## Measurement of branching fraction and direct $\boldsymbol{C P}$ asymmetry in $B^{\mathbf{0}} \rightarrow \rho^{\mathbf{0}} \boldsymbol{\pi}^{\mathbf{0}}$ decays

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We report a measurement of the branching fraction of the decay $B^{0} \rightarrow \rho^{0} \pi^{0}$, using $386 \times 10^{6} B \bar{B}$ pairs collected at the $\mathrm{Y}(4 S)$ resonance with the Belle detector at the KEKB asymmetric-energy $e^{+} e^{-}$collider. We detect $51_{-13}^{+14}$ signal events with a significance of 4.2 standard deviations, including systematic uncertainties, and measure the branching fraction to be $\mathcal{B}\left(B^{0} \rightarrow \rho^{0} \pi^{0}\right)=\left(3.12_{-0.82}^{+0.88}(\right.$ stat $) \pm$ $0.33(\text { syst })_{-0.68}^{+0.50}($ model $\left.)\right) \times 10^{-6}$. We also perform the first measurement of direct $C P$ violating asymmetry in this mode.

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Tests of the Kobayashi-Maskawa model [1] for $C P$ violation are ongoing. In particular, $B$-factories are directing focus towards measurements of the lesser known angles of the Cabibbo-Kobayashi-Maskawa (CKM) triangle, $\phi_{2}$ and $\phi_{3}$. Measurements of $\phi_{2}$ typically rely on timedependent $C P$-violation studies of $B$ meson decays to $\pi^{+} \pi^{-}, \rho^{ \pm} \pi^{\mp}$ and $\rho^{+} \rho^{-}[2,3]$, since the leading tree amplitudes for these processes involve the relevant CKM phases. However, penguin amplitudes may also contribute significantly in these decays and-via introducing additional unknown phases-greatly impair $\phi_{2}$ constraints from the time-dependent measurements. In such cases, isospin analyses can be employed to separate the tree-level process from penguin contamination [4].

Measurements of $\phi_{2}$ from the $\rho \pi$ system rely on knowledge of the $B^{0} \rightarrow \rho^{0} \pi^{0}$ branching fraction [4,5]. Since the tree amplitude of $B^{0} \rightarrow \rho^{0} \pi^{0}$ decay is color suppressed, the decay rate is sensitive to the penguin amplitude contribution. Thus, the $\rho^{0} \pi^{0}$ branching fraction plays a critical role in constraining the $\phi_{2}$ uncertainty due to penguin pollution from time-dependent $B^{0} \rightarrow \rho^{ \pm} \pi^{\mp}$ measurements [3,4]. Furthermore, measurement of $\phi_{2}$ from the full $B \rightarrow \rho \pi$ isospin analysis requires the $\rho^{0} \pi^{0}$ branching fraction along with its $C P$ asymmetry. Since the branching fractions and $C P$ asymmetries of all the other $\rho \pi$ final states have been measured [6], $\rho^{0} \pi^{0}$ is the only channel that remains to complete the isospin pentagons. A simpli-
fication, whereby the pentagons collapse into quadrangles, is also possible if the $\rho^{0} \pi^{0}$ amplitude is sufficiently small.

An alternative technique to measure $\phi_{2}$ from the $\rho \pi$ system, even if penguin contamination is large, is a timedependent amplitude analysis of $B^{0} \rightarrow \pi^{+} \pi^{-} \pi^{0}$ [5]. Here, the interferences between $\rho^{+} \pi^{-}, \rho^{0} \pi^{0}$ and $\rho^{-} \pi^{+}$contributions to the $\pi^{+} \pi^{-} \pi^{0}$ final state provide the critical information on the unknown phases introduced by penguin amplitudes. Recently, the first time-dependent studies of the $\pi^{+} \pi^{-} \pi^{0}$ Dalitz plot have been performed [7]. In these studies, a simplification is made with the assumption that the $\rho^{0} \pi^{0}$ contribution is small. A more complex timedependent Dalitz analysis is required if this is not the case.

The Belle Collaboration reported first evidence of the $B^{0} \rightarrow \rho^{0} \pi^{0}$ decay [8] with a branching fraction larger than most predictions [9], and a central value above the $90 \%$ confidence-level upper limit set by the BABAR Collaboration [10]. In this paper, we report an improved measurement of the $B^{0} \rightarrow \rho^{0} \pi^{0}$ branching fraction [11], using 2.5 times more data, and perform a first direct $C P$ violation search in this mode. The results are consistent with and supersede those reported in our previous publication. The analysis is based on $(385.8 \pm 4.8) \times 10^{6} B \bar{B}$ pairs, collected with the Belle detector at the KEKB asymmetric-energy $e^{+} e^{-}$collider [12] that operates at the $\Upsilon(4 S)$ resonance. The production rates of $B^{+} B^{-}$and $B^{0} \bar{B}^{0}$ pairs are assumed to be equal.

