

Electric and magnetic screening masses at finite temperature from generalized Polyakov-line correlations in two-flavor lattice QCD

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Screenings of the quark-gluon plasma in electric and magnetic sectors are studied on the basis of generalized Polyakov-line correlation functions in lattice QCD simulations with two flavors of improved Wilson quarks. Using the Euclidean-time reflection (\mathcal{R}) and the charge conjugation (\mathcal{C}), electric and magnetic screening masses are extracted in a gauge-invariant manner. Long distance behavior of the standard Polyakov-line correlation in the quark-gluon plasma is found to be dictated by the magnetic screening. Also, the ratio of the two screening masses agrees with that obtained from the dimensionally-reduced effective field theory and the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory.

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I. INTRODUCTION

Screenings inside the quark-gluon plasma at high temperature are dictated by thermal fluctuations of quarks and gluons, and are characterized by the electric and magnetic screening masses [1]. From the phenomenological point of view, screening of color charge affects the force between heavy-quarks and determines the fate of heavy-quark bound states such as J/ψ and Υ inside hot QCD matter produced in relativistic heavy-ion collisions at RHIC and LHC [2,3].

Nonperturbative determinations of the screening masses have been attempted from various approaches in the past: use of Polyakov-line correlations with appropriate projection in color space [4–7], use of correlations of spatially local operators classified in terms of Euclidean-time reflection [8], use of dimensionally-reduced effective field theory [9,10], and a direct lattice evaluation of the gluon propagator under gauge fixing [11]. The screening masses of the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory for large 't Hooft coupling have been also analyzed on the basis of the AdS/CFT correspondence [12].

In this paper, we extract electric and magnetic screening masses from gauge-invariant Polyakov-line correlations in two-flavor lattice QCD simulations (a preliminary account has been reported in [13]). According to the symmetry properties under the Euclidean-time reflection \mathcal{R} and the charge conjugation \mathcal{C} [8], we decompose the standard Polyakov-line operator Ω into four independent types, $\Omega_{M\pm}$ and $\Omega_{E\pm}$. Here, M and E stand for the \mathcal{R} -even magnetic sector and the \mathcal{R} -odd electric sector, respectively, while \pm stands for the even or odd under \mathcal{C} . Then,

the magnetic (electric) screening mass is extracted from the correlation of $\text{Tr}\Omega_{M+}$ ($\text{Tr}\Omega_{E-}$). In our simulations on a $16^3 \times 4$ lattice, we employ two-flavor lattice QCD with improved Wilson quarks coupled to an renormalization group improved glue (Iwasaki gauge action) for the temperatures $T/T_{\text{pc}} \simeq 1-4$ with T_{pc} being the pseudocritical temperature. We take the quark masses corresponding to $m_{\text{PS}}/m_{\text{V}} = 0.65$ and 0.80 , where m_{PS} (m_{V}) is pseudoscalar (vector) meson mass at $T = 0$.

This paper is organized as follows: In Sec. II, we discuss the properties of the Polyakov-line operators under \mathcal{R} and \mathcal{C} . Numerical simulations and the results of the screening masses are shown in Sec. III. Comparison of our results with those obtained from the dimensionally-reduced effective field theory and the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory are also given. Section IV is devoted to summary and concluding remarks.

II. GENERALIZED POLYAKOV-LINE CORRELATIONS

We start with the standard Polyakov-line operator

$$\Omega(\mathbf{x}) = P \exp \left[ig \int_0^{1/T} d\tau A_4(\tau, \mathbf{x}) \right]. \quad (1)$$

Its gauge-invariant correlation function reads

$$C_{\Omega}(r, T) \equiv \langle \text{Tr}\Omega^{\dagger}(\mathbf{x}) \text{Tr}\Omega(\mathbf{y}) \rangle - |\langle \text{Tr}\Omega \rangle|^2, \quad (2)$$

where $r \equiv |\mathbf{x} - \mathbf{y}|$ and we define $\text{Tr}\Omega \equiv \frac{1}{N_c} \sum_{\alpha=1}^{N_c} \Omega^{\alpha\alpha}$ with α being color indices. The screening mass is defined from the exponential falloff of this gauge-invariant corre-

lation,

$$C_{\Omega}(r, T) \xrightarrow{r \rightarrow \infty} \gamma_{\Omega}(T) \frac{e^{-m_{\Omega}(T)r}}{rT}. \quad (3)$$

Since the magnetic gluon is less screened at finite T , the dominant contribution to $m_{\Omega}(T)$ would come from non-perturbative magnetic sector of QCD as pointed out in [8].

