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Horizontal Merger and Merger Remedies in Antitrust Law

— A Unified Explanation of Regulatory Constraints under the Linear Demand —

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Abstract

The antitrust regulator often imposes a merger remedy on a horizontally merged firm in order to remove the anti-competitive effect of the merger. From the economic viewpoint, a merger remedy is a kind of regulatory constraint. This paper analyzes how various kinds of regulatory constraints affect the merged firm's profit in the linear demand model. We show that various forms of regulatory constraints are replaced by the simple total supply constraint. We graph the relationship between regulatory constraints and the corresponding total supply constraint.

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1 Introduction

In the drastic change to the current economic environment caused by globalization, borderless mergers and acquisitions (M&As) have tended to increase since the 1990s, and antitrust regulators have been concerned about their anticompetitive effect. In many cases, in exchange for permission for multiple firms in the same market to merge horizontally, the antitrust authority implements a merger remedy in order to remove the anti-competitive effect of the merger.

Merger remedies are structural and behavioral means available to the antitrust authority to remedy competitive detriments resulting from mergers. If the merged firm obeys merger remedies under which the same competitive conditions as before can be guaranteed, the merger is conditionally allowed by the antitrust authority.

In real accepted practices for horizontal mergers, there are many cases in which the antitrust authority imposes merger remedies. For example, the Japanese Fair Trade Commission (FTC, *Kousei-torihiki-iinkai*) has publicly revealed several issues it raised regarding mergers from 1998 to 2006. Out of a total of 27 cases, 6 merger plans were not approved and 21 cases were permitted with implementation of a merger remedy.

A representative example of the implementation of merger remedies is the merger between JAL (Japan Airlines) and JAS (Japan Air System) in the Japanese airline industry. In October 2002, an announcement of the implementation plan for management integration between JAL and JAS was presented. If this merger is realized, the three flag carriers in the Japanese airline industry will decrease to two, including another flag carrier, ANA (All Nippon Airways), and Japan's international airline market becomes a duopoly. The Japanese FTC notified both parties to the merger that it may possibly infringe on antitrust laws (*Dokusen-kinshi-hou*). In response to the notification, JAL and JAS jointly proposed the following improvement plan to the FTC in order to avoid the competition-reducing effect of the merger.

1. *To reduce the normal fares of all main airlines by a uniform 10% and to freeze price increases for at least three years*
2. *To expand the number of airline routes in which discount fares are applied by advance purchase of discount fares*

The above improvement plan consists of two merger remedies. The first remedy prevents price increases, and the second prevents the reduction of airline routes. Scrutinizing this improvement plan, the FTC judged that there was no need for apprehension about substantial competitive restriction, and the FTC approved the merger. In this M&A, although the firms devised a voluntary improvement plan to avoid competitive restrictions, the firms could never merge if they had not submitted a proper and substantial improvement plan to the FTC. Thus, it is a necessary condition for the merging firms to submit merger remedies in order to allow the FTC to integrate the firms.

In another case in the international airline industry, although not a horizontal merger, the European Commission approved the alliance between Austrian Airlines and Lufthansa by implementing merger remedies in May 2001. This was similar to the improvement plan imposed on JAL and JAS. The Commission Notice of the European Commission guides merger remedies in the EU. In the US, the Horizontal Merger Guidelines of the Department of Justice (DOJ) and Federal Trade Commission (FTC) guide merger remedies. In almost all countries where the post-merger firm carries out merger remedies, horizontal mergers that might restrict market competition are approved by the competition authority.¹

From the economic viewpoint, a merger remedy is a kind of regulatory constraint. As most actual merger remedies are status quo provisions, we can regard the merger remedy as a welfare constraint on the post-merger firm, under which the level of social welfare that existed before the merger is guaranteed. We examine how the merger approval under the welfare constraint affects the merged firm's profit in the linear demand model.

In general, the antitrust authority imposes various forms of regulatory constraints on the merged firm, because it has additional concerns besides social welfare. In the case of the merger between JAL and JAS, price regulation and quantity constraint are imposed in order to maintain airfares and service levels for passengers. As the consumer's surplus must be maintained at least the level before the merger, this merger remedy is considered as a status quo provision regarding consumer surplus. The form of the merger remedies or regulatory constraints that the competition authority implements will influence the decisions of firms that attempt to integrate as well as the merged firm's profits. We analyze how various forms of regulatory constraints affect

¹ See Motta (2004, Ch.5) for the survey and case studies on merger remedies in the US and the EU.

the merged firm's profit in the linear demand model. We show that various forms of regulatory constraints are replaced by the simple total supply constraint. We graph the relationship between the regulatory constraints and the corresponding total supply constraint.

The remainder of this article is organized as follows. Section 2 describes the model. Section 3 presents the Cournot-Nash equilibria under the pre-merger, the post-merger without any regulatory constraint, and the post-merger with the total supply constraint. In Section 3, we make the comparison between the pre-merger equilibrium and the post-merger one. Section 4 examines how various regulatory constraints affect the firm's profit and social welfare in the post-merger equilibrium. Section 5 presents the concluding remarks.

2 The Model

In this section, we present a model for a horizontal merger under the Cournot-quantity competition, using standard oligopoly theory. Suppose there is a market for a homogeneous product with n identical firms.² Firms engage in market competition in the form of the Cournot-quantity competition. When a horizontal merger occurs, the number of firms that participate in the merger is denoted by $k \in [2, n]$.

Let us specify notations. q_i denotes firm i 's individual output and $Q = \sum_{i=1}^n q_i$ is the total supply. It is assumed that the demand function and the cost function are linear. Suppose that the utility level of the representative consumer in the market is defined by $U(Q) \equiv aQ - \frac{b}{2}Q^2$; $a > 0, b > 0$. The inverse demand function is denoted by $p(Q) = a - bQ$.³

The firms have identical linear cost functions, $C_i(q_i) = cq_i$, where c is the constant marginal cost and we assume $a > c$. There is no fixed cost; that is, there is no economy of scale for firms.

Firm i 's profit is denoted by $\pi_i(q_i) = (p(Q) - c)q_i$, where $Q_{-i} \equiv \sum_{j \neq i} q_j = Q - q_i$. The consumer's surplus (CS) and the producer's surplus (PS) are defined by $CS \equiv U(Q) - p(Q)Q = \frac{b}{2}Q^2$ and $PS \equiv \sum_{i=1}^n \pi_i = (p(Q) - c)Q$, respectively. The social welfare, which is the sum of CS and PS, is denoted by $W \equiv CS + PS = (a - c)Q - \frac{b}{2}Q^2$.

