Numerical Study of Electromagnetic Surface Wave on Corrugated Metal Surface

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The dispersion characteristics of surface wave propagated on a deeply corrugated metal surface are analyzed. Surface wave becomes slow wave, because surface wave is evanescent wave to the direction away from the corrugated metal surface. The surface wave is possible to be backward wave, due to the periodic nature of the corrugation of metal surface. Because the Rayleigh hypothesis has failed for the analysis of an electromagnetic field upon a deeply corrugated surface, the dispersion relations are solved by a direct numerical computation of wave equations.

Keywords: surface wave, slow wave structure, backward wave, Rayleigh hypothesis

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A wave propagating along the surface of a medium is called surface wave. This wave decays exponentially in the direction away from the surface. Surface waves are currently applied in various areas such as plasma processing and in acoustic wave filters. However, a flat metal surface cannot propagate an electromagnetic surface wave. In this paper, it is shown that a deeply corrugated metal surface can propagate surface waves due to the waves' diffraction effect of the wave. We analyze an electromagnetic wave in vacuum area in the vicinity of a periodically corrugated metal surface.

The dispersion relation of the normal mode in the vacuum is given by $\omega^2 = c^2(k_\parallel^2 + k_\perp^2)$, where c and ω are the light velocity and the frequency, respectively. $k_\parallel$ and $k_\perp$ are the wavenumbers in directions parallel and perpendicular to the metal surface, respectively. The surface wave is always slow wave and is evanescent wave to the direction away from the corrugated metal surface. On a periodically corrugated structure, a dispersion relation becomes periodic due to the Floquet theorem [1]. Therefore, the dispersion relation of this surface wave has backward wave regions.

The periodically corrugated slow wave structure (SWS) has been extensively used in microwave devices [2]. Usually, these devices are analyzed by the Fourier expansion scheme based on the Rayleigh hypothesis. However, on a deeply corrugated structure, the Fourier expansion scheme looses accuracy due to the breakdown of the convergence of the Fourier series. The criterion of the Rayleigh hypothesis is given by $H/L < 0.0713$ ($2\pi H/L < 0.448$ in Ref. [3]), where H and L are the ripple amplitude and the period of the corrugation, respectively. In Ref. [3], the existence of locked oscillation mode is pointed out a bottom of a ravine of the corrugation. The surface wave is not the locked oscillation.

A deep corrugation is indispensable to the existence of a surface wave. In order to ensure the calculation results, dispersion characteristics are directly computed by wave equations. This method avoids the limitation given with Rayleigh hypothesis. Therefore, a set of partial differential equations of a vector potential for a magnetic field are derived. The equations are solved numerically using the HIDM (Higher order Implicit Difference Method) [4, 5].

Figure 1 shows an analytical model for the study of surface wave. The surface of the SWS is given by $X(z) = x_0 + H\cos(k_0 z)$, where $k_0 = 2\pi/L$. The magnetic field is expressed by a vector potential as follows:

$$\boldsymbol{B}(x, y, z, t) = \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \nabla \times \begin{bmatrix} \Phi(x, y, z, t) \\ \Psi(x, y, z, t) \\ 0 \end{bmatrix}. \quad (1)$$

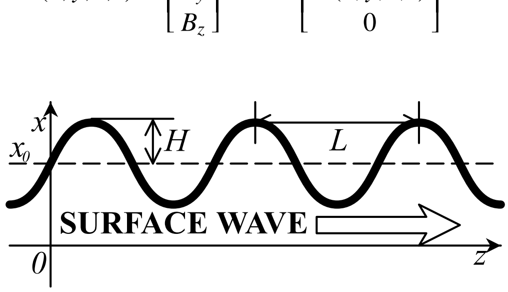

Fig. 1 Analytical model. The SWS is the metal surface which is corrugated sinusoidally in the z-direction. H and L are the ripple amplitude and the period of the corrugation, respectively.

