Higher Order Mode Radiations of Weakly Relativistic Oversized Backward Wave Oscillator[*)]

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Higher order mode radiations of oversized K-band backward wave oscillator (BWO) are examined experimentally. By injecting an annular beam in the weakly relativistic region less than 100 kV, a beam mode with harmonic number $n = 1$ excites BWO radiations based on the TM_{01} surface wave. The same beam mode excites higher order modes leading to radiations in the U- and E-bands. The power levels of the higher order modes are hundreds of times smaller than that of BWO due to the surface wave. For a beam mode with $n = 2$, higher order mode radiations are in the F- and D-bands, up to about 100 GHz, which is about four times higher than the frequency of the TM_{01} surface wave. The power level of higher order mode radiation by the $n = 2$ beam mode decreases by a factor of hundreds from that of the $n = 1$ beam mode.

Keywords: oversized periodical corrugation, surface wave, higher order mode, backward wave oscillator, Smith-Purcell radiation, weakly relativistic region, cold cathode

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1. Introduction

Electromagnetic wave sources based on Smith-Purcell (SP) radiations have been studied more than half a century as compact and tunable sources in the visible, far-infrared and THz wave regions [1–5]. An electron beam moving above a metal grating structure can excite higher order modes of the grating. Higher order modes in the fast-wave region have phase velocities equal to or larger than the light velocity and can propagate in free space, leading to SP radiation. The original SP radiations are spontaneous emissions and their corresponding radiation intensity is essentially weak [1, 3]. In 1998, stimulated radiation or "super-radiant SP radiation" was first reported in Ref. [2]. Because the grating used in that study was of a slow-wave structure (SWS), it has a slow wave which has an attenuation length of the order of the wavelength. So, the slow wave of the grating in the high frequency region such as the THz region, concentrates near the grating surface forming a surface wave. The resonant interaction between the beam and the surface wave is essential to stimulated SP radiation. Beam interaction may result in backward wave oscillation (BWO) or traveling wave tube (TWT) operation depending on the group velocity of the surface wave. Additionally, beam bunch via surface wave interaction may lead to stimulated SP emission. Stimulated SP radiations allow the development of SP free electron lasers (SP-FELs) to generate intense THz waves.

BWO and TWT operations themselves have been extensively studied as high-power microwave sources in the frequency region lower than SP-FEL. Typical SWS in high-power BWO is a periodically corrugated cylindrical waveguide. In X-band BWO, a slow wave expands into a waveguide and is converted to radiation modes via reflections at the SWS ends [6]. To increase frequency and power handling capability, oversized hollow SWSs are successfully used [7–12]. The diameter of the oversized SWS is larger than the wavelength of the output electromagnetic wave. The field properties in the oversized BWO become quite different from the non-oversized X-band case. The slow wave concentrates in the vicinity of the oversized SWS wall and forms a surface wave similar to SP-FEL.

So far, the two radiation mechanisms have been studied separately, i.e., radiation due to surface waves in BWO and radiation due to higher order modes in SP-FEL. However, it is very natural to consider that the two mechanisms coexist in the periodic slow-wave device The two mechanisms may be effective as described in the pioneering work of the Ledatron [13]. In the Ledatron, higher order modes are formed in a Fabry-Perot resonator and are called "Fabry-Perot modes". By using the Fabry-Perot resonator, the two mechanisms are controlled and the mode pulling in which the higher order mode interaction suppresses the surface wave interaction is demonstrated. Previous works including the Ledatron and non-oversized and oversized BWOs show that the effectiveness of the two mechanisms in radiation processes seems to depend on corrugation size and surrounding waveguide configuration The responsible mechanism for the radiations may be determined by device

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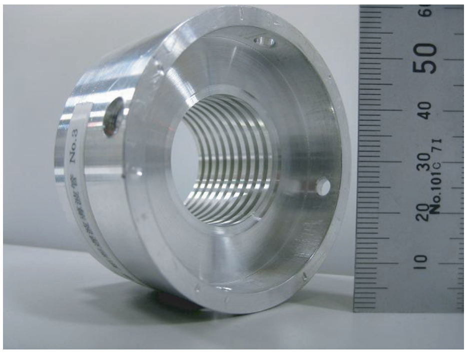

Fig. 1 One modular section of the oversized SWS.

geometry when the frequency of the oversized BWO increases to the sub-millimeter and THz wave region. Hence, the two mechanisms should be examined more thoroughly in order to develop compact sub-millimeter and THz wave sources using periodic SWS.