² For analytical simplification, we abstract the integer problem on the number of firms from the following analysis. Although n is dealt with as a positive real number in our paper, the essential results remain unchanged even when n is limited to being an integer.

³ It is immediately satisfied that $U(Q) = \int_0^Q p(q) dq$.

3 Derivation of the Equilibrium

In subsection 3.1, the pre-merger Cournot-Nash equilibrium is derived. In subsection 3.2, the post-merger equilibrium without any regulatory constraint is derived and in subsection 3.3, the post-merger equilibrium with the total supply constraint is derived.

3.1 The pre-merger equilibrium

There are n identical firms before the merger. The first-order condition (f.o.c.) of firm $i \in \{1, \dots, n\}$, which maximizes its profit, $\pi_i(q_i; Q_{-i}) = (p(Q) - c)q_i$, is as follows:

$$\frac{\partial \pi_i(q_i; Q_{-i})}{\partial q_i} = a - bQ - c - bq_i = 0. \quad (1)$$

Equation (1) is the reaction function of firm i . As n firms are identical, the individual output level is identical, which is defined by $q \equiv q_i$; (1) is rewritten as follows:

$$a - bQ - c - bq = 0. \quad (2)$$

Multiplying (2) by n and substituting $Q = nq$ into (2), total supply is obtained:

$$Q = \frac{n(a - c)}{b(n + 1)}. \quad (3)$$

By $q = \frac{Q}{n}$, the individual output level is as follows:

$$q = \frac{a - c}{b(n + 1)}. \quad (4)$$

The pre-merger equilibrium is calculated as the Cournot-Nash equilibrium with n identical firms, which depends on the total number of firms, n . In the following, the variables in the pre-merger equilibrium are denoted by a superscript, as in $x^{pre} \equiv x(n)$. The calculation result is shown in Table 1.

3.2 The post-merger equilibrium without any regulatory constraint

If k firms participate in the merger, there are $(n - k + 1)$ identical firms after the merger under no regulatory constraint. The post-merger equilibrium with $(n - k + 1)$ identical firms is different from the pre-merger one only in the number of firms. Thus, the post-merger equilibrium without

Table 1: The pre-merger equilibrium

output	$q^{pre} \equiv q(n) = \frac{a-c}{b(n+1)}$
total supply	$Q^{pre} \equiv Q(n) = \frac{n(a-c)}{b(n+1)}$
price	$p^{pre} \equiv p(n) = \frac{a-c}{n+1} + c$
profit	$\pi^{pre} \equiv \pi(n) = \frac{(a-c)^2}{b(n+1)^2}$
consumer's surplus	$CS^{pre} \equiv CS(n) = \frac{n^2(a-c)^2}{2b(n+1)^2}$
producer's surplus	$PS^{pre} \equiv PS(n) = \frac{n(a-c)^2}{b(n+1)^2}$
social welfare	$W^{pre} \equiv W(n) = \frac{n(n+2)(a-c)^2}{2b(n+1)^2}$

any regulatory constraint can be immediately calculated by replacing the number of firms in the pre-merger equilibrium from n to $(n - k + 1)$. As regarding the number of firms in market competition, see Figure 1.

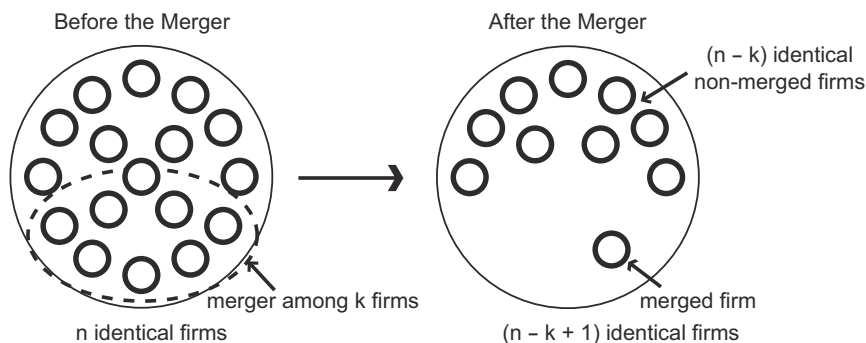


Figure 1: The number of firms before and after the merger without any regulatory constraint

We summarize the calculated result of the post-merger equilibrium without any regulatory constraint in Table 2. The variables in the post-merger equilibrium without any regulatory constraint are denoted by a superscript, as in $x^{post} \equiv x(n - k + 1)$.

We examine whether or not the horizontal merger is profitable for merging firms under no regulatory constraint. Salant, et al. (1983) first show the well-known result of the profitability of horizontal merger under the Cournot oligopoly. Hamada and Takarada (2007) reconsider this

Table 2: The post-merger equilibrium without any regulatory constraints

output	$q^{post} \equiv q(n - k + 1) = \frac{a-c}{b(n-k+2)}$
total supply	$Q^{post} \equiv Q(n - k + 1) = \frac{(n-k+1)(a-c)}{b(n-k+2)}$
price	$p^{post} \equiv p(n - k + 1) = \frac{a-c}{n-k+2} + c$
profit	$\pi^{post} \equiv \pi(n - k + 1) = \frac{(a-c)^2}{b(n-k+2)^2}$
consumer's surplus	$CS^{post} \equiv CS(n - k + 1) = \frac{(n-k+1)^2(a-c)^2}{2b(n-k+2)^2}$
producer's surplus	$PS^{post} \equiv PS(n - k + 1) = \frac{(n-k+1)(a-c)^2}{b(n-k+2)^2}$
social welfare	$W^{post} \equiv W(n - k + 1) = \frac{(n-k+1)(n-k+3)(a-c)^2}{2b(n-k+2)^2}$

result and amplify the so-called at least 80% market share rule and its derivation in detail.

The firms have an incentive to merge if the profits of the post-merger firm exceed the sum of the pre-merger firms' profits. If the merger of k firms is profitable, the following equation must be satisfied:

$$\pi(n - k + 1) \geq k\pi(n) \Leftrightarrow (n + 1)^2 \geq k(n - k + 2)^2 \Leftrightarrow n \leq k + \sqrt{k} - 1. \quad (5)$$

From (5), the following proposition is obtained.

