Absolute Instability for Enhanced Radiation from a High-Power Plasma-Filled Backward-Wave Oscillator

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(Received 16 January 1990)

The linear theory of electromagnetic radiation from a backward-wave oscillator with a plasma-filled, sinusoidally corrugated waveguide driven by a relativistic electron beam has been derived and analyzed numerically. The presence of plasma can cause a substantial increase in the spatial growth rate of the absolute instability. For high plasma densities, however, the absolute instability is suppressed and only the convective instability remains. The predicted radiation enhancement can be attributed to a decrease in the group velocity of the backward wave in the presence of a plasma.

PACS numbers: 52.40.−w, 41.80.Ee, 52.25.Sw, 85.10.Jz

In recent years, a dramatic resurgence in research activities on relativistic electronics for high-power microwave sources has been taking place all over the world, driven by the development of new technologies and the requirements of present and future applications.[1] Gyrotrons and free-electron lasers are examples of such new devices. The backward-wave oscillator (BWO) driven by an intense relativistic electron beam is another microwave source on which research has been conducted primarily in the U.S. and U.S.S.R. for the last fifteen years.[2-10] Previously, gas-filled BWOs were observed to yield significant microwave output power at X-band frequencies.[3,9] Recently, however, considerable increases in both the output power and the efficiency of a BWO due to plasma injection from an external plasma gun have been observed.[10] Lin and Chen[11] have recently reported numerical simulations of the experiments, although they assumed an artificial periodic boundary condition in the axial direction which did not correspond to the experiments. They reported that a parametric beam-plasma wave interaction could not be the dominant mechanism for the efficiency enhancement and that the enhancement could be attributed to a decrease in the phase velocity of the most unstable beam mode in the presence of a background plasma. In order to understand analytically the physical reason for the observed enhancement, we have developed a linear theory of the beam-plasma interaction and have analyzed numerically the absolute instability[12] in plasma-filled BWOs.

First, we consider an infinitely long axisymmetric waveguide, i.e., slow-wave structure, whose wall radius varies with the axial coordinate z according to $R(z) = R_0 + h\cos(k_0 z)$, where $k_0 = 2\pi/z_0$. Here R_0, h, and z_0 are, respectively, the average waveguide radius, the amplitude of corrugation, and its period. A uniform, cold, and collisionless plasma with density N_p is assumed to be present, and a beam with uniform electron density N_b, a longitudinal velocity v, and radius $R_b < R_0 - h$ is present in the slow-wave structure. An infinitely strong external magnetic field which confines electron motion to be strictly along the field lines is applied in the axial direction z. We have obtained the linear dispersion relation $D(k,\omega) = 0$ for the symmetric TM modes with field components E_r, E_z, and B_θ.[13] Here, D is the value of the determinant of a matrix for which the element of the mth column and nth row, D_{mn}, is given by

$$D_{mn} = [1 + (n-m)Q_n](K_n C^J_{mn} + L_n C^N_{mn}) , \tag{1}$$

$$\begin{aligned} C^J_{mn} &= \sum_{q=0}^{\infty} \frac{(Y_n \alpha)^{2q+|n-m|} J_0^{(2q+|n-m|)}(Y_n)}{2^{2q+|n-m|} q!(q+|n-m|)!} , \\ K_n &= Y_n J_0(X_n\delta) N_1(Y_n\delta) - X_n J_1(X_n\delta) N_0(Y_n\delta) , \\ L_n &= X_n J_0(Y_n\delta) J_1(X_n\delta) - Y_n J_0(X_n\delta) J_1(Y_n\delta) , \\ X_n^2 &= R_0^2 \left(\frac{\omega^2}{c^2} - k_n^2 \right) \left[1 - \frac{\omega_p^2}{\omega^2} - \frac{\omega_b^2}{\gamma^3(\omega - k_n v)^2} \right] , \\ Y_n^2 &= R_0^2 \left(\frac{\omega^2}{c^2} - k_n^2 \right) \left(1 - \frac{\omega_p^2}{\omega^2} \right) , \\ Q_n &= \frac{k_0 k_n}{w^2/c^2 - k_n^2} , \quad \alpha = \frac{h}{R_0} , \quad \delta = \frac{R_b}{R_0} , \quad k_n = k + nk_0 . \end{aligned} \tag{2}$$

A similar expression for C^N_{mn} holds, with J_0 replaced by N_0 in C^J_{mn} given by (2). Here, we have assumed a phase factor $\exp[-i(\omega t - k_n z)]$ for time and spatial variations of the rf electric fields, and the other notations are standard. We confine our analysis here to the case of the high-frequency electromagnetic TM_{01} mode with $\omega > \omega_p$. Our dispersion relation (1) tends to previously obtained results[4,5] in the limit of small α.