In order to extract the electric and magnetic screening masses separately, we classify the Polyakov-line operator into different classes on the basis of the symmetries under \mathcal{R} and \mathcal{C} which are good symmetries of QCD at zero chemical potential [8]. Under \mathcal{R} and \mathcal{C} , the gluon fields transform as

$$A_i(\tau, \mathbf{x}) \xrightarrow{\mathcal{R}} A_i(-\tau, \mathbf{x}), \quad A_4(\tau, \mathbf{x}) \xrightarrow{\mathcal{R}} -A_4(-\tau, \mathbf{x}), \quad (4)$$

$$A_{\mu}(\tau, \mathbf{x}) \xrightarrow{\mathcal{C}} -A_{\mu}^*(\tau, \mathbf{x}). \quad (5)$$

We call an operator magnetic (electric) if it is even (odd) under \mathcal{R} . Under these transformations, the standard Polyakov-line operator transforms as

$$\Omega \xrightarrow{\mathcal{R}} \Omega^{\dagger}, \quad \Omega \xrightarrow{\mathcal{C}} \Omega^*. \quad (6)$$

Then, we can define magnetic (electric) operator which is \mathcal{R} even (\mathcal{R} odd) as

$$\Omega_M \equiv \frac{1}{2}(\Omega + \Omega^{\dagger}), \quad \Omega_E \equiv \frac{1}{2}(\Omega - \Omega^{\dagger}). \quad (7)$$

Furthermore, they can be decomposed into \mathcal{C} -even and \mathcal{C} -odd operators as

$$\Omega_{M\pm} \equiv \frac{1}{2}(\Omega_M \pm \Omega_M^*), \quad \Omega_{E\pm} \equiv \frac{1}{2}(\Omega_E \pm \Omega_E^*). \quad (8)$$

Using these operators and noting the fact that $\text{Tr}\Omega_{M-} = \text{Tr}\Omega_{E+} = 0$, we can define two generalized gauge-invariant Polyakov-line correlation functions,

$$C_{M+}(r, T) \equiv \langle \text{Tr}\Omega_{M+}(\mathbf{x}) \text{Tr}\Omega_{M+}(\mathbf{y}) \rangle - |\langle \text{Tr}\Omega \rangle|^2, \quad (9)$$

$$C_{E-}(r, T) \equiv \langle \text{Tr}\Omega_{E-}(\mathbf{x}) \text{Tr}\Omega_{E-}(\mathbf{y}) \rangle. \quad (10)$$

Note that $\text{Tr}\Omega_{M+}$ ($\text{Tr}\Omega_{E-}$) is nothing but the real (imaginary) part of $\text{Tr}\Omega$, and that $\langle \text{Tr}\Omega \rangle$ is real due to the \mathcal{C} symmetry.

The electric and magnetic screening masses can be defined from the above gauge-invariant correlation functions through their long distance behavior:

$$C_{M+}(r, T) \xrightarrow{r \rightarrow \infty} \gamma_{M+}(T) \frac{e^{-m_{M+}(T)r}}{rT}, \quad (11)$$

$$C_{E-}(r, T) \xrightarrow{r \rightarrow \infty} \gamma_{E-}(T) \frac{e^{-m_{E-}(T)r}}{rT}. \quad (12)$$

Notice that the standard Polyakov-line correlation function and the above generalized correlation functions are simply related as

$$C_{\Omega}(r, T) = C_{M+}(r, T) - C_{E-}(r, T). \quad (13)$$

Thus the separate determination of $C_{M+}(r, T)$ and

$C_{E-}(r, T)$ on the lattice enables us to study the relative importance of the magnetic and electric sector in a non-perturbative manner.

III. LATTICE SIMULATIONS

A. Lattice setup

We employ a renormalization group improved gauge action S_g and a clover-improved Wilson quark action with two flavors S_q :

$$S_g = -\beta \sum_x \left(c_0 \sum_{\mu < \nu, \mu, \nu=1}^4 W_{\mu\nu}^{1 \times 1}(x) + c_1 \sum_{\mu \neq \nu, \mu, \nu=1}^4 W_{\mu\nu}^{1 \times 2}(x) \right), \quad (14)$$

$$S_q = \sum_{f=1,2} \sum_{x,y} \bar{q}_x^f D_{x,y} q_y^f, \quad (15)$$

where $\beta = 6/g^2$, $c_1 = -0.331$, $c_0 = 1 - 8c_1$, and $W_{\mu\nu}^{n \times m}$ is a $n \times m$ rectangular shaped Wilson loop and

$$D_{x,y} = \delta_{xy} - K \sum_{\mu} \{ (1 - \gamma_{\mu}) U_{x,\mu} \delta_{x+\hat{\mu},y} + (1 + \gamma_{\mu}) U_{x,\mu}^{\dagger} \delta_{x,y+\hat{\mu}} \} - \delta_{xy} c_{SW} K \sum_{\mu < \nu} \sigma_{\mu\nu} F_{\mu\nu}. \quad (16)$$