Proposition 1. *No merger involving less than 80% of the firms in the industry is privately profitable.*

Proof. The proof is consistent with that in Hamada and Takarada (2007). (5) is the necessary and sufficient condition for a profitable merger. The market share of the merged firm which is evaluated under the pre-merger equilibrium is represented as $\frac{k}{n}$ by using the number of firms. By (5), the market share for the profitable merger must satisfy the following inequality:

$$\frac{k}{k + \sqrt{k} - 1} \leq \frac{k}{n}. \quad (6)$$

Denote the least share for a profitable merger by $s(k) \equiv \frac{k}{k + \sqrt{k} - 1}$. The first and second derivatives of $s(k)$ are $s'(k) = \frac{\frac{1}{2}\sqrt{k}-1}{(k+\sqrt{k}-1)^2}$ and $s''(k) = \frac{-3k^{-\frac{1}{2}}(k-1)+7}{4(k+\sqrt{k}-1)^3}$. Solving the first-order condition on $s(k)$ with regard to k , $s'(k) = 0$, $\widehat{k} = 4$ is obtained. Because the second-order condition, $s''(k) > 0$, is satisfied under any k in the neighborhood of \widehat{k} , \widehat{k} is a local minimum value. Moreover, $s(k)$ is strictly decreasing when $0 \leq k < \widehat{k}$ ($s'(k) < 0$) and it is strictly increasing when $k > \widehat{k}$ ($s'(k) > 0$ respectively). Thus, $\widehat{k} = 4$ is

the unique global minimum value. The minimum of the share is $s(\widehat{k}) = 4/5$, where is obtained when the number of firms is $n = 5$ and the number of merging firms is $\widehat{k} = 4$. \square

It is obvious that any horizontal merger is welfare-deteriorating. As the social welfare is $W(Q) = (a - c)Q - \frac{b}{2}Q^2$, it is satisfied that $W'(Q) = a - bQ - c = p(Q) - c > 0$ under the positive profit margin when the market is an imperfect competition. Any merger decreases with total supply because $Q(n) = \frac{n(a-c)}{b(n+1)}$ is an increasing function with regard to n and any merger decreases with the number of firms.

Therefore, without any regulatory constraint, all horizontal mergers decrease social welfare as well as consumer's surplus. From the viewpoint of antitrust law, no mergers should be permitted.

3.3 The post-merger equilibrium with the total supply constraint

Suppose that the antitrust authority imposes a total supply constraint on the post-merger firm. Under the total supply constraint, a horizontal merger is permitted by the antitrust authority if the total supply is maintained at the same level as it was before the merger.

If k firms participate in the merger, there are a merged firm and $(n - k)$ identical non-merged firms after the merger under this regulatory constraint. The merged firm is not identical with other non-merged firms because the merged firm maximizes its profit only by taking the total supply constraint into consideration. The post-merger equilibrium with the total supply constraint is different from the pre-merger one in both the number of firms and the heterogeneity of the post-merger firm. Thus, if k firms participate in a merger, the number of firms becomes $(n - k + 1)$ in the post-merger equilibrium, although the homogeneity between the merged and non-merged firms is lost after the merger. As regarding the number of firms in market competition, see Figure 2.

Since the firms are identical before the merger, suppose that in firm $i \in \{1, \dots, n\}$, firm $i \in \{1, n - k\}$ denotes non-merged firms and firm $i \in \{n - k + 1, \dots, n\}$ denotes the merger participants, without loss of generality. The merged firm and $(n - k)$ identical non-merged firms compete in a Cournot-quantity manner.

The total supply constraint that is imposed on the merged firm by the antitrust authority is as follows: $Q \geq Q^{pre}$. The merged firm solves the following profit maximization problem with

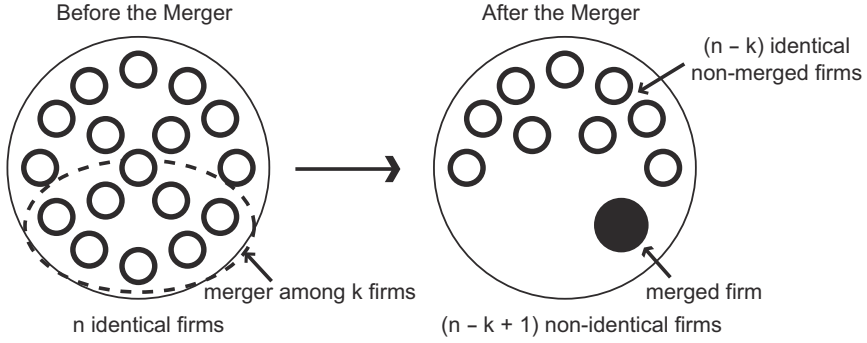


Figure 2: The number of firms before and after the merger with the total supply constraint

the welfare constraint with regard to q^m as follows:

$$\max_{q^m \geq 0} \pi^m(q^m; Q_{-m}) = (a - c - b(q^m + Q_{-m}))q^m; \quad \text{s.t.} \quad Q \geq Q^{pre} \equiv Q(n), \quad (7)$$

where $Q_{-m} \equiv \sum_{i=1}^{n-k} q_i = Q - q^m$. Total supply is $Q = q^m + Q_{-m}$.

By using the Lagrange multiplier method, the f.o.c. of the merged firm is written as follows:

$$L(q^m; \lambda) \equiv (a - c - b(q^m + Q_{-m}))q^m + \lambda((q^m + Q_{-m}) - Q^{pre}); \quad (8)$$

$$\frac{\partial L(q^m; \lambda)}{\partial q^m} = a - c - b(q^m + Q_{-m}) - bq^m + \lambda = 0, \quad (9)$$

$$\lambda(Q - Q^{pre}) = 0, \quad (10)$$

where $\lambda \geq 0$ is a Lagrange multiplier. (10) is a complementary condition.

By (9), the reaction function of the merged firm is given as follows:

$$q^m \equiv R^m(Q_{-m}) = \frac{a - c - bQ_{-m} + \lambda}{2b}. \quad (11)$$

On the other hand, the identical non-merged firm $i \in \{1, \dots, n - k\}$ solves the following profit maximization problem with regard to q_i as follows:

$$\max_{q_i \geq 0} \pi_i(q_i; Q_{-i}) = (a - c - b(q_i + Q_{-i}))q_i, \quad (12)$$

where $Q_{-i} \equiv \sum_{j \neq i, j \in \{1, \dots, n-k\}} q_j + q^m = Q - q_i$. By the f.o.c. for the non-merged firm, the reaction function of the non-merged firm is obtained as follows:

$$q_i \equiv R_i(Q_{-i}) = \frac{a - c - bQ_{-i}}{2b}. \quad (13)$$

Since $(n - k)$ non-merged firms are identical, the non-merged firm's output level is also identical. Thus, the identical output level and the sum of the non-merged firm's output are denoted as $q \equiv q_i$ and $Q_{-i} = (n - k - 1)q + q^m$, respectively. The reaction function of the representative non-merged firm to q^m is rewritten as follows:

$$q \equiv R(q^m) = \frac{a - c - bq^m}{b(n - k + 1)}. \quad (14)$$

$Q_{-m} = (n - k)q = Q_{-m} = \frac{(n-k)(a-c-bq^m)}{b(n-k+1)}$ is satisfied.