The Belle detector $[13,14]$ is a large-solid-angle magnetic spectrometer that consists of a silicon vertex detector, a 50 -layer central drift chamber (CDC), an array of aerogel threshold Cherenkov counters (ACC), a barrel-like arrangement of time-of-flight scintillation counters (TOF), and an electromagnetic calorimeter comprised of $\mathrm{CsI}(\mathrm{Tl})$ crystals (ECL) located inside a superconducting solenoid coil that provides a 1.5 T magnetic field. An iron fluxreturn located outside of the coil is instrumented to detect $K_{L}^{0}$ mesons and to identify muons.

The $B$ reconstruction procedure is identical to our previously published analysis [8]. Charged tracks are required to originate from the interaction point and have transverse momenta greater than $100 \mathrm{MeV} / c$. Pions are identified by combining information from the ACC, TOF and the CDC $d E / d x$ measurements. We further reject tracks that are consistent with an electron hypothesis. Pairs of photons with invariant masses in the range $0.115 \mathrm{GeV} / c^{2}<m_{\gamma \gamma}<$ $0.154 \mathrm{GeV} / c^{2}$ are used to form the $\pi^{0}$ mesons. The photon energy in the laboratory frame is required to be greater than $50(100) \mathrm{MeV}$ in the barrel (endcap) region of the ECL. The $\pi^{0}$ candidates are required to have transverse momenta greater than $100 \mathrm{MeV} / c$ in the laboratory frame and a loose requirement is made on $\chi_{\pi^{0}}^{2}$, the goodness of fit of a $\pi^{0}$ mass-constrained fit of the two photons. We also veto possible contributions to $\pi^{+} \pi^{-} \pi^{0}$ from charmed ( $b \rightarrow c$ ) decays: $B^{0} \rightarrow D^{-} \pi^{+}, \bar{D}^{0} \pi^{0}$ and $J / \psi \pi^{0}$.

Signal $B$ candidates are identified with two kinematic variables: the beam-energy constrained mass $M_{\mathrm{bc}} \equiv$ $\sqrt{E_{\text {beam }}^{2} / c^{4}-p_{B}^{2} / c^{2}}$ and the energy difference $\Delta E \equiv$ $E_{B}-E_{\text {beam }}$. Here, $E_{B}\left(p_{B}\right)$ is the reconstructed energy (momentum) of the $B$ candidate, and $E_{\text {beam }}$ is the beam energy, all expressed in the center-of-mass (CM) frame. We consider candidate events in the region $-0.2 \mathrm{GeV}<$ $\Delta E<0.4 \mathrm{GeV}$ and $M_{\mathrm{bc}}>5.23 \mathrm{GeV} / c^{2}$; and define signal regions in $\Delta E$ and $M_{\mathrm{bc}}$ as $-0.135 \mathrm{GeV}<\Delta E<$ 0.082 GeV and $5.269 \mathrm{GeV} / c^{2}<M_{\mathrm{bc}}<5.290 \mathrm{GeV} / c^{2}$. To select $\rho^{0} \pi^{0}$ from the $\pi^{+} \pi^{-} \pi^{0}$ candidates, we require the $\pi^{+} \pi^{-}$invariant mass to be in the range $0.5 \mathrm{GeV} / c^{2}<$ $m_{\pi^{+} \pi^{-}}<1.1 \mathrm{GeV} / c^{2}$ and the $\rho^{0}$ helicity angle to satisfy $\left|\cos \theta_{\text {hel }}^{\rho}\right|>0.5$, where $\theta_{\text {hel }}^{\rho}$ is defined as the angle between the negative pion direction and the opposite of the $B$ direction in the $\rho$ rest frame. We explicitly veto contributions from $B^{0} \rightarrow \rho^{ \pm} \pi^{\mp}$ by the requirement $m_{\pi^{ \pm} \pi^{0}}>$ $1.1 \mathrm{GeV} / c^{2}$. This requirement also vetoes the region of the Dalitz plot where the interference between $\rho^{0} \pi^{0}$ and $\rho^{ \pm} \pi^{\mp}$ is strongest. After all selection requirements, $11 \%$ of events have more than one candidate. Among those candidates the one with the smallest $\chi_{\mathrm{vt}}^{2} / \mathrm{ndf}+\chi_{\pi^{0}}^{2} / \mathrm{ndf}$ is selected, where $\chi_{\mathrm{vtx}}^{2}$ is the goodness of fit of a vertexconstrained fit of $\pi^{+} \pi^{-}$.