The following parameters[9,10] are adopted unless specified otherwise; the relativistic factor $\gamma=2.23$, $R_0=1.445$ cm, $R_b=0.9$ cm, $h=0.445$ cm, $z_0=1.67$ cm, and $N_b=2.09\times10^{11}$ cm^{-3}. The plasma density N_p is considered to be a variable. For these parameters, we must calculate a 9×9 determinant equation, with the terms up to the order of α^{10} in (2), to obtain 1% numerical accuracy in periodicity for two periods of wave number k_0.[13]

The spatial growth rate is a more practical measure of the strength of instabilities than is the temporal growth rate,[12] as will be discussed later in detail. There can be many roots of $D(k,\omega)=0$ that have complex $k=k_r+ik_i$ for a real ω. In order to sort out the spatially growing waves of convective and absolute instabilities from other roots of evanescent or stable waves, we deform the path of integration in the inverse Laplace transformation in the complex $\omega=\omega_r+i\omega_i$ plane downward to the real axis. If k_i has different signs when ω_i takes a large positive value and zero, then the wave is convectively unstable; otherwise, it is an evanescent wave. If the deformation of the path of integration towards the real axis is prevented by merging of two roots (saddle point) for some positive ω_i from both sides of the k half planes, then the instability is absolute which corresponds to a BWO. In general, ω and k at the saddle point are both complex[12,14] ($k_s=k_{rs}+ik_{is}$, $\omega_s=\omega_{rs}+i\omega_{is}$), and the absolute instability causes electromagnetic radiation from an infinitely long slow-wave structure even if a feedback mechanism does not exist. The spatial growth rate k_{is} and the real part k_{rs} of the wave number at the saddle point versus N_p are plotted in Fig. 1(a), while the temporal growth rate $\omega_{is}/2\pi$ and the oscillation frequency $\omega_{rs}/2\pi$ are shown in Fig. 1(b). As N_p increases the saddle point goes down, and the spatial growth rate is enhanced. As the point moves into the $\omega_i<0$ region as shown in Fig. 1(b), the absolute instability disappears suddenly and only the convective instabilities continue to exist for exceedingly large N_p. These convective instabilities [corresponding to a plasma traveling-wave tube (TWT)] cannot give rise to any radiation unless a feedback mechanism exists in the slow-wave structure. The maximum spatial growth rates k_{im} for $\omega_i=0$ are a measure of the amplification in the plasma BWO and are shown in Fig. 1(a). The group velocity,[15] approximately given by ω_{is}/k_{is}, and the phase velocity ω_{rs}/k_{rs} vs N_p are shown in Fig. 2 for an infinitely long ($L=\infty$) plasma-loaded, slow-wave structure. The absolute value of the group velocity decreases with N_p while the phase velocity slightly increases with N_p. The group velocity, however, changes direction and becomes positive for plasma density $N_p>1.6\times10^{12}$ cm^{-3} as shown in Fig. 2 (solid circles), in which case the system works as a plasma TWT ($\omega_i<0$), because the wave is still growing spatially.

In our linear theory, the output radiation is proportional to $\exp(2\omega_{is}t)$, where t is the interaction time between the beam and the backward wave. In real experiments, the interaction time $t\gg1$ nsec is given by the ratio of a finite L and the group velocity. Then, $\exp(2\omega_{is}t)=\exp(2k_{is}L)\gg1$. The increase in k_{is} with N_p as shown in Fig. 1(a) may thus enhance the radiation. This is the reason why the spatial growth rate k_{is} is a more practical and critical measure of the instabilities than the temporal growth rate $\omega_{is}/2\pi$ for a slow-wave structure with finite axial length. The predicted enhancement can be attributed to an increase in the interaction time, i.e., a decrease in the group velocity of the backward wave. As shown in Fig. 1(a), our result predicts linear enhancement for values of N_p 1 order greater than that observed in Ref. 11.