By solving simultaneous equations (11) and (14), the post-merger equilibrium with the welfare constraint is obtained as follows:

$$(q^m, q) = \left(\frac{(1 + \lambda)(a - c)}{b(n - k + 2 + \lambda)}, \frac{a - c}{b(n - k + 2 + \lambda)} \right). \quad (15)$$

Total supply is $Q = q^m + (n - k)q = \frac{(n-k+1+\lambda)(a-c)}{b(n-k+2+\lambda)}$. If the total supply constraint is binding, λ is determined as a non-negative value by the complementary condition:

$$\begin{aligned} Q = Q^{pre} &\Leftrightarrow \frac{(n - k + 1 + \lambda)(a - c)}{b(n - k + 2 + \lambda)} = \frac{n(a - c)}{b(n + 1)} \\ &\Leftrightarrow \lambda = k - 1. \end{aligned} \quad (16)$$

Substituting $\lambda = k - 1$ into (15), we obtain the post-merger equilibrium with the welfare constraint.

$$(q^m, q) = \left(\frac{k(a - c)}{b(n + 1)}, \frac{a - c}{b(n + 1)} \right). \quad (17)$$

The merged firm's output and profit are denoted as \hat{q}^m and $\hat{\pi}^m$. Other variables in the post-merger equilibrium with the total supply constraint are denoted by the superscript with a hat, as in \hat{x}^{post} .

The post-merger equilibrium with the total supply constraint is shown in Table 3.

The merged firm's output and profit are greater than those of non-merged firms, $\hat{q}^m = k\hat{q}^{post}$ and $\hat{\pi}^m = k\hat{\pi}^{post}$. As the number of merger participants increases, the merged firm's output and profit increase because \hat{q}^m is a strictly increasing function with regard to k . Moreover, if and only if $\hat{q}^m \geq \hat{Q}_{-m}^{post}$ and $\hat{\pi}^m \geq (n - k)\hat{\pi}^{post}$, $k \geq \frac{n}{2}$. Thus, if and only if the number of merger participants is larger than half of the total number of firms before the merger, the merged firm's output and profit are greater than the sum of the output and profit of the non-merged firms.

Table 3: The post-merger equilibrium with the total supply constraint

merged firm's output	$\widehat{q}^m = \frac{k(a-c)}{b(n+1)}$
non-merged firm's output	$\widehat{q}^{post} = \frac{a-c}{b(n+1)}$
total supply	$\widehat{Q}^{post} = \frac{n(a-c)}{b(n+1)}$
price	$\widehat{p}^{post} = \frac{a-c}{n+1} + c$
merged firm's profit	$\widehat{\pi}^m = \frac{k(a-c)^2}{b(n+1)^2}$
non-merged firm's profit	$\widehat{\pi}^{post} = \frac{(a-c)^2}{b(n+1)^2}$
consumer's surplus	$\widehat{CS}^{post} = \frac{n^2(a-c)^2}{2b(n+1)^2}$
producer's surplus	$\widehat{PS}^{post} = \frac{n(a-c)^2}{b(n+1)^2}$
social welfare	$\widehat{W}^{post} = \frac{n(n+2)(a-c)^2}{2b(n+1)^2}$

Note that the equilibrium variables except for the merged firm's output and profit do not depend on the number of merger participants. Under the total supply constraint, the merged firm must recover the pre-merger total supply level by its own output expansion against the output reduction by the non-merged firms. This implies that the merged firm imitates the pre-merger individual firms in order to recover the pre-merger output completely.

Now, we examine whether or not the merger is profitable under the total supply constraint. The firms have an incentive to merge if the profit of the merged firm exceeds the sum of the pre-merger firm's profit. If the merger by k firms is profitable, the following inequality must be satisfied:

$$\widehat{\pi}^m = (p(\widehat{Q}^{post}) - c)\widehat{q}^m \geq k\pi^{pre} = k(p(Q^{pre}) - c)q^{pre}. \quad (18)$$

From the above result with regard to profits, the following proposition is immediately obtained.

Proposition 2. *In the case of the total supply constraint, it is always indifferent for any firms to merge.*

Whether firms merge or not is always indifferent. If the antitrust authority imposes the total supply constraint for the merged firm to maintain the pre-merged welfare and this constraint is binding, any merger is indifferent for the merged entity and leads to the same welfare as before. This result implies that even if the merger may not be profitable, without the total supply

constraint (in a possibly realistic case in which the “at least 80% rule” is not satisfied), the firms can always avoid the unprofitable merger in a weak sense by imposing the constraint. Thus the total supply constraint imposed by the antitrust authority may enlarge the opportunity for the firms to merge, in a paradoxical way in which Salant, et al. (1983) suggest that most mergers are unprofitable.

As (18) is always satisfied with equality, whether or not to merge is always indifferent for firms under the total supply constraint. As a result, unlike the first impression, it is possible that the merged firm’s profit with the total supply constraint is greater than that without the constraint.

3.4 The comparison among the equilibria

In this subsection, we compare the pre-merger equilibrium and the post-merger equilibrium with and without the total supply constraint. The comparison is summarized in Table 4.

Table 4: The comparison among the equilibria

output	$q^{pre} = \widehat{q}^{post} < q^{post} < \widehat{q}^m$
total supply	$Q^{pre} = \widehat{Q}^{post} > Q^{post}$
price	$p^{pre} = \widehat{p}^{post} < p^{post}$
profit	$\pi^{pre} = \widehat{\pi}^{post} < \pi^{post}, \pi^{pre} < \widehat{\pi}^m$
consumer’s surplus	$CS^{pre} = \widehat{CS}^{post} > CS^{post}$
producer’s surplus	$PS^{pre} = \widehat{PS}^{post} < PS^{post}$
social welfare	$W^{pre} = \widehat{W}^{post} > W^{post}$

It is obvious that the pre-merger equilibrium is equivalent to the post-merger one with the total supply constraint. As total supply remains unchanged by the merger under this constraint, that is, $\widehat{Q}^{post} = Q^{pre}$, price, CS, PS, and welfare in the post-merger equilibrium with the total supply constraint are the same as that of the pre-merger equilibrium. As the merged firm’s output increases and the non-merged firm’s output remains unchanged, the increase of production by the merged firm just countervails the reduced production due to the decrease of

the number of firms as a result of the merger. The merged firm obtains greater profit compared with the pre-merger profit.

