LETTERS

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Anomaly of excess pressure drops of the flow through very small orifices

Tomiichi Hasegawa,[a] Masaki Suganuma, and Hiroshi Watanabe
Department of Mechanical and Production Engineering, Faculty of Engineering, Niigata University, Ikarashi-2, Niigata-shi, 950-21 Japan

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Excess pressure drops are measured for the flow through very small orifices whose diameter ranges from the order of 1 mm to 10 μm using water, silicon oils, and solutions of glycerin in water: For larger orifices it is almost the same as, but for smaller orifices several times higher than, that of the numerical analysis of Newtonian fluid. Water gives the highest among the liquids used. Velocities are also measured along the centerline of a small orifice and found inconsistent with the result of the numerical analysis. A different mechanism may be dominant in the flow through very small orifices.
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The excess pressure drop (epd) Δp of the flow through apertures is defined as the pressure difference obtained by subtracting the pressure drop that would exist if only losses from fully developed flow were present, from the total pressure drop necessary to eject the fluid through the aperture. Here Δp is customarily represented as

$$\frac{2\,\Delta p}{\rho V^2} = K + K'/\mathrm{Re}, \tag{1}$$

where V is the mean velocity, ρ is the density, K and K' are constants, and Re is Reynolds number ($=\rho VD/\mu$; μ is the viscosity and D is the aperture diameter). The creeping flow for an infinitely thin orifice was theoretically studied and 37.7 was obtained as K' by researchers,[1–3] and experiments[4,5] gave nearly the same value of K'. Numerical analyses[6–8] made for creeping pipe flows provided the values around 27 of K', and this was experimentally confirmed.[9] Dagan and others theoretically treated the creeping motion through an orifice of finite length and found that the excess pressure drop across the orifice can be closely approximated (less than 1% error) by Sampson's result theoretically obtained for an infinitely thin orifice.[10] All the experiments mentioned above were conducted using fluids of increased viscosity such as oils or a mixture of water and syrup or glycerin for apertures larger than the order of 1 mm. But such previous works may not provide information of flows through smaller orifices for water and other liquids of low viscosity, because an orifice flow has a character of elongational flow and the elongational rate is higher for smaller orifices than for larger orifices, even at the same Reynolds number. The fluid mechanics of such flows has been scarcely investigated, although recently it is considered fundamentally important concerning micromechanics.

In the present study we measure the total pressure drops and calculate the excess pressure drops for the flow through

