccounted factors in forecast period of 1997-2002 is estimated by formula:

$$\Theta_t^{**} = \Theta_t^{(acc)} + \Theta_t^{**(un)}. \tag{54}$$

When we insert the estimated value of value Θ_t^{**} in formula (52) instead of Θ_t^*, we get the formula for calculation of GDP in perspective period 1997-2002:

$$Y_{t+1}^{**} = A_0 \cdot K_{t+1}^{\alpha(0)} \cdot L_{t+1}^{\beta(0)} \cdot e^{\Theta(t+1,**)}. \tag{55}$$

Calculations in table 10 show, that forecast method of PF (14) describes actual economic processes more accurately than traditional PF. The main difficulty in forecasting the PF (14) is the right selection of hypothesis for the perspective period on the dynamics of $\Theta_{t+1}^{**(un)}$ unaccounted factors' influence upon economic growth.

Table 10. Forecasting of GDP of Japan by Data of Retrospective Period 1990-1995

$(\nu = (\alpha + \beta) = 1)$

Year	*(29)* Θ_t^*	*(78)* $\Theta_t^{(acc)}$	*(80)* $\Theta_t^{*(un)}$	*(43)* $Y_t^{(acc)}$	*(53)* $\Theta_t^{**(un)} =$ -33,066+ 0.017·t	*(54)* $\Theta_t^{**} = \Theta_t^{(acc)} + \Theta_t^{**(un)}$	*(55)* Y_t^{**}	*(54)* $Y_t^* = A \cdot K^{\alpha} \cdot L^{\beta} \cdot e^{\lambda \cdot t}$ (1980-1996)	$Y_t^{(act)}$
1980	0.000	0.000	0.000	312929.9	-0.040	0.000	312929.9	316906.6	313140.1
1981	0.017	0.001	0.016	317011.7	-0.032	-0.031	307077.3	325041.1	322325.9
1982	0.041	0.006	0.035	319943.7	-0.024	-0.018	312326.8	331240.7	331236.1
1983	0.055	0.026	0.029	326782.5	-0.016	0.010	321483.0	336976.2	336575.0
1984	0.074	0.029	0.045	331832.6	-0.009	0.020	328989.2	345664.9	347072.5
1985	0.094	0.043	0.052	346586.5	-0.001	0.042	346288.2	360111.5	364712.2
1986	0.100	0.051	0.049	357614.5	0.007	0.058	360084.6	372998.7	375502.9
1987	0.106	0.063	0.043	373346.5	0.015	0.077	378847.9	389179.8	389753.2
1988	0.117	0.083	0.034	401895.3	0.022	0.105	410988.1	414308.8	416119.1
1989	0.128	0.093	0.035	422823.9	0.030	0.123	435751.8	436467.6	438135.7
1990	0.140	0.101	0.038	443522.1	0.038	0.139	460636.6	459150.3	460925.2
1991	0.153	0.101	0.051	452618.9	0.046	0.147	473739.1	475046.4	476369.4
1992	0.164	0.119	0.044	460146.1	0.053	0.173	485362.0	481707.2	480999.6
1993	0.175	0.111	0.064	452456.3	0.061	0.172	480961.2	485015.6	482190.5
1994	0.191	0.125	0.065	456610.2	0.069	0.194	489150.5	489680.0	487488.0
1995	0.207	0.127	0.080	458658.0	0.077	0.203	495164.2	497912.5	496911.5
1996	0.216	0.139	0.077	475989.7	0.084	0.224	517870.6	516146.2	513893.1
1997	0.224	0.139	0.085	480951.2	0.092	0.231	527336.8	528879.0	523421.1
1998	0.231	0.133	0.097	469521.3	0.100	0.233	518807.0	527447.5	517515.2
1999	0.239	0.133	0.106	465663.0	0.108	0.241	518544.1	530484.5	517810.6
2000	0.260	0.142	0.118	473222.1	0.115	0.257	531058.5	541429.6	532541.8
2001	0.271	0.156	0.115	476811.0	0.123	0.279	539246.1	545263.4	534851.5
2002	0.297	0.148	0.149	458859.0	0.131	0.279	522978.0	537648.3	532961.9

6. Conclusion

Transition from traditional PF with constant parameters to modified functions with variable parameters lets solve methodological problems on the equivalence of production functions' characteristics, the selection of PF and estimation of variable contribution of factors into output growth.

Methods of estimation the parameters of modified PF let reduce the number of estimated parameters and increase their stability and significance by excluding the value of unaccounted factors' influence.

Transition from basic data *Y, K* and *L* to modified data $Y_t^{(0)}$, $K_t^{(0)}$ and $L_t^{(0)}$, reduced to the common base year $t=0$, lets build a common PF for heterogeneous periods which have both the periods of growth and decline.

The accuracy of economic processes' description with the help of traditional dynamic Tinbergen's PF will be lower in comparison with the forecast method which uses modified PF (14) for heterogeneous periods which do not have clear tendency of economic growth.

Forecast method with the help of PF (14) lets estimate separately the influence of both accounted and unaccounted factors on economic growth. Usage of this method lets estimate the tendency of technical progress' and unaccounted factors' influence on economic growth of Japan.

Thanks to the positive influence of new unaccounted factors, the actual level of GDP of Japan in 2002 increased in average 1.161 times regarding the potential production volume. Application of the elaborated methodology of selection and estimation of PF will let determine the most effective variants of economic development of Japan by doing various calculations.

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