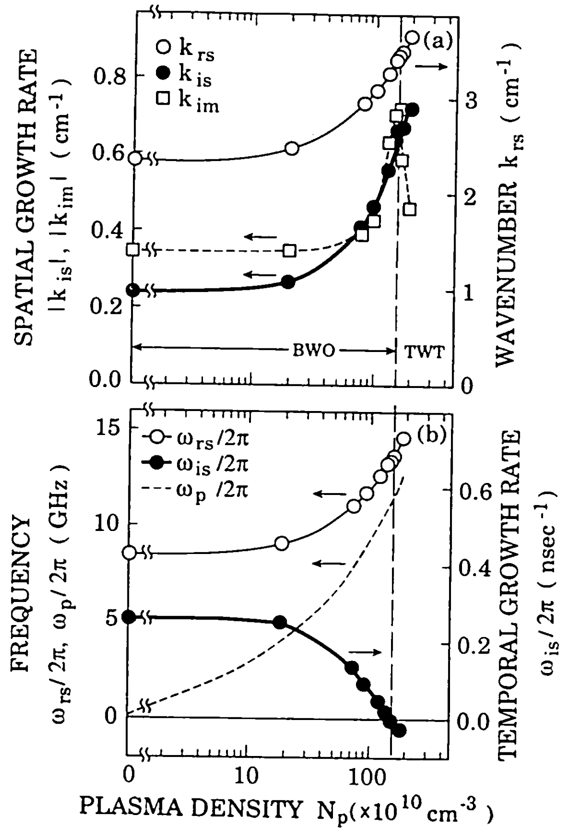

FIG. 1. (a) The spatial growth rate k_{is}, the real part k_{rs}, at the saddle point, and the maximum spatial growth rate for $\mathrm{Im}(\omega)=0$, k_{im}, vs plasma density N_p. (b) Temporal growth rate $\omega_{is}/2\pi$, oscillation frequency $\omega_{rs}/2\pi$, at the saddle point, and plasma frequency $\omega_p/2\pi$ vs N_p.

In order to elucidate the physical meaning of the radiation enhancement, the maximum spatial growth rate k_{im} of the plasma BWO in the limit of small α has been analyzed in a qualitative way. The procedure used here is not very different from that in Ref. 11. A 2×2 determinant equation $D(k,\omega)=0$, with $-1\le m,n\le0$, is used, and only the terms up to order α are taken into account in (2). We assume that the beam term in X_n [see Eq. (2)] is small, and the spatial growth rate ($\omega_i=0$) at the intersection of the beam space-charge wave $\omega=kv$

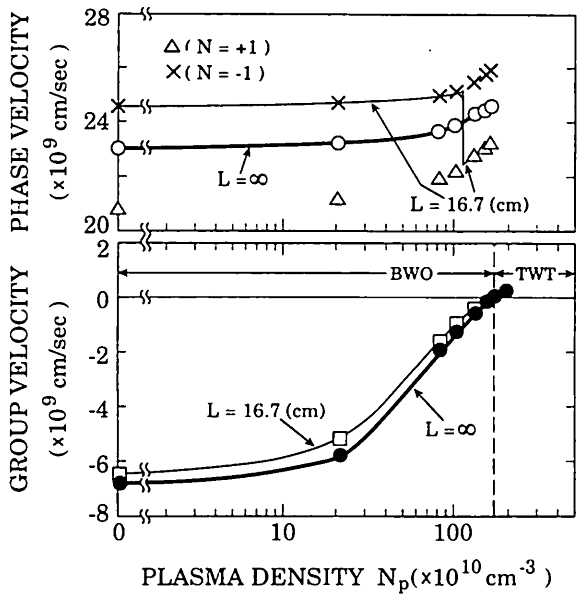

FIG. 2. Phase and group velocities of the plasma-loaded slow-wave structure in the case of $L=\infty$ vs N_p, shown, respectively, by solid and open circles. Those for a slow-wave structure with finite axial length $L=16.7$ cm are shown by thin lines.