Here, K is the hopping parameter and $F_{\mu\nu}$ is the lattice field strength, $F_{\mu\nu} = 1/8i(f_{\mu\nu} - f_{\mu\nu}^{\dagger})$, with $f_{\mu\nu}$ being the standard clover-shaped combination of gauge links. For the clover coefficient c_{SW} , we adopt the mean field value using $W^{1 \times 1}$ which was calculated in the one-loop perturbation theory,

$$c_{SW} = (W^{1 \times 1})^{-3/4} = (1 - 0.8412\beta^{-1})^{-3/4}. \quad (17)$$

Our simulations are performed on a lattice with a size of $N_s^3 \times N_t = 16^3 \times 4$ along the lines of constant physics, i.e. the lines of constant m_{PS}/m_V (the ratio of pseudoscalar and vector meson masses) at $T = 0$ in the space of simulation parameters. Details on the lines of constant physics and the phase diagram for $N_t = 4$ with the same actions as above are given in Refs. [7,14]. We take two values, $m_{PS}/m_V = 0.65$ and 0.80 , with the temperature range of $T/T_{pc} \sim 1.0-4.0$ (10 points) and $1.0-3.0$ (7 points), respectively, where T_{pc} is the pseudocritical temperature along the line of constant physics [7,14]. The number of trajectories for each run after thermalization is 5000–6000, and we measure physical quantities at every 10 trajectories. The statistical errors are estimated by a jackknife method with a bin size of 100 trajectories.

B. Screening masses

Results of the generalized Polyakov-line correlations $C_{M+(E-)}(r, T)$ are shown in Fig. 1 for $m_{PS}/m_V = 0.65$ (upper panel) and 0.80 (lower panel). These figures show that (i) the magnetic correlation and the electric correlation have opposite signs, and (ii) the magnetic correlation has

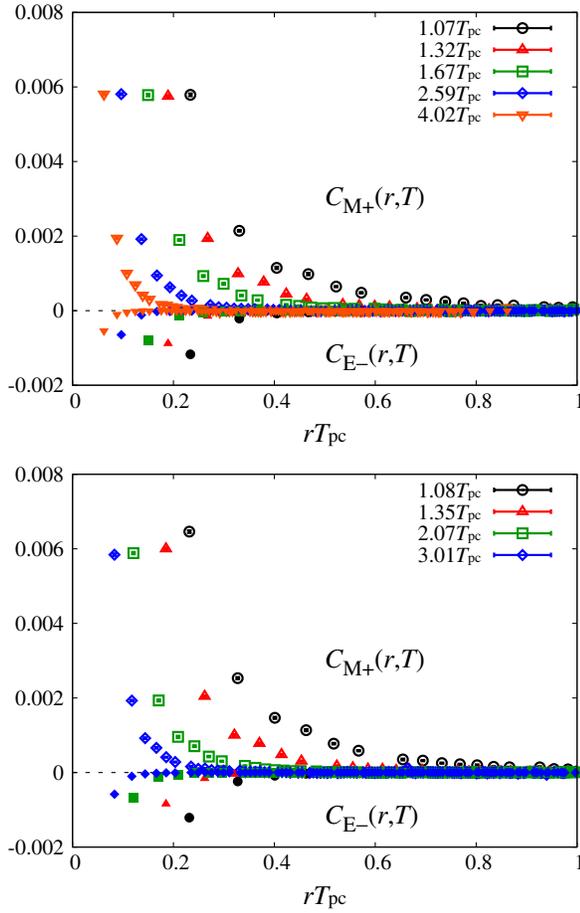


FIG. 1 (color online). Results of $C_{M+}(r, T)$ and $C_{E-}(r, T)$ for several temperatures as a function of rT_{pc} at $m_{PS}/m_V = 0.65$ (upper panel) and 0.80 (lower panel).

larger magnitude and longer range than the electric correlation at long distances. The latter implies that the standard Polyakov-line correlation is dominated by the magnetic sector. In Fig. 2, we show $C_{M+}(r, T)$ (upper panel) and $-C_{E-}(r, T)$ (lower panel) in a logarithmic scale as a function of rT for $m_{PS}/m_V = 0.80$. These correlators behave linearly and are scaled well by rT at high temperature, so that the screening masses can be extracted by fitting the correlators using the Yukawa form defined in Eqs. (11) and (12).