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For simplification, here we assume that the system is uniform in the y-direction, and the electromagnetic field is the transverse magnetic wave mode (TM mode). In this case, from the Maxwell equations, $\Psi(x, y, z, t) = 0$ and the electric field becomes as follows:

$$\boldsymbol{E}(x,z,t) = \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \frac{\exp(-\mathrm{i}\omega t)}{\mathrm{i}\omega\epsilon_0\mu_0} \begin{bmatrix} \dfrac{\partial^2\Phi(x,z)}{\partial z^2} \\ 0 \\ -\dfrac{\partial^2\Phi(x,z)}{\partial x\partial z} \end{bmatrix}. \quad (2)$$

The tangential components of the electric field on the SWS should be zero. Thus $0 = E_z + E_x X'(z)$, where $X'(z) \equiv \mathrm{d}X(z)/\mathrm{d}z$. For accurate manipulation of this boundary condition, we introduce a new coordinate system given by $(\xi, z) = (x/X(z), z)$. In this (ξ, z) system, the surface of the SWS is considered a plane ($\xi = 1$). On the new coordinate system of the SWS with period L, for the wave propagating in the z-direction with wavenumber k, $\Phi(x, z)$ is transformed as $\Phi(x, z) = \exp(\mathrm{i}kz)\chi(\xi, z)$, where $\chi(\xi, z)$ is a periodic function of z with the period L. Thus, the wave equation can be derived as Eq. (3):

$$0=\left(\frac{\omega^2}{c^2}-k^2\right)\chi+\frac{1+\xi^2X'^2}{X^2}\frac{\partial^2\chi}{\partial\xi^2}+2\mathrm{i}k\frac{\partial\chi}{\partial z}+\frac{\partial^2\chi}{\partial z^2}$$
$$-2\xi\frac{X'}{X}\left\{\mathrm{i}k\frac{\partial\chi}{\partial\xi}+\frac{\partial^2\chi}{\partial\xi\partial z}\right\}+\xi\frac{2X'^2-XX''}{X^2}\frac{\partial\chi}{\partial\xi}. \quad (3)$$

Equation (3) was computed using the HIDM and the dispersion relation $\omega = \omega(k)$ is obtained.

A dispersion relation of the surface wave mode was computed as shown in Fig. 2 for the case of $H/L = 5/4$. The very deep corrugation causes even a third harmonic wave to occur in the surface wave, as seen in Fig. 2. Three surface modes (solid black lines) appear in the slow wave region under the light velocity line (dashed lines). The dispersion relation of the surface wave has the period k_0 in terms of wavenumber k, and has backward wave regions.

It is confirmed the existence of the electromagnetic surface wave in the vacuum on the corrugated metal surface.

At the wavenumber $k_z = k_0/2$, the group velocity of the surface wave becomes zero. In this case, each corrugations are considered as a repetition of open-ended cavities with depth $2H$. The eigen frequency of the wave can be approximated by $\omega/(2\pi c) = (2n+1)/(8H)$, where n is the harmonic number ($n = 0, 1, \cdots$). This approximate eigen frequencies are also plotted in Fig. 2 by the chained lines. On the other hand, these frequencies $\omega/(2\pi c)$ should be less than $1/(2L)$ and the harmonic number n should be $n < (2H/L - 1/2)$. Therefore, the surface wave exists when $H/L \geq 1/4 > 0.0713$ [3].

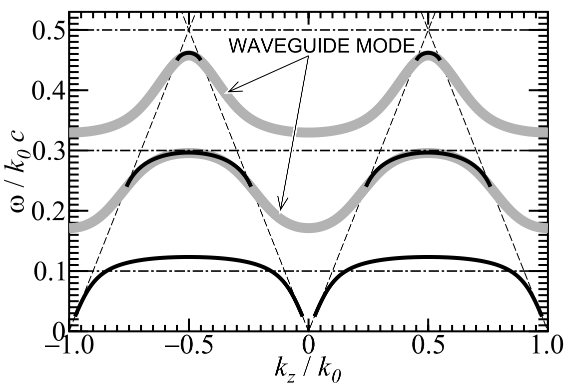

Fig. 2 Dispersion relation of the surface wave mode. The corrugation factor of the SWS is $H/L = 5/4$. The surface wave modes are presented by solid black lines. Heavy gray lines show waveguide modes transmitted in the space separated by the same two corrugated metal surface ($x_0 = \pm 2L$). The dashed lines represent spatial harmonic modes of the light velocity. The chained lines represent analytically estimated eigen frequency of the surface wave $\omega/(2\pi c) = (2n + 1)/(8H)$, ($n = 0, 1, 2$).

There is a possibility that the surface wave propagated in the vacuum area in the vicinity of the corrugated metal surface can be applied to a power supply, transmission path, and signal filter of the microwave.

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