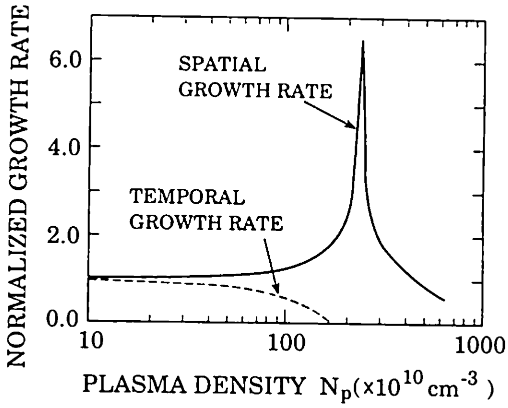

FIG. 3. Simplified theoretical result of the maximum spatial growth rate (solid curve) for $\mathrm{Im}(\omega)=0$ normalized to the vacuum case vs plasma density N_p; $N_b=5.15\times10^{10}$ cm^{-3}, $\alpha=0.1$, and other parameters are the same as those in Figs. 1 and 2. The maximum temporal growth rates for $\mathrm{Im}(k)=0$ normalized to the vacuum case are also shown by the dashed curve.

and the backward wave $Y_1=2.405$ is calculated. The final dispersion relation results in a quartic equation. It must be emphasized here that the previous analytical result[5] for $\gamma\gg1$ gives no enhanced growth rates for the parameters in the experiments[9,10] with $\gamma=2.23$. The spatial growth rate versus N_p calculated from the quartic equation is shown in Fig. 3. A resonant increase in the spatial growth rates is found and the result is qualitatively similar to k_{im} in Fig. 1(a). If one ignores the fourth-order term in the quartic equation assuming that it is small, the peak in Fig. 3 goes to infinity at $k_r=k_0$ and at the cutoff frequency, where the group velocity of the backward wave is zero. The decrease in the group velocity enhances the interaction between the backward wave and the beam. This is the physical reason for the enhanced radiation from the plasma BWO predicted in the present analysis. The temporal growth rate ($k_i=0$) which corresponds to $\omega_{is}/2\pi$ in Fig. 1(b) has also been calculated in the same 2×2 determinant equation, and the result is shown by a dashed line in Fig. 3. It should be noted that the temporal growth rate is zero for $N_p>1.7\times10^{12}$ cm^{-3}.

So far we have treated an infinitely long slow-wave structure. For a finite-length device, different modes may be linked. In our analysis we considered four modes—two structure modes (forward and backward TM_{01}) and two beam space-charge modes (four roots in the dispersion relation). The oscillation is affected by the reflection coefficients R_1 and R_2 at both ends.[14] The oscillation does not occur at the k_s of the saddle point, but at a growing root k_+ near k_s, where k_+ is found from the following simultaneous equations:

$$D(k_+,\omega)=0,\quad D(k_++\epsilon,\omega)=0\,,$$

$$\epsilon=k_--k_+=-2\pi N/L-i\ln(R)/L\,,$$

$$R=|R_1R_2|,\text{ and } N=\pm1,\pm2,\ldots\,.$$

Here, k_- is the evanescent root propagating in the opposite direction to k_+ in order to feed back the oscillation energy to the original position. In this work, we neglect mode conversion at both ends and consider only the growing wave with highest growth rate k_+ and the evanescent wave with smallest damping rate k_- because the main contribution comes from these two waves. Note that, to have oscillation, in addition to $\omega_i>0$, the spatial growth rate $\mathrm{Im}(k_+)$ must surpass the spatial damping rate $\mathrm{Im}(k_-)$, because $R\leq1.0$. This imposes an additional restriction to backward-wave oscillation. We do not find k_+ in the case of convective instabilities. The phase and group velocities for $L=16.7$ cm and $R=1.0$ are shown in Fig. 2. The thin line for phase velocity denotes the physical solutions which change from $N=-1$ to $N=1$ as N_p increases. The plasma BWOs will not operate ($\omega_i<0$) for L and R less than the critical values[13] for given N_p and N_b.

In conclusion, we present a small-signal-gain calculation for a plasma-loaded slow-wave structure immersed in an infinitely large axial magnetic field for frequencies $\omega>\omega_p$. It was found that the presence of the plasma modifies both the dispersion relation and the nature of the vacuum linear instability. For low plasma densities, the instability is absolute. For large plasma densities, the group velocity changes sign, in which case the instability became convective. The present analysis predicts a considerable increase both in the spatial growth rate and in the oscillation frequency with increased N_p [Fig. 1(b)]. The saturated levels and efficiencies[11] of plasma

BWOs are beyond the scope of the present linear analysis. Nonlinear calculations are underway.

The authors would like to acknowledge helpful discussions with Dr. J. Swegle, Dr. B. Levush, and Dr. T. Antonsen. This work is partly supported by a cooperative research program, National Institute for Fusion Science, Japan, and by the Air Force Weapons Laboratory under a contract administered by the Naval Research Laboratory.

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