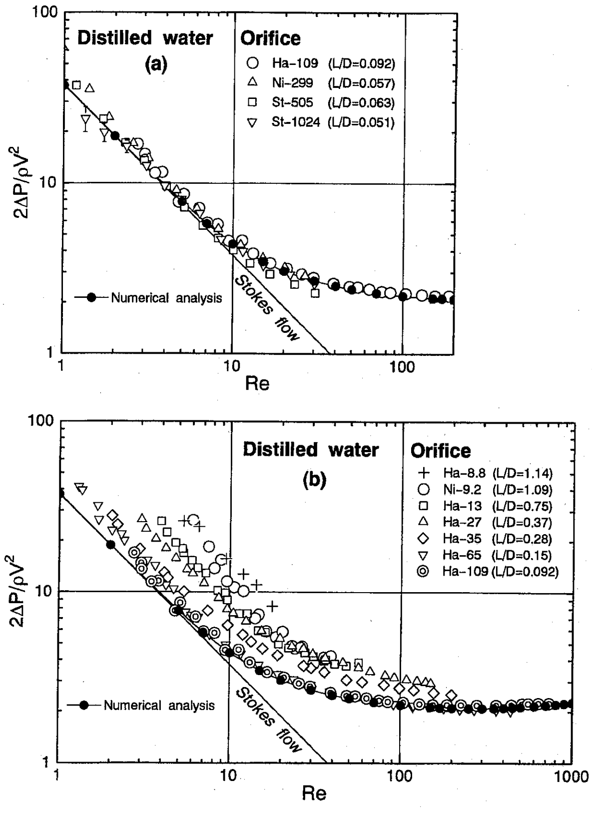

FIG. 1. Dimensionless excess pressure drop ($2\,\Delta p/\rho V^2$) against Reynolds number (Re) for distilled water. Here L is the thickness of the orifice and D is its diameter. Ha, Ni, and St represent Harver, nickel, and stainless Steel of the orifice material respectively. The numeral following Ha, Ni, and St means the orifice diameter (μm): For instance, Ni-299 means that the orifice material is nickel and the diameter is 299 μm. –●– illustrates the numerical analysis, and—gives the theoretical result of Stokes flow. (a) ○ (D=109 μm; L=10 μm); △ (D=299 μm, L=17 μm); □ (D=505 μm, L=32 μm); ▽ (D=1024 μm, L=52 μm). (b) + (D=8.8 μm); ○ (D=9.2 μm); □ (D=13.4 μm); △ (D=27.4 μm); ◇ (D=35.3 μm); ▽ (D=64.9 μm); ◎ (D=109 μm). All the orifices in (b) have the thickness of 10 μm.

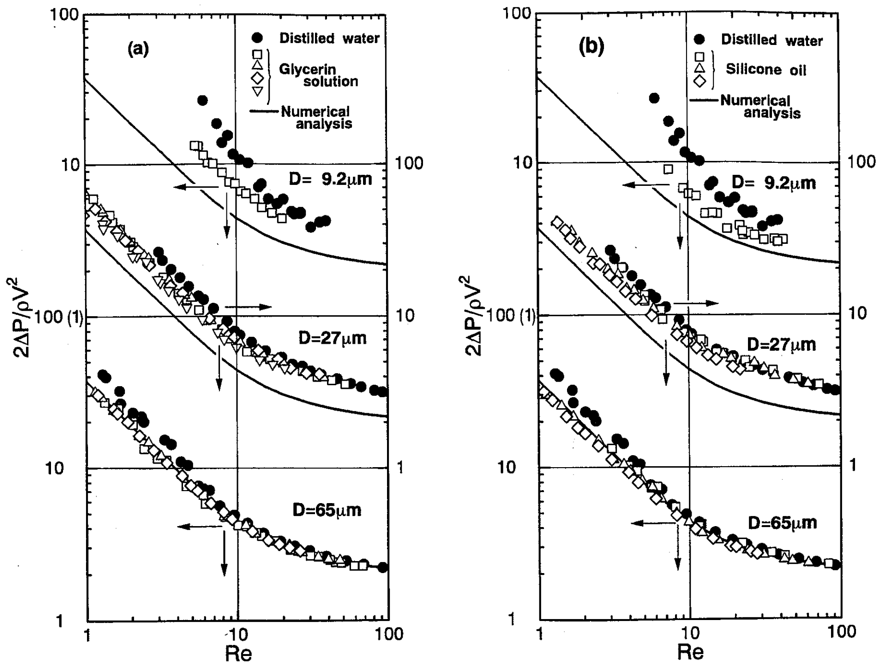

FIG. 2. (a) Comparison of water and glycerin solutions in water; ● is for water (ν=1.00), □ for glycerin 20% in water (ν=1.69), △ for 30% (ν=2.39), ◇ for 40% (ν=3.44), ▽ for 50% (ν=5.28). Here ν is the kinematic viscosity ($\times 10^{-6}$ m^2 s^{-1}) at 20 °C. (b) Comparison of water and silicon oils; ● is for water (ν=1.00$\times 10^{-6}$ m^2 s^{-1}), □ for silicon oil (ν=1.10), △ for silicon oil (ν=2.22), and ◇ for silicon oil (ν=5.13).

small orifices of the order from mm to tens of μm's in a range of relatively low Reynolds numbers using water and other liquids of low viscosity. The velocity along the centerline is also measured, and these results are compared with the result of numerical analysis.