The dominant background originates from continuum $e^{+} e^{-} \rightarrow q \bar{q}(q=u, d, s, c)$ production. To separate the jetlike $q \bar{q}$ events, we use event shape variables: five modified Fox-Wolfram moments [15], combined into a Fisher
discriminant. We further combine the cosine of the $B$ meson flight direction in the CM system with the output of the Fisher discriminant into a signal/background likelihood variable, $\mathcal{L}_{s / b}$, and define the likelihood ratio $\mathcal{R}=$ $\mathcal{L}_{s} /\left(\mathcal{L}_{s}+\mathcal{L}_{b}\right)$. Additional discrimination against continuum is achieved through use of the $b$-flavour tagging algorithm [16]. We use the parameter $r$, with values between 0 and 1 , as a measure of the confidence that the remaining particles in the event (other than $\pi^{+} \pi^{-} \pi^{0}$ ) originated from a flavour specific $B$ meson decay andas a corollary - not from a continuum process.

We use an iterative procedure to find the optimal contiguous area in $r$ - $\mathcal{R}$ space by maximising $N_{s} / \sqrt{N_{s}+N_{b}}$, where $N_{s}\left(N_{b}\right)$ is the expected number of signal (background) events in the $\Delta E$ and $M_{\mathrm{bc}}$ signal regions. Here, the optimisation procedure assumes a branching fraction for $B^{0} \rightarrow \rho^{0} \pi^{0}$ of $3.3 \times 10^{-6}$ [17]. Anticipating the use of $r$ for its primary purpose of flavour tagging in $C P$ asymmetry fits, the borders of the contiguous area were constrained to match the six $r$ bins employed in previous analyses. The result of the optimisation procedure is that we select events within the region shown in Fig. 1(a). We find 1397 candidates remain in the data.

We obtain the signal yield using an extended unbinned maximum-likelihood fit to the $\Delta E-M_{\mathrm{bc}}$ distribution of the selected candidate events. The likelihood function is defined as

$$
\begin{equation*}
\mathcal{L}=\exp \left(-\sum_{j, l} N_{j, l}\right) \prod_{i}\left(\sum_{j, l} N_{j, l} \mathcal{P}_{j}^{i}\right) . \tag{1}
\end{equation*}
$$

Here, the index $i$ is the event identifier; $l$ distinguishes events in various $r$ bins; and $j$ runs over all six components included in the fitting function - one for the signal, and the others for continuum, $b \rightarrow c$ combinatorial, and the charmless $B$ backgrounds: $B^{+} \rightarrow \rho^{+} \rho^{0}, B^{+} \rightarrow \rho^{+} \pi^{0}$ and $B^{+} \rightarrow$ $\pi^{+} \pi^{0} . N_{j, l}$ represents the number of events, and $\mathcal{P}_{j}^{i}=$ $P_{j}\left(M_{\mathrm{bc}}^{i}, \Delta E^{i}\right)$ are two-dimensional probability density functions (PDFs).

The PDFs for signal, $b \rightarrow c$ and charmless $B$ backgrounds are taken from smoothed two-dimensional histograms obtained from Monte Carlo (MC) simulations. For the $B^{+} \rightarrow \rho^{+} \rho^{0}$ channel, we assume a $100 \%$ longitudinally polarised decay [18]. Small corrections to MC peak positions and widths are applied to the signal PDF. These factors are derived from control samples of reconstructed decays $\quad B^{0} \rightarrow D^{*-} \rho^{+} \quad\left(D^{*-} \rightarrow \bar{D}^{0} \pi^{-}, \quad \bar{D}^{0} \rightarrow K^{+} \pi^{-} ;\right.$ $\rho^{+} \rightarrow \pi^{+} \pi^{0}$ ) and $B^{+} \rightarrow \bar{D}^{0} \rho^{+}\left(\bar{D}^{0} \rightarrow K^{+} \pi^{-} ; \rho^{+} \rightarrow\right.$ $\pi^{+} \pi^{0}$ ), in which we require that the $\pi^{0}$ momentum be greater than $1.8 \mathrm{GeV} / c$ in order to mimic the high momentum $\pi^{0}$ in our signal. The two-dimensional PDF for the continuum background is described as the product of a first-order polynomial in $\Delta E$ and an ARGUS function [19] in $M_{\mathrm{bc}}$. All of the shape parameters describing the continuum background are free parameters in the fit. The normalisations of $B^{+} \rightarrow \rho^{+} \pi^{0} \quad(21.7 \pm 4.4$ events) and