In this paper, higher order mode radiations of K-band oversized BWO are examined by comparing BWO radiations based on the surface wave. K-band BWO are driven by a weakly relativistic electron beam less than 100 kV. Surface wave BWO radiations are measured by a K-band waveguide detecting system. Higher order mode radiations are measured by waveguide systems from the U- to G-bands Higher order mode radiations obtained are examined by comparing with numerical dispersion curves of oversized BWO.

2. Rectangulary Corrugated SWS

The wall of the oversized SWS is rectangularly corrugated as shown in Fig. 1. The size parameters of the rectangular corrugation are average radius R_0, corrugation amplitude h, corrugation width d and periodic length z. The corrugation wave number is given by $k_0 = 2\pi/z_0$. The dispersion characteristics of the SWS are controlled by changing R_0, h, d and z_0. In our experiments, $R_0 = 15.1$ mm, h is 1.1 mm, $d = 1.5$ mm and $z_0 = 3.0$ mm. According to Floquet's theorem, the dispersion curves of spatially periodic SWSs are periodic in wave number space (k_z-space) with a period of $k_0 = 20.9\,\mathrm{cm}^{-1}$. Figure 1 is one modular section of the oversized SW including ten periods ($10z_0$). In the experiments, the total length of the oversized SWS is changed by changing the number of modular sections In this paper, the length is $20z_0$.

Figure 2 shows the dispersion curves in the reduced zone scheme from $k_z = -k_0/2$ to $k_z = k_0/2$. The lowest mode is TM_{01}, which is a surface wave in the slow-wave region near the upper cuttof at $k_z = \pm k_0/2$. The higher order modes are TM_{02}, TM_{03}, TM_{04} … and complicated higher order modes composed of the multiple spatial harmonics of TM. Higher order modes are in the fast-wave regions and are excited by beam modes $\omega = k_n v_0$ with any interger n. Here ω, $k_n = k_z + nk_0$ and v_0 are the angular frequency, the wave number of the nth harmonoic beam mode and the beam velocity, respectively.

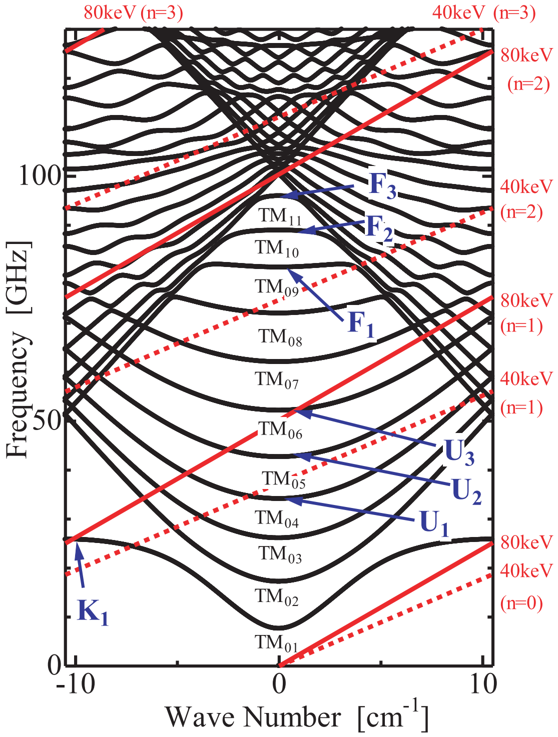

Fig. 2 Dispersion curves of the oversized SWS. Solid and dotted lines are beam modes $\omega = k_n v_0$ with 80 kV and 40 kV, respectively. Approximate beam interaction points are shown by K_1 for Fig. 5, by U_1, U_2 and U_3 for Fig. 6 and by F_1, F_2 and F_3 for Fig. 7.

3. Experimental Setup

Experimental setup of our oversized BWO is schematically shown in Fig. 3. A disk cold cathode proposed in Refs. [12,14] is used. It can generate uniformly distributed annular beams with currents of hundreds of amperes in a weakly relativistic region. An injected electron beam has an annular shape, which is examined by observing burn patterns in thermally sensitive paper obtained by intersecting the beam. The diameter of the annular beam is nearly the same as the diameter of the cathode. The thickness of the annular beam is typically 2-3 mm. A beam limiter with diameter ϕ28 mm is placed to protect the corrugation surface. For beam prorogation, a uniform axial magnetic field B_0 is provided by 10 solenoid coils. The value of B_0 is about 0.8 T in this study.