A test liquid was supplied with a head tank and made to flow in a channel and through a small orifice set in the midst of the channel. The flow rate was measured by weighing the liquid during a time with a digital balance. It was confirmed that vaporization during weighing of the liquid gives no virtual effect on the result. In the case that the orifice diameter is relatively large, namely, for the orifices larger than 299 μm, the liquid was supplied with a displacement pump specially designed, whose discharge was obtained by push of a plunger into a liquid pool with a turning screw. The filter is used only for silicon oils. The pressure differential was measured between the upstream and downstream positions relative to the orifice using pressure transducers of different capacities according to the magnitude of the pressure measured. Each transducer was precisely and carefully calibrated by applying the definite heads of water. The orifice is made of a thin foil of metal and an aperture is made by spark machining. Water as a test liquid was refined through both ion exchange resin and distillation. Because the mixing of dust into the test liquid causes choke, great care was taken for cleanliness in setting an orifice or a channel and in storing the liquid. Velocities were measured along the centerline of the orifice in the upstream and downstream regions using a laser-Doppler velocimeter.

We carried out a numerical analysis for the orifice flow of a Newtonian fluid by the finite element method (FEM). The FEM program employed is FIDAP of Fluid Dynamics International Inc. Flow conditions for calculation were made to coincide with that for the present experiment; that is, in addition to the range of Reynolds numbers, orifice thicknesses different for each orifice (see Fig. 1) were given as a condition in the numerical analysis.

In Fig. 1 we have the dimensionless epds $(2\,\Delta p/\rho V^2)$ plotted against the Reynolds number (Re) for various orifice diameters obtained by the experiment, the numerical analysis, and the theory of creeping flow. The experimentally decided epd occupies at least 60% of the measured total pressure drop in the 8.8 μm orifice (the thickest orifice), and the portion of epd increases as the orifice diameter increases, or, L/D decreases. Because the present numerical analysis gives nearly the same dimensionless epd regardless of the various ratios of thickness to diameter adopted in this experiment, it is represented as single points in the figure. We see that the epd of numerical analysis at a low Reynolds number coincides with the theoretical one of Stokes flow, and that for the orifices larger than 65 μm the experimental epd takes almost the same value as the numerical result except the higher deviation in the range of Reynolds number less than 5 for the 65 μm orifice [Figs. 1(a) and 1(b)]. On the other hand, it should be noted that the orifices below 35 μm provide the experimental data higher in value than the numerical analysis [Fig. 1(b)]: Smaller orifices give the higher excess pressure drop than the numerical analysis, especially the orifices of 8.8 and 9.2 μm giving the values several times larger than the numerical one. Considering that the difference between the results of the experiment and numerical analysis corresponds to the pressure differential larger than 1000 Pa with the 9 μm orifice, and the flow rate is measured under the error of several %s, and further the experimental reproducibility is good, we can conclude that this increase in epd is not an error in measurement.

The effect of the kind of the liquids on epd was examined and the result is given in Fig. 2. We see the glycerin solutions in water and silicon oils tested also give higher epds for the orifices below 35 μm than by the numerical

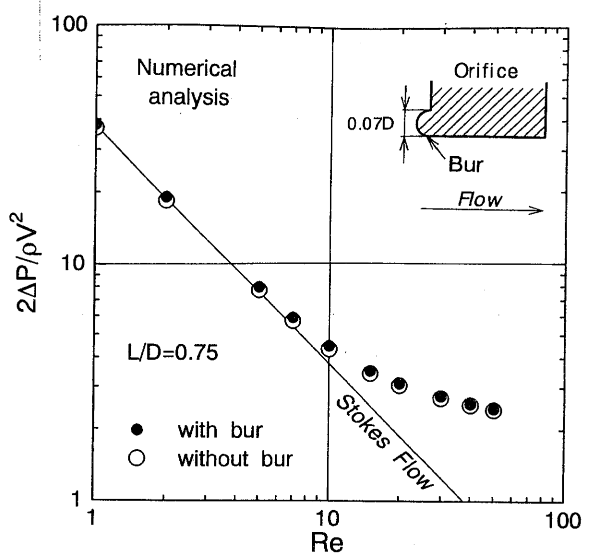

FIG. 3. Examination of the effect of a bur on the excess pressure drop by numerical analysis. The assumed bur is $0.07D$ in magnitude.