FIG. 1 (color online). (a) Distribution of signal (continuum) events in $r-\mathcal{R}$ space shown with open (shaded) proportional boxes; the marked region (top-right) indicates the area selected. (b) (c) Distributions of $\Delta E\left(M_{\mathrm{bc}}\right)$ in the signal region of $M_{\mathrm{bc}}(\Delta E)$. Projection of the fit result is shown as the thick solid curve; the thin solid line represents the signal component; the dashed, dotted and dash-dotted curves represent, respectively, the cumulative background components from continuum processes, $b \rightarrow c$ decays, and charmless $B$ backgrounds. (d), (e) Distributions of fit yields in $m_{\pi^{+}} \pi^{-}$and $\cos \theta_{\text {hel }}^{\rho}$ variables for $\rho^{0} \pi^{0}$ candidate events. Points with error bars represent data fit results, and the histograms show signal MC expectation; the selection requirements described in the text are shown as dashed lines.
$B^{+} \rightarrow \pi^{+} \pi^{0}(21.0 \pm 5.5$ events $)$ are fixed in the fit according to previous measurements $[6,20]$, and that of $b \rightarrow c$ background ( $62 \pm 62$ events) according to MC expectation (assigning a conservative error); the normalisations of all other components are allowed to float.

The fit result is shown in Fig. 1(b) and 1(c). The signal yield is found to be $50.9_{-13.4}^{+14.3}$ with $4.5 \sigma$ significance. The significance is defined as $\sqrt{-2 \ln \left(\mathcal{L}_{0} / \mathcal{L}_{\max }\right)}$, where $\mathcal{L}_{\max }$ ( $\mathcal{L}_{0}$ ) denotes the likelihood with the signal yield at its nominal value (fixed to zero). The contribution from $B^{+} \rightarrow$ $\rho^{+} \rho^{0}$ decays (which peaks in the low $\Delta E$ region) is obtained from the fit as $43.1_{-12.1}^{+13.2}$ events; this value is consistent with the MC expectation ( $33.9_{-9.8}^{+8.1}$ events) based on our branching fraction measurement of $B^{+} \rightarrow \rho^{+} \rho^{0}$ [18]. A possible contribution from $B \rightarrow \omega\left(\pi^{+} \pi^{-} \pi^{0}\right) \pi^{0}$ decays is also accounted for by floating the $B^{+} \rightarrow \rho^{+} \rho^{0}$ PDF, since the two decays have similar distributions in $\Delta E$ and $M_{\mathrm{bc}}$. To verify that the signal candidates originate from $B^{0} \rightarrow \rho^{0} \pi^{0}$ decays, we change the criteria on $m_{\pi^{+} \pi^{-}}$and $\cos \theta_{\text {hel }}^{\rho}$ in turn, and repeat fits to the $\Delta E-M_{\mathrm{bc}}$ distribution. The yields obtained in each $m_{\pi^{+} \pi^{-}}$and $\cos \theta_{\text {hel }}^{\rho}$ bin are shown in Fig. 1(d) and 1(e).

The $\cos \theta_{\text {hel }}^{\rho}$ distribution is used to limit contributions from $B^{0} \rightarrow \sigma \pi^{0}, f_{0}(980) \pi^{0}, \eta^{\prime} \pi^{0}, K_{S} \pi^{0}$ and $\pi^{+} \pi^{-} \pi^{0}$ (nonresonant), which are expected to be flat in this variable. We perform a $\chi^{2}$ fit including components for pseudoscalar $\rightarrow$ pseudoscalar vector $\left(P V \sim \cos ^{2} \theta_{\text {hel }}^{\rho}\right)$, and pseudoscalar $\rightarrow$ pseudoscalar scalar ( $P S \sim$ flat) decays, for which the shapes are obtained from our $\rho^{0} \pi^{0}$ signal MC , and a sample of $\sigma \pi^{0} \mathrm{MC}$ [21], respectively. We also include a linear term to allow for possible interference. We find that the $P S$ level is consistent with zero; taking its uncertainty into account, we assign a model error of $+0.0-15.0 \%$ to the $P V$ component. The $m_{\pi^{+}} \pi^{-}$distribution is consistent with the expectation from $B^{0} \rightarrow \rho^{0} \pi^{0}$ production.