The radiated microwaves are picked up by rectangular horn antennas connected to waveguides of the K-, U-, E-, F-, D- and G-bands, which are the EIA standard rectangular waveguides with cut-off frequencies of 14, 31, 48, 74, 91 and 116 GHz, respectively [15]. The K-, U- and E-band antennas are placed 600 mm from the output window and an adequate amount of attenuation is provided in the detecting systems, typically 33 dB (K-band), 10 dB (U-

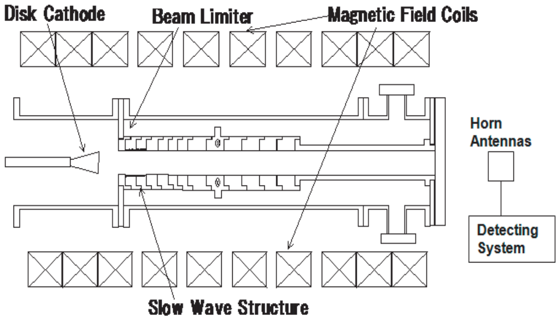

Fig. 3 Schematic diagram of the experimental setup.

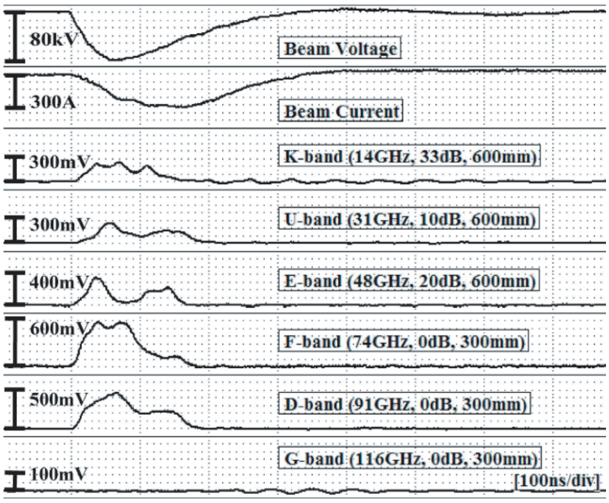

Fig. 4 Waveform of measurement signals. The beam voltage and current have the peak values of about 80 kV and 300 A, respectively. The cutoff frequency, the attenuation and the distance from the output window are written for each microwave signal.

band) and 20 dB (E-band). The crystal detectors are Agilent HP8474E for the K- and U-bands and ELVA-1 ZBD-12 for the E-band. HP8474E crystal detectors have fairly flat sensitivities in a wide frequency range up to 50 GHz and their voltage is calibrated to absolute powers using Gunn oscillators at 24 GHz and 45 GHz.

The F-, D- and G-band antennas are placed 300 mm from the output window and no attenuation is provided. The crystal detectors are ELVA-1 ZBD-08 and ZBD-06 for the F- and D-bands, respectively, and Pacific Millimeter Product GD for the G-band.

In the K-band system, the detected microwave signal is split into two branches by a multi-hole directional coupler. One consists of a short waveguide and forms a prompt signal. The other branch is connected to a delay line composed of Nihon Koshuha WRJ-260, which is the JIS designation. The delay line has a cutoff frequency of 17.4 GHz and forms a delayed signal to measure frequency.

4. Experimental Results

Typical signals are shown in Fig. 4. The K-band signal reaches a peak value in the first half. The frequency is estimated to be about 26 GHz by the delay time measurement, which is in agreement with the numerical value of TM_{01} excitation by BWO interaction of an 80 kV beam near K_1 in Fig. 2. The U-, E-, F- and D-band signals are attributed to higher order modes. No G-band signal is detected. Higher order mode radiations are confirmed up to around 100 GHz.

Two kinds of waveforms can be seen in higher order mode radiations. One is that of the U- and E-band waveforms, which have two humps in the first and second halves. The radiations in this frequency range can be explained by interactions between beam mode with $n = 1$ and the TM_{04}, TM_{05} and TM_{06} modes. The other waveform is that of the F- and D-band waveforms, which reach the peak values in the first half, like the K-band signal. In this frequency range, the responsible interactions are those between beam mode with $n = 2$ and the TM_{09}, TM_{10} and TM_{11} modes.