Comparing the post-merger equilibria with and without the total supply constraint, we obtain $q^{post} < \hat{q}^m$.⁴ The merged firm's output with the total supply constraint is greater than that without the constraint. On the contrary, the non-merged firm's output with the total supply constraint is smaller than that without the constraint. As the constraint requires $\hat{Q}^{post} = Q^{pre}$, it is immediately obtained that $Q^{post} < \hat{Q}^{post}$. By $Q^{post} < \hat{Q}^{post}$, the profit margin, CS, PS, and welfare can be easily compared.

Regarding the profit of the merged firm, if and only if $(n+1)^2 \leq k(n-k+2)^2$, $\pi^{post} \leq \hat{\pi}^m$ is obtained. Note that this condition is the converse of (5). Which profit of the merged firm is greater in both cases depends on the relative size of (n, k) . The larger the value of k , the smaller is the profit of the merged firm under the constraint. If the market share of the merged firm is greater than 80%, as shown in Proposition 1, the merged firm's profit under the constraint is smaller than that under no constraint. In other words, if many firms do not participate in merger, the merged firm prefers enforcing the antitrust authority to impose the total supply constraint to no regulation. Contrary to our first impression, it is shown that the merged firm's profit with the constraint is greater than that without the constraint.

As $\pi^{post} > \hat{\pi}^{post}$, the non-merged firm's profit necessarily decreases by imposing the total supply constraint.

4 Various forms of regulatory constraints

In this section, we examine what happens when the antitrust authority imposes various forms of regulatory constraints on the merged firm. We show that various forms of regulatory constraints can be rewritten by the total supply constraint.

4.1 Direct control of the merged firm's output or profit

Suppose that the antitrust regulator controls the merged firm's output or profit directly. If the minimum output level of the merged firm is regulated to $q \geq kq^{pre} = \frac{k(a-c)}{b(n+1)}$, the merged firm

⁴ $\hat{q}^m - q^{post} = \frac{(k-1)(n-k+1)(a-c)}{b(n+1)(n-k+2)} > 0$.

chooses the output level $\widehat{q}^{post} = kq^{pre}$ in order to maximize its profit. The total supply becomes $\widehat{Q}^{post} = Q^{pre}$ and the same post-merger equilibrium as that under the total supply constraint is replicated.

Likewise, if the regulator designates a ceiling profit level for the merged firm, by imposing $\widehat{\pi}^{post} \geq k\pi^{pre} = \frac{k(a-c)^2}{b(n+1)^2}$, the merged firm chooses the output level $\widehat{q}^{post} = kq^{pre}$ in order to maximize its profit. The total supply becomes $\widehat{Q}^{post} = Q^{pre}$ and the same post-merger equilibrium as the one that existed under the total supply constraint is replicated.

4.2 Price constraint

Suppose that the regulator imposes a price constraint on the merged firm. As $p(Q) = a - bQ$, apparently, the same post-merger equilibrium as the one that existed under the total supply constraint is replicated by imposing the price ceiling constraint, $p \leq p^{pre}$, because $p(Q) \leq p^{pre}$, if and only if $Q \geq Q^{pre}$.

4.3 CS constraint

Suppose that the regulator imposes a consumer's surplus (CS) constraint on the merged firm. In other words, when a regulator who respects only the consumers approves mergers, the post-merger CS must not fall below the pre-merger one. The CS constraint is rewritten as follows:

$$CS(Q) \geq CS^{pre} \Leftrightarrow \frac{b}{2}Q^2 \geq \frac{b}{2}(Q^{pre})^2 \Leftrightarrow Q \geq Q^{pre}. \quad (19)$$

Thus, the CS constraint is equivalent to the total supply constraint, which is analyzed in the above section. The CS constraint is graphed in Figure 3.

4.4 PS constraint

Suppose that the regulator imposes a producer's surplus (PS) constraint on the merged firm. When a regulator who respects only the producers approves mergers, the post-merger PS must not decline with respect to the pre-merger one. The PS constraint is rewritten as follows:

$$PS(Q) \geq PS^{pre} \Leftrightarrow (a-c)Q - bQ^2 \geq \frac{n(a-c)^2}{b(n+1)^2} \Leftrightarrow \frac{a-c}{b(n+1)} \leq Q \leq \frac{n(a-c)}{b(n+1)}. \quad (20)$$

The lower bound of the total supply constraint is always less than the monopoly output, $\frac{a-c}{b(n+1)} < Q(1) = \frac{a-c}{2b}$. As the PS constraint is not actually binding, the post-merger equilibrium under the PS constraint is equivalent to the post-merger equilibrium without any regulatory constraint. See Table 2 and Figure 4.

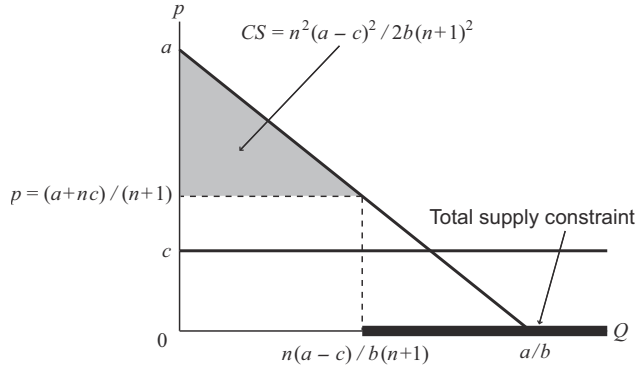


Figure 3: CS constraint

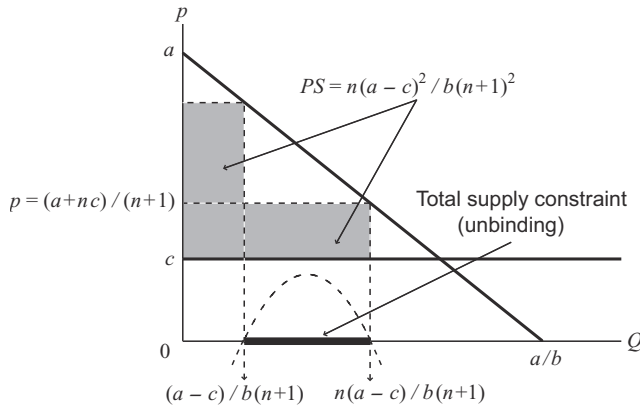


Figure 4: unbinding PS constraint

4.5 Welfare constraint

Suppose that the antitrust regulator imposes a welfare constraint on the merged firm. The post-merger social welfare must not decline with respect to the pre-merger one when a regulator who is concerned about social welfare approves mergers. The welfare constraint is rewritten as follows:

$$W(Q) \geq W^{pre} \Leftrightarrow (a-c)Q - \frac{b}{2}Q^2 \geq \frac{n(n+2)(a-c)^2}{2b(n+1)^2} \Leftrightarrow \frac{n(a-c)}{b(n+1)} \leq Q \leq \frac{(n+2)(a-c)}{b(n+1)}. \quad (21)$$