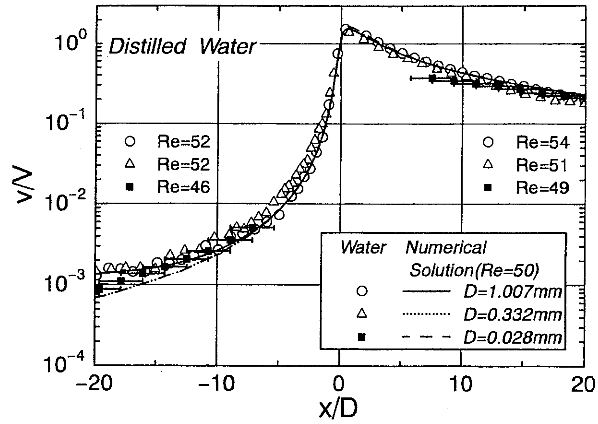

FIG. 4. Centerline velocities made dimensionless with mean velocity (v/V) upstream and downstream of the orifice plotted against the dimensionless distance (x/D; $x<0$ indicates the upstream region and $x>0$ the downstream region). The velocity was measured separately upstream and downstream and consequently Reynolds number is slightly different on inlet and outlet sides. The measuring position is indeterminate by a breadth of the finite measuring volume consisting of obliquely crossed laser beams. The breadth is not neglected for a 0.028 mm orifice and denoted by line segments in the figure.

analysis, but they are lower than that of water.

Further examinations were made on the material of the orifice and the condition of the orifice hole: Orifices of the same diameter made of Harver and nickel foils were used, but any difference of epd was not found between them and therefore the effect of orifice material may be small for metal, if it exists.

The orifice was examined with an electron microscope and a bur produced at the periphery in machining was found. The largest bur was about 10% of the orifice diameter in size. In order to examine the influence of the bur on the epd, a numerical analysis was done: we assumed a bur attached to the periphery of an orifice as shown in Fig. 3, analyzed the flow through the orifice by FIDAP and calculated the epd. The result is shown in Fig. 3 and we have the increase of only several percentages in epd by this analysis. Therefore we conclude that the bur cannot be a cause of the great increase of epd in the experiment.

It is well known that a surface dipped in a solution is usually charged with ions, which leads to the formation of an electric double layer. The measure of the layer is represented by the Debye length. For a univalent electrolyte the Debye length is about 1 nm for a concentration of 10^2 mol/m^3 and 10 nm for 1 mol/m^3,[11] which is much smaller than the smallest orifice applied in the present experiment. Therefore the effect may be small. To confirm this conjecture, we again measured the excess pressure drop using 10^2 and 10^3 mol/m^3 NaCl solutions in water. If the increase in the epd had been caused by the electric double layer, the epds measured would have decreased in the solutions of NaCl used. But the experiment clarified that there is no discernible difference of epds between water and NaCl solutions used. As a result, it is thought that the electric double layer is not a cause of an increase of the epd in small orifices.

Figure 4 gives the result of the velocity measurement on the centerline for three orifices. We see a good agreement between the measured and numerical results for the 1.01 mm orifice but smaller experimental values for the 0.028 mm orifice in the downstream region.

These results show that the flow through very small orifices is different from that through the orifice of ordinary size, which can be solved with a Navier–Stokes equation. Another fluid mechanics may be needed to analyze this kind of flow.

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[a]Telephone: +81-25-262-7009; Fax: +81-25-263-3174; Electronic mail:hasegawa@eng.niigata-u.ac.jp

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