Peak values of K-band signals are plotted versus beam voltage in Fig. 5 with cathodes of diameters $\phi 27$ mm and $\phi 23$ mm For the $\phi 20$ mm cathode, meaningful radiations are not detected. K-band signals start at about 40 kV and increase as beam voltage increases. The maximum radiation power is estimated to be about 100 kW in the voltage region of 60-80 kV, which is the same level as a previous oversized BWO in Ref. [12]. The power decreases by decreasing the cathode diameter from $\phi 27$ mm to $\phi 23$ mm and no-signals are detected with the $\phi 20$ mm cathode.

For the $\phi 27$ mm cathode, beam diameter is close in size to the inner diameter of corrugation $\phi 28$ mm For the $\phi 23$ mm cathode, the beam is about 2 mm away from the corrugation surface. For the $\phi 20$ mm cathode, the distance between beam and corrugation surface is estimated as 4 mm and the radiations disappear in accordance with the decaying length of the surface wave, which is about 2 mm at the beam interaction point near $k_z = k_0/2$.

The peaks of the U-band signals are plotted versus beam voltage in Fig. 6, which are excited by the $n = 1$ beam mode. They start at about 40 kV, the same voltage as the K-band. The power increases sharply once decreases above 60 kV and again increases above 80 kV. According to Fig. 2, radiations above about 80 kV are attributed to the TM_{06} mode near U_3 and those in the range of 40-60 kV to the TM_{04} mode near U_1 and TM_{05} mode near U_2. As shown in Fig. 7, F-band signals are small around 40 kV and increase sharply above around 6 kV. These signals are caused by the $n = 2$ beam mode and the former small signals are attributed to TM_9 mode near F_1 of Fig. 2 and the latter radiations to the TM_{10} mode near F_2 and TM_{11} mode near F_3 of Fig. 2.

Higher order mode radiations are excited around U_1, U_2, U_3, F_1, F_2 and F_3 of Fig. 2. These regions are near the lower cutoff at $k_z = 0$ and correspond to the operation

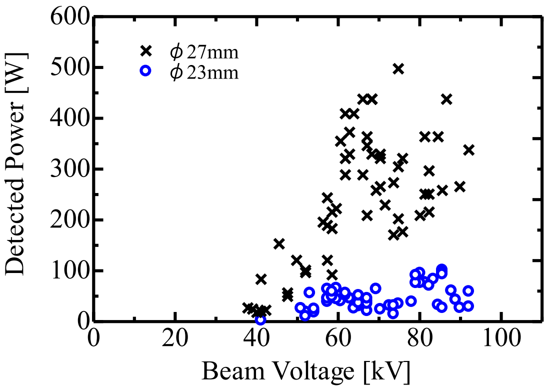

Fig. 5 Peak value of K-band signal versus beam voltage. The beam voltages at the radiation peaks are used. Magnetic field is about 0.8 T. No meaningful radiations are detected with the ϕ20 mm cathode.

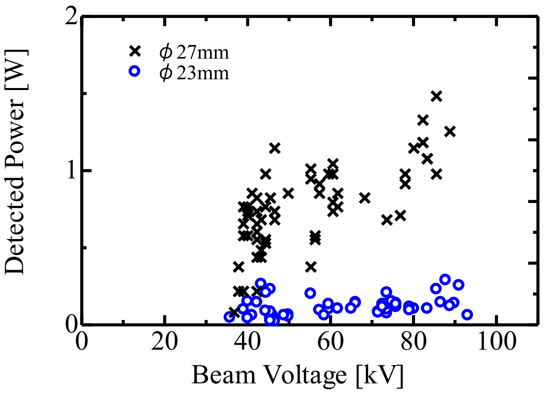

Fig. 6 Peak value of the U-band signal versus beam voltage. The beam voltages at the radiation peaks are used. Magnetic field is about 0.8 T. No meaningful radiations are detected with the ϕ20 mm cathode.

point of the relativistic diffraction generator, which is also called the orotron [16]. Near the cutoff group velocity of the higher order modes becomes small and beam interaction may be enhanced. A similar enhancement attributed to a decrease in group velocity can be seen for BWO operation, where the interaction point in the slow-wave region is shifted toward the upper cutoff point at $k_z = k_0/2$ [9–12].

In BWO operation a beam should propagate within the decaying length of the surface wave. On the contrary, higher order modes are volumetric fast waves with strong fields inside the waveguide. From Figs. 6 and 7, it is seen that higher order mode radiations decrease by decreasing the cathode diameter from ϕ27 mm to ϕ23 mm Although the decreasing rate seen in Fig. 7 is less than that of Figs. 5 and 6, all signals disappear with the ϕ20 mm cathode To excite higher order modes, harmonic beam mode $\omega = k_n v_0$ needs to be formed on the annular beam by interacting with the corrugation as shown by Fig. 2. Experimental results show that this interaction length is comparable to the attenuation length of the surface wave responsible for BWO operation.