However, as the welfare is maximized at the quantity level of the perfectly competitive market, the interval of total supply that the regulator sets does not exceed the total supply under perfect competition, i.e., $Q \leq \frac{a-c}{b}$. In the interval of $0 \leq Q \leq \frac{a-c}{b}$, $W(Q)$ is a strictly increasing function for all $Q \in [0, \frac{a-c}{b})$. Thus, under the welfare $W(Q) \geq W^{pre}$, the proper interval of total supply is $\frac{n(a-c)}{b(n+1)} \leq Q \leq \frac{a-c}{b}$. Under this interval of total supply, profit margin is positive, i.e., $p(Q) - c \geq 0$. Since each firm prefers that total supply be as small as possible, the binding welfare constraint is reduced to the total supply constraint, $Q = Q^{pre}$. Thus, the profit maximization problem under the welfare constraint is the same as the one that existed under the total supply constraint. The total supply constraint is graphed in Figure 5.

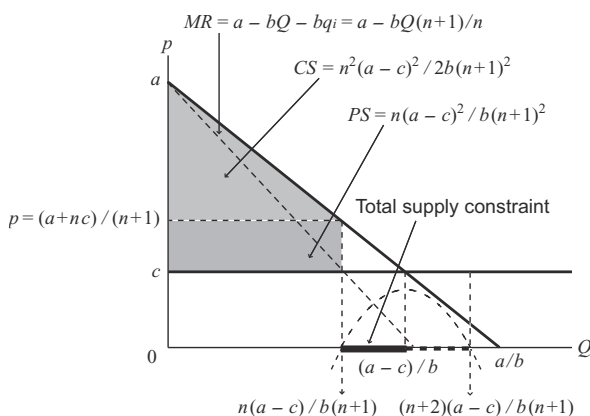


Figure 5: Welfare constraint

4.6 Generalized welfare constraint

Suppose that the regulator imposes the constraint that the post-merger welfare must exceed the $100 \times \alpha$ percentage of the welfare before the merger when it approves mergers. The generalized welfare constraint that the regulator imposes on the merged firm is denoted as follows: $W \geq \alpha W^{pre}$, where the weight, $\alpha (> 0)$, is exogenously determined by reflecting public opinion.

Note that if α is sufficiently larger than unity, there is no equilibrium under the generalized welfare constraint. If the severer welfare constraint exceeds the maximum value of welfare under the perfect competitive market, i.e., $\alpha W^{pre} \geq \max_Q W(Q)$, there is no equilibrium that satisfies the profit maximization problem under this constraint. In order that there exists an equilibrium, it is necessary that α is not sufficiently large. It is assumed that $\alpha < \frac{\max_Q W(Q)}{W^{pre}} = \frac{(n+1)^2}{n(n+2)}$. On the other hand, if α is sufficiently small, $W^{post} \geq \alpha W^{pre}$ is automatically satisfied and the constraint does not bind. As $W^{post} \geq \alpha W^{pre} \Leftrightarrow \frac{(n-k+1)(n-k+3)(a-c)^2}{2b(n-k+2)^2} \geq \alpha \frac{n(n+2)(a-c)^2}{2b(n+1)^2} \Leftrightarrow \alpha \leq \frac{(n+1)^2(n-k+1)(n-k+3)}{n(n+2)(n-k+2)^2}$, if $\alpha \leq \alpha(n, k) \equiv \frac{(n+1)^2(n-k+1)(n-k+3)}{n(n+2)(n-k+2)^2}$, the welfare constraint is unbinding.

The generalized welfare constraint can be interpreted as a total supply constraint as follows:

$$W(Q) \geq \alpha W^{pre} \Leftrightarrow (a-c)Q - \frac{b}{2}Q^2 \geq \frac{\alpha n(n+2)(a-c)^2}{2b(n+1)^2} \Leftrightarrow \underline{Q} \leq Q \leq \overline{Q}, \quad (22)$$

where $\underline{Q} \equiv \frac{(n+1-\sqrt{(n+1)^2-\alpha n(n+2)})(a-c)}{b(n+1)}$ and $\overline{Q} \equiv \frac{(n+1+\sqrt{(n+1)^2-\alpha n(n+2)})(a-c)}{b(n+1)}$.

By the similar argument of subsection 4.5, as the welfare is maximized at the quantity level of the perfectly competitive market, the proper interval of total supply is $\underline{Q} \leq Q \leq \frac{a-c}{b}$. Under this interval of total supply, the welfare constraint is reduced to the total supply constraint, $Q \geq \underline{Q}$. Thus, the profit maximization problem under the welfare constraint is the same as that under the modified total supply constraint. The total supply constraint is graphed in Figure 6.

Note that $Q^{pre} = \frac{n(a-c)}{b(n+1)} \geq \underline{Q} \Leftrightarrow \alpha \leq 1$. Thus, when $\alpha < 1$ or $\alpha > 1$, the generalized welfare constraint becomes looser or tighter, respectively, than the full-fledged welfare constraint. Under the looser welfare constraint ($\alpha < 1$), total supply is smaller than that before the merger. On the other hand, under the tighter welfare constraint ($\alpha > 1$), total supply is greater than that before the merger. The qualitative characteristics of the constraint remain unchanged.

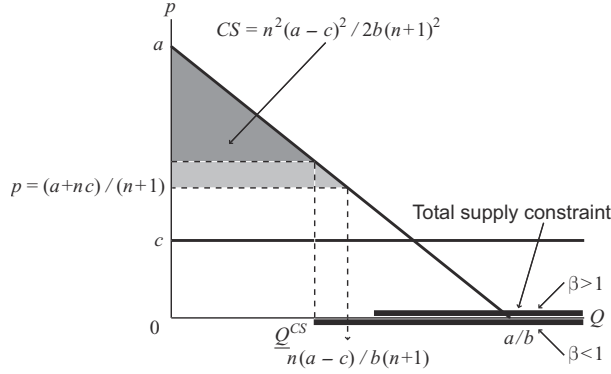


Figure 7: Generalized CS constraint

4.8 Generalized PS constraint

Suppose that the regulator imposes a generalized PS constraint on the merged firm. When the regulator who primarily interested in the effect of the merger on producers approves mergers, the post-merger PS must not decline $100 \times \gamma$ percent of the pre-merger one. The generalized PS constraint is as follows:

$$PS(Q) \geq \gamma PS^{pre} \Leftrightarrow (a-c)Q - bQ^2 \geq \frac{\gamma n(a-c)^2}{b(n+1)^2} \Leftrightarrow \underline{Q}^{PS} \leq Q \leq \overline{Q}^{PS}, \quad (24)$$

where $\underline{Q}^{PS} \equiv \frac{(n+1 - \sqrt{(n+1)^2 - 4\gamma n})(a-c)}{2b(n+1)}$ and $\overline{Q}^{PS} \equiv \frac{(n+1 + \sqrt{(n+1)^2 - 4\gamma n})(a-c)}{2b(n+1)}$.