To extract the branching fraction, we determine the reconstruction efficiency, $(4.99 \pm 0.03) \%$, from MC and
correct for small differences between data and MC in the pion identification and continuum suppression requirements. The correction factor due to charged pion identification ( 0.872 ) is obtained in bins of track momentum and polar angle from an inclusive $D^{*}$ control sample ( $D^{*-} \rightarrow$ $\bar{D}^{0} \pi^{-}, \bar{D}^{0} \rightarrow K^{+} \pi^{-}$). The corresponding systematic error is $\pm 3.1 \%$. For the continuum suppression requirement on $r$ and $\mathcal{R}$, we use the control sample $B^{0} \rightarrow D^{-} \rho^{+}\left(D^{-} \rightarrow\right.$ $K^{+} \pi^{-} \pi^{-} ; \rho^{+} \rightarrow \pi^{+} \pi^{0}$ ) to obtain an efficiency correction factor of 0.972 and a corresponding systematic error of $\pm 6.0 \%$.

We calculate additional systematic errors from the following sources: PDF shapes by varying parameters by $\pm 1 \sigma(+0.9-2.0 \%) ; \pi^{0}$ reconstruction efficiency by comparing the yields of $\eta \rightarrow \pi^{0} \pi^{0} \pi^{0}$ and $\eta \rightarrow \gamma \gamma$ between data and MC ( $\pm 4.0 \%$ ); track finding efficiency from a study of partially reconstructed $D^{*}$ decays ( $\pm$ $2.4 \%$ ); and data-MC efficiency differences due to the $\Delta E>-0.2 \mathrm{GeV}$ requirement ( $\pm 2.0 \%$ ). We repeat the fit after changing the normalization of the fixed background components according to the given errors and obtain a systematic error of $+2.0-1.8 \%$. Using a large MC sample, the total systematic error from possible charmless $B$ decays not otherwise included, $B^{0} \rightarrow K^{* 0} \pi^{0}$ ( $5.4 \%), B^{+} \rightarrow K^{*+} \pi^{0}(1.5 \%)$ and $B^{0} \rightarrow K^{+} \rho^{-}(0.5 \%)$, is $\pm 5.6 \%$.

When the normalizations of all the backgrounds fixed in the fit are simultaneously increased by $1 \sigma$, the statistical significance decreases from $4.5 \sigma$ to $4.2 \sigma$, we interpret the latter value as the significance of our result. Finally, we estimate the uncertainty due to possible interference with $B^{0} \rightarrow \rho^{ \pm} \pi^{\mp}$ by varying the $m_{\pi^{ \pm} \pi^{0}}$ veto requirement from $m_{\pi^{ \pm} \pi^{0}}>0 \mathrm{MeV} / c^{2}$ (no veto) to $m_{\pi^{ \pm} \pi^{0}}>1.7 \mathrm{GeV} / c^{2}$. We find the largest change in the result to be within $\pm 16 \%$, and we include this value in the model error, so that the obtained $B^{0} \rightarrow \rho^{0} \pi^{0}$ branching fraction is

$$
\mathcal{B}=\left(3.12_{-0.82}^{+0.88}(\text { stat }) \pm 0.33(\text { syst })_{-0.68}^{+0.50}(\text { model })\right) \times 10^{-6}
$$

Having observed a significant $B^{0} \rightarrow \rho^{0} \pi^{0}$ signal, we utilize the $B^{0} / \bar{B}^{0}$ separation provided by the flavour tagging to measure the $C P$ asymmetry. For this purpose we replace $\mathcal{P}_{j}^{i}$ of Eq. (1) with the expression

$$
\begin{equation*}
\mathcal{P}_{j, l}^{i}=\frac{1}{2}\left[1+q^{i} \cdot\left(\mathcal{A}_{C P}^{\prime}\right)_{j, l}\right] P_{j}\left(M_{\mathrm{bc}}^{i}, \Delta E^{i}\right), \tag{2}
\end{equation*}
$$