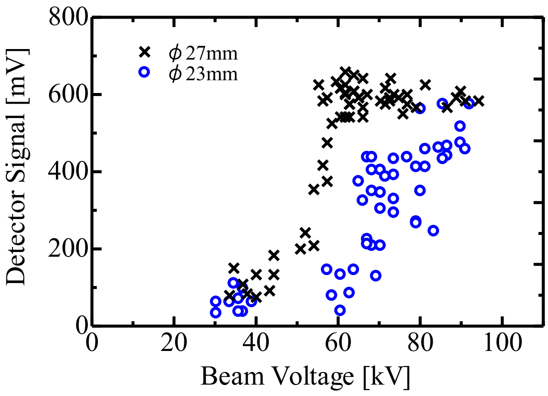

Fig. 7 Peak value of the F-band signal versus beam voltage. The beam voltages at the radiation peaks are used. Magnetic field is about 0.8 T. No meaningful radiations are detected with the ϕ20 mm cathode.

The electron beam mode $\omega = k_1 v_0$ excites the BWO of the TM$_{01}$ surface wave (K-band) and the higher order mode radiations of the TM$_{04}$, TM$_{05}$ and TM$_{06}$ modes (U- and E-bands). Since the crystal detectors for the K- and U-bands are the calibrated HP8474E detectors, the vertical axes of Figs. 5 and 6 are the same. Hence, it can be said that the detected power level of the U-band is about 500 times smaller than that of the K-band. The BWO of the TM$_{01}$ mode and higher order mode radiations of the TM$_{04}$, TM$_{05}$ and TM$_{06}$ modes are excited by the same beam mode $\omega = k_1 v_0$. Our experiments show that the radiations based on the surface wave are hundreds times larger than the higher order mode radiations. Higher order mode radiations excited by the beam mode with $n = 2$ ($\omega = k_2 v_0$) are in the F- and D-bands. The power levels of the F- and D-bands are evaluated based on antenna size position and the sensitivity given by the makers. F- and D-band radiations are hundreds times smaller than radiations in the U- and E-bands. Radiations by the $n = 2$ beam mode are much less than those of the $n = 1$ beam mode.

In the K-band oversized BWO, BWO radiations are superior to higher order mode radiations. This is a good result for a BWO aiming for surface wave operation. However, corrugation size becomes too small to be fabricated when the BWO is developed for the THz wave region. Higher order mode radiations are inevitable. To intensify higher order mode radiations, the surface wave needs to be controlled in addition to the higher order mode itself. Additionally, appropriate mechanisms are required to intensify higher order mode radiations. The mode pulling in the Ledatron [13] is a very attractive mechanism. Coaxial SWSs are used as double corrugation or the Fabry-Perot resonator to control beam interactions [17, 18]. Stimulated radiation by the bunching beam like super-radiant SP radiation is also a useful mechanism to enhance higher order mode radiations [2, 4]. Moreover, the combined resonance between Cherenkov and slow-wave cyclotron interactions is also a hopeful candidate for controlling surface wave oscillation [7,8].

5. Conclusions

Higher order mode radiations from the oversized K-band BWO are studied experimentally. By injecting the annular beam in the weakly relativistic region less than 100 kV, the BWO radiations based on the TM_{01} surface wave are observed at about 26 GHz. The radiation power is about 100 kW. The beam mode $\omega = k_1 v_0$ excites the surface wave via BWO operation. The same beam mode excites higher order modes leading to radiations in the U- and E-bands. The power level of this higher order mode radiation is hundreds times smaller than that of the BWO. The higher order mode radiations in the F- and D-bands are excited by the beam mode $\omega = k_2 v_0$ and are hundreds times smaller than the radiations by the beam mode $\omega = k_1 v_0$. Although power level decreases by increasing the harmonic number n of beam mode, higher order mode radiations are observed up to $n = 2$, up to about 100 GHz. It is about four times higher than the frequency of the TM_{01} surface wave.

In the K-band oversized BWO, BWO radiations are superior to higher order mode radiations. When BWO is developed in to the THz wave region, the corrugation size becomes small enough higher order mode operation to be necessary. For compact sub-millimeter and THz wave sources using periodic SWS, mechanisms such as double corrugation and stimulated radiation via the bunching beam may be required to intensify higher order mode radiations.

Acknowledgments

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