Note that if γ is sufficiently larger than unity, there is no equilibrium under the generalized PS constraint. If a tight generalized PS constraint exceeds the monopoly profit, i.e., $\gamma \overline{PS} > PS(1)$, there is no equilibrium. In order that there exists an equilibrium, it is necessary that $\gamma \overline{PS} \leq PS(1)$, if and only if $\gamma \leq \frac{PS(1)}{\overline{PS}} = \frac{(n+1)^2}{4n}$. It is assumed that $\gamma < \frac{PS(1)}{\overline{PS}} = \frac{(n+1)^2}{4n}$.

If and only if $\gamma < 1$, $\underline{Q}^{PS} < \frac{a-c}{b(n+1)}$ and $\overline{Q}^{PS} > \frac{n(a-c)}{b(n+1)}$ are satisfied. Thus, when $\gamma < 1$ or $\gamma > 1$, the generalized PS constraint becomes looser or tighter, respectively, than the full-fledged PS constraint. When $\gamma < 1$, the generalized PS constraint is not binding and the equilibrium is the same as that without any regulatory constraint. When $\gamma > 1$, the lower bound of the total supply constraint, $\underline{Q}^{PS} < Q(1) = \frac{a-c}{2b}$, is always satisfied. As $PS(1) = \frac{(a-c)^2}{4b} > \frac{\gamma n(a-c)^2}{b(n+1)^2}$ under $\gamma < \frac{(n+1)^2}{4n}$, the generalized PS constraint is not binding if $Q(n-k+1) = \frac{(n-k+1)(a-c)}{b(n-k+2)} < \overline{Q}^{PS}$. $Q(n-k+1) = \frac{(n-k+1)(a-c)}{b(n-k+2)} < \overline{Q}^{PS}$, if and only if $\gamma < \frac{(n+1)^2(n-k+1)}{n(n-k+2)^2} (< \frac{(n+1)^2}{4n})$. Thus, only if

γ is sufficiently high, i.e., $\gamma > \frac{(n+1)^2(n-k+1)}{n(n-k+2)^2}$, the PS constraint is binding. The generalized PS constraint is rewritten by $Q = \bar{Q}^{PS}$.⁶ See also Figure 8.

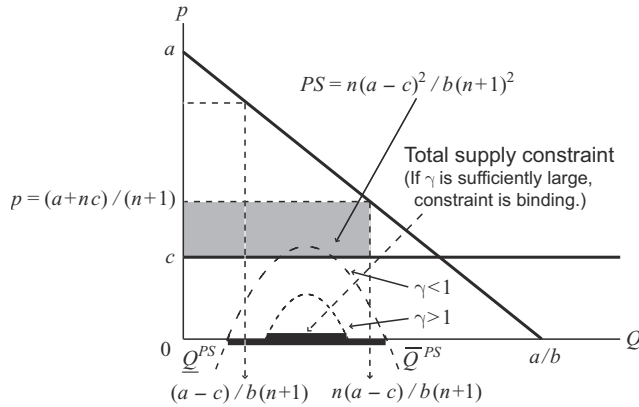


Figure 8: Generalized PS constraint

4.9 Weighted CS and PS constraints

Finally, we consider the weighted CS and PS constraints. Suppose that the objective of the antitrust authority is the average-weighted function between CS and PS. The averaged weight between CS and PS after the merger must not decline to what it was before the merger when it approves mergers. The weighted CS and PS constraints are as follows:

$$\beta CS(Q) + \gamma PS(Q) \geq \beta CS^{pre} + \gamma PS^{pre}. \tag{25}$$

When $(\beta, \gamma) = (\beta, \beta)$, this constraint is equivalent to the welfare constraint: $W(Q) \geq W^{pre}$. When $(\beta, \gamma) = (\beta, 0)$, this constraint is equivalent to the CS constraint: $CS(Q) \geq CS^{pre}$. When $(\beta, \gamma) = (0, \gamma)$, this constraint is equivalent to the PS constraint: $PS(Q) \geq PS^{pre}$. We can confine the argument to $0 \leq \beta \leq 1, \gamma = 1 - \beta$. The reason is that under any $(\beta, \gamma) \gg (0, 0)$, by redefining $\hat{\beta} \equiv \frac{\beta}{\beta+\gamma}$ and $\hat{\gamma} \equiv \frac{\gamma}{\beta+\gamma}$, $(\hat{\beta}, \hat{\gamma})$; $\hat{\beta} + \hat{\gamma} = 1$ is satisfied under the redefined constraint:

⁶However, such a situation is quite a specific and unrealistic one, because the regularity authority has the regulation policy under which total supply is required for all firms to be under-provision up to the neighborhood of monopoly output level. It seems that such a regulatory policy is hardly realistic. The situation in this subsection is only a virtual calculating example but does not have any real economic reasoning.

$\beta CS(Q) + \gamma PS(Q) \geq \beta CS^{pre} + \gamma PS^{pre} \Leftrightarrow \hat{\beta} CS(Q) + \hat{\gamma} PS(Q) \geq \hat{\beta} CS^{pre} + \hat{\gamma} PS^{pre}$. Thus, it is assumed that $0 \leq \beta \leq 1, \gamma = 1 - \beta$ w.l.o.g.

The weighted CS and PS constraints are rewritten as follows:

$$\begin{aligned} \beta CS(Q) + (1 - \beta) PS(Q) &\geq \beta CS^{pre} + (1 - \beta) PS^{pre} \\ \Leftrightarrow (1 - \beta)(a - c)Q + \frac{b}{2}(3\beta - 2)Q^2 &\geq \frac{n((n - 2)\beta + 2)(a - c)^2}{2b(n + 1)^2}. \end{aligned} \quad (26)$$

In the following, we classify (26) with regard to the relative size of β into five cases in order to examine the weighted CS and PS constraints. First, when $\beta \in (\frac{2}{3}, 1]$ (Case (i)), where the weight to CS is relatively large, (26) is rewritten as follows: $Q \leq \underline{Q}^w$ or $Q \geq \overline{Q}^w$, where $\underline{Q}^w \equiv -\frac{((n-2)\beta+2)(a-c)}{b(3\beta-2)(n+1)} < 0$ and $\overline{Q}^w \equiv \frac{n(a-c)}{b(n+1)} > 0$. As $\underline{Q}^w < 0$ and $\overline{Q}^w > 0$, the binding constraint is $Q \geq \overline{Q}^w = \frac{n(a-c)}{b(n+1)}$. Thus, in Case (i), the constraint is the same as the total supply constraint, $Q \geq Q^{pre}$.