in which the indices keep the same meaning. In this equation, $q$ represents the $b$-flavour charge $[q=+1(-1)$ when the tagging $B$ meson is a $\left.B^{0}\left(\bar{B}^{0}\right)\right]$ and $\mathcal{A}_{C P}^{\prime}$ denotes the effective charge asymmetry, such that $\left(\mathcal{A}_{C P}^{\prime}\right)_{j, l}=$ $\left(\mathcal{A}_{C P}\right)_{j}\left(1-2 \chi_{d}\right)\left(1-2 w_{l}\right)$. Here, $\left(\mathcal{A}_{C P}\right)_{j}$ are the charge asymmetries for the signal and the background components. Further, $\chi_{d}=0.182 \pm 0.015$ [22] is the timeintegrated mixing parameter and $w_{l}$ is the wrong-tag fraction. For continuum background, $\chi_{d}$ and $w_{l}$ are set to zero. The data is divided into the six $r$-bins, and the $r$-dependent wrong-tag fractions, $w_{l}(l=1, \ldots, 6)$, are determined using a high statistics sample of self-tagged $B^{0} \rightarrow D^{(*)-} \pi^{+}$, $D^{*-} \rho^{+}$and $D^{*-} \ell^{+} \nu$ events [16].

The total number of signal, continuum background and $\rho^{+} \rho^{0}$ events are free parameters in the fit, and the remaining background components (from $b \rightarrow c, \rho^{+} \pi^{0}$ and $\pi^{+} \pi^{0}$ decays) are fixed. Also, the relative fractions for the signal and continuum background components in different $r$ bins are allowed to float in the fit; for the $b \rightarrow c$ and charmless $B$ decay backgrounds, they are fixed. The only free $\mathcal{A}_{C P}$ parameter in the nominal fit is that of our signal; the others are fixed to be zero (for continuum and $b \rightarrow c$ ) or at their previously measured values (for charmless $B$ backgrounds) [20]. We measure the direct $C P$ asymmetry in $B^{0} \rightarrow \rho^{0} \pi^{0}$ decays to be

$$
\mathcal{A}_{C P}=-0.53_{-0.84}^{+0.67}(\text { stat })_{-0.15}^{+0.10}(\text { syst }) .
$$

The impact of background asymmetry ( $+0.058-0.127$ ) is the largest contribution to the systematic error; it is estimated by releasing, in turn, all of the background $\mathcal{A}_{C P}$ parameters (limiting them within $\pm 1 \sigma$ range of their measured values for the charmless $B$ decays), and summing in quadrature the differences obtained from the central $\mathcal{A}_{C P}$ value. A similar sum gives $+0.059-0.057$ as the systematic uncertainty obtained by varying all other fixed parameters in the fit, including $\chi_{d}$ and $w_{l}$ values, by $\pm 1 \sigma$. Finally a systematic error of $\pm 0.058$ is obtained as a result of a null asymmetry test, when the same analysis procedure is applied to the $B^{0} \rightarrow D^{-} \rho^{+} \quad\left(D^{-} \rightarrow\right.$ $K^{+} \pi^{-} \pi^{-} ; \rho^{+} \rightarrow \pi^{+} \pi^{0}$ ) control sample. To illustrate the asymmetry, we show the results separately for $\rho^{0} \pi^{0}$ candidate events tagged as $q=+1$ and $q=-1$ in Fig. 2.

In summary, using $386 \times 10^{6} B \bar{B}$ pairs, we confirm evidence of $B^{0} \rightarrow \rho^{0} \pi^{0}$ decays with a branching fraction higher than most theoretical predictions [9]. The central value remains only slightly above the $90 \%$ confidence-


FIG. 2 (color online). $\Delta E$ and $M_{\mathrm{bc}}$ distributions (with projections of the fit results) shown separately for events tagged as $q=$ +1 (left) and $q=-1$ (right).
level upper limit set by the BABAR Collaboration [10], and is in agreement with the upper limit set by the CLEO Collaboration [6]. Our measurement is consistent with, and supersedes, our previous result [8]. We have also performed a first measurement of direct $C P$ violation in the $B^{0} \rightarrow \rho^{0} \pi^{0}$ mode and find no statistically significant asymmetry.
The large $\rho^{0} \pi^{0}$ branching fraction suggest that one can only impose a loose constraint on penguin uncertainty in the determination of $\phi_{2}$ from time-dependent $B^{0} \rightarrow \rho^{ \pm} \pi^{\mp}$ measurements. It also implies that a useful measurement of $\phi_{2}$ from the full $\rho \pi$ isospin analysis may be impractical even with super $B$-factory like luminosities [23]. Therefore, we can expect that the best measurements of $\phi_{2}$ from the $\rho \pi$ system will come from the full timedependent amplitude analysis of $B^{0} \rightarrow \pi^{+} \pi^{-} \pi^{0}$.

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