We fit these correlators by Eqs. (11) and (12) in the same interval of $0.5 \leq rT \leq 1.0$ by minimizing χ^2/N_{dof} .¹ Upper (lower) panel of Fig. 3 shows results of screening masses in the magnetic and electric sector at $m_{PS}/m_V = 0.65$ (0.80) as a function of temperature. Numerical results are summarized in Tables I and II for $m_{PS}/m_V = 0.65$ and 0.80 , respectively. The screening mass obtained from the standard Polyakov-line correlation function $m_{\Omega}(T)$ using Eq. (3) is also shown. As expected, the magnetic sector has

¹We study the fit range dependence of the results, and find that the magnitude of systematic error due to the fit range is smaller than or comparable to the statistical errors at $T \gtrsim 1.2T_{pc}$.

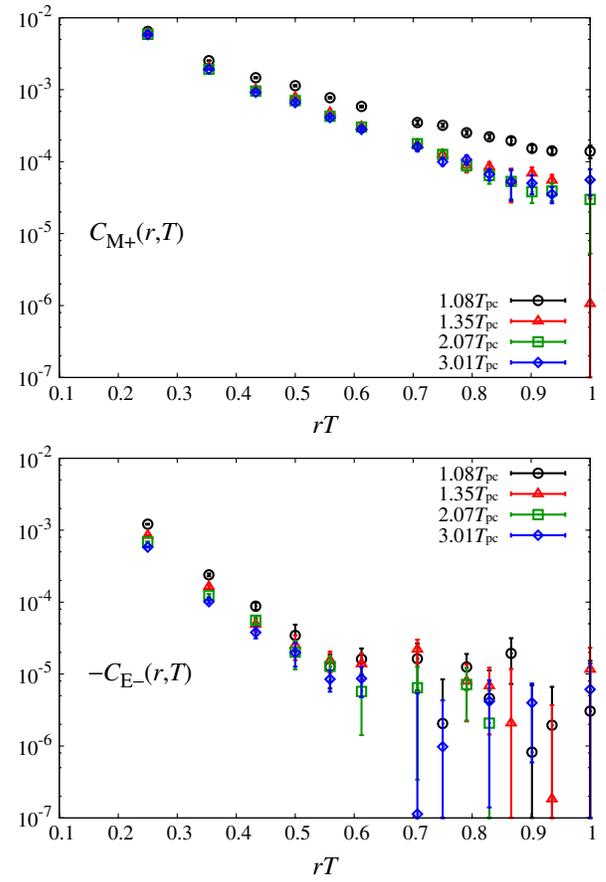


FIG. 2 (color online). Results of $C_{M+}(r, T)$ (upper panel) and $-C_{E-}(r, T)$ (lower panel) in a logarithmic scale as a function of rT at $m_{PS}/m_V = 0.80$.

longer range than the electric sector [$m_{M+}(T)$ is smaller than $m_{E-}(T)$] and also the standard screening mass is dominated by the magnetic mass [$m_{\Omega}(T) \approx m_{M+}(T)$].

C. Comparison to other approaches

Let us compare our results with the predictions by the dimensionally-reduced effective field theory [three-dimensional (3D)-effective field theory(EFT)] and the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory (SYM).

In the 3D-EFT approach [9], the screening masses in various channels with the quantum numbers $J_{\mathcal{R}}^{PC}$ are calculated: Here, \mathcal{P} is a parity in a two-dimensional plane perpendicular to \mathbf{x} - \mathbf{y} , and J is an angular momentum in the two-dimensional plane. Since the lowest masses for $(\mathcal{R}, \mathcal{C}) = (+, +)$ and $(\mathcal{R}, \mathcal{C}) = (-, -)$ are in $(J, \mathcal{P}) = (0, +)$ channels, we extract m_{M+} from the 0_{++} channel and m_{E-} from the 0_{+-} channel. Tables 6 and 8 of Ref. [9] for the lattice coupling $\beta = 21$, lattice size $L = 30$, $N_f = 2$ and $T \approx 2\Lambda_{\overline{\text{MS}}}$ lead to

3D-EFT($N_f = 2$):

$$\begin{aligned} m_{M+}/T &= 3.96(5), & m_{E-}/T &= 7.01(10), \\ m_{E-}/m_{M+} &\approx 1.77. \end{aligned} \quad (18)$$