When $\beta = \frac{2}{3}$ (Case (ii)), (26) is rewritten as follows: $Q \geq \overline{Q}^w$. Thus, also when $\beta = \frac{2}{3}$, the constraint is the same as the total supply constraint.

When $\beta \in [0, \frac{2}{3})$ (Case (iii)), where the weight to PS is relatively large, $\underline{Q}^w \equiv -\frac{((n-2)\beta+2)(a-c)}{b(3\beta-2)(n+1)} > 0$ and $\overline{Q}^w \equiv \frac{n(a-c)}{b(n+1)} > 0$ are satisfied. If and only if $\underline{Q}^w \geq \overline{Q}^w$, $(2\beta - 1)n + (1 - \beta) \geq 0$, which is equivalent to $\beta \geq \frac{n-1}{2n-1}$. As $\frac{n-1}{2n-1} < \frac{1}{2}$ for all $n \geq 2$, if $\beta \geq \frac{1}{2}$, $\underline{Q}^w > \overline{Q}^w$ is satisfied regardless of n .⁷ When $\beta \in (\frac{n-1}{2n-1}, \frac{2}{3})$ (Case (iii)-1), the constraint is rewritten as $\overline{Q}^w \leq Q \leq \underline{Q}^w$. As the upper bound is not actually binding, the constraint is $Q \geq \overline{Q}^w = \frac{n(a-c)}{b(n+1)}$, which is equivalent to the total supply constraint. When $\beta = \frac{n-1}{2n-1}$ (Case (iii)-2), $\underline{Q}^w = \overline{Q}^w$ is satisfied and the constraint is $Q = \overline{Q}^w = \frac{n(a-c)}{b(n+1)}$. When $\beta \in [0, \frac{n-1}{2n-1})$ (Case (iii)-3), $\underline{Q}^w < \overline{Q}^w$ is satisfied and the constraint is $\underline{Q}^w \leq Q \leq \overline{Q}^w$. As the lower bound is not actually binding, the constraint is $Q \leq \overline{Q}^w = \frac{n(a-c)}{b(n+1)}$, which is equivalent to the PS constraint. In Case (iii)-3, the equilibrium is equivalent to that without any regulatory constraint.

As a result, according to the value of β , the weighted CS and PS constraints are replaced in the total supply constraint as follows: in Case (i) where $\beta \in (\frac{2}{3}, 1]$, $Q \geq \overline{Q}^w$; in Case (ii) where $\beta = \frac{2}{3}$, $Q \geq \overline{Q}^w$; in Case (iii)-1 where $\beta \in (\frac{n-1}{2n-1}, \frac{2}{3})$, $\overline{Q}^w \leq Q \leq \underline{Q}^w$; in Case (iii)-2 where $\beta = \frac{n-1}{2n-1}$, $Q = \overline{Q}^w$; in Case (iii)-3 where $\beta \in [0, \frac{n-1}{2n-1})$, $\underline{Q}^w \leq Q \leq \overline{Q}^w$. Under the weighted CS and PS constraints, the total supply constraint is graphed in Figure 9.

⁷Note that $\lim_{n \rightarrow \infty} (\frac{n-1}{2n-1}) = \frac{1}{2}$.

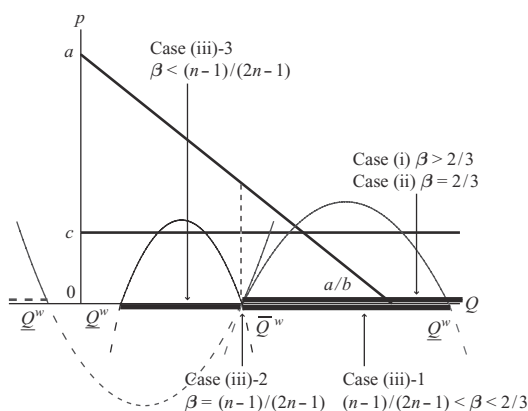


Figure 9: Weighted CS and PS constraints

Although the generalized weighted CS and PS constraints can also be analyzed, we do not present the generalized result here because of the complexity of the analysis.

From the above examination, various regulatory constraints can be replicated by the corresponding total supply constraints. Various forms of regulatory constraints are basically replicated by certain total supply constraints.

5 Concluding remarks

A merger remedy that the antitrust regulator imposes on a merged firm is a kind of regulatory constraint after the merger. In this article, we examined how a total supply constraint imposed on the merged firm affects the merged firm's profit and the merger incentive in the linear demand model. We show that when there is a total supply constraint, the merged firm's profit may be larger than that under no constraint. Paradoxically, the introduction of the constraint possibly increases the incentive for the firms to merge. Moreover, we show that various forms of regulatory constraints are replaced by the simple total supply constraint. We graph the relationship between regulatory constraints and the corresponding total supply constraint.

In the article, we focus on the analysis under the linear demand function. However, even if the general demand function is considered, under certain conditions, the total supply constraint

corresponds one-to-one with other regulatory constraints. Under the general demand function, if the demand function is a strictly decreasing function, $p'(Q) < 0$, the price constraint corresponds one-to-one with the total supply constraint. Since the consumer's surplus is $CS(Q) = U(Q) - p(Q)Q$ and $U'(Q) = p(Q)$, $CS'(Q) = -p'(Q)Q$ is satisfied. Thus, if $p'(Q) < 0$, $CS'(Q) > 0$ is satisfied and the consumer's surplus corresponds one-to-one with the total supply constraint. Likewise, if the producer's surplus is $PS(Q) = (p(Q) - c)Q$, $PS'(Q) = (p(Q) - c) + p'(Q)Q$ is satisfied. Under the interval of total quantity in which the marginal revenue exceeds the marginal cost, $PS'(Q) > 0$ is satisfied. Thus, if $p(Q) + p'(Q)Q > c$, $PS'(Q) > 0$ is satisfied and the producer's surplus corresponds one-to-one with the total supply constraint. Moreover, since the welfare is $W(Q) = U(Q) - cQ$, $W'(Q) = p(Q) - c$ is satisfied. Thus, when $p(Q) > c$, $W'(Q) > 0$ is satisfied and the producer's surplus corresponds one-to-one with the total supply constraint. Therefore, if there is a one-to-one correspondence from total supply to other regulatory objectives, the regulator can control the merged firm by implementing some kind of total supply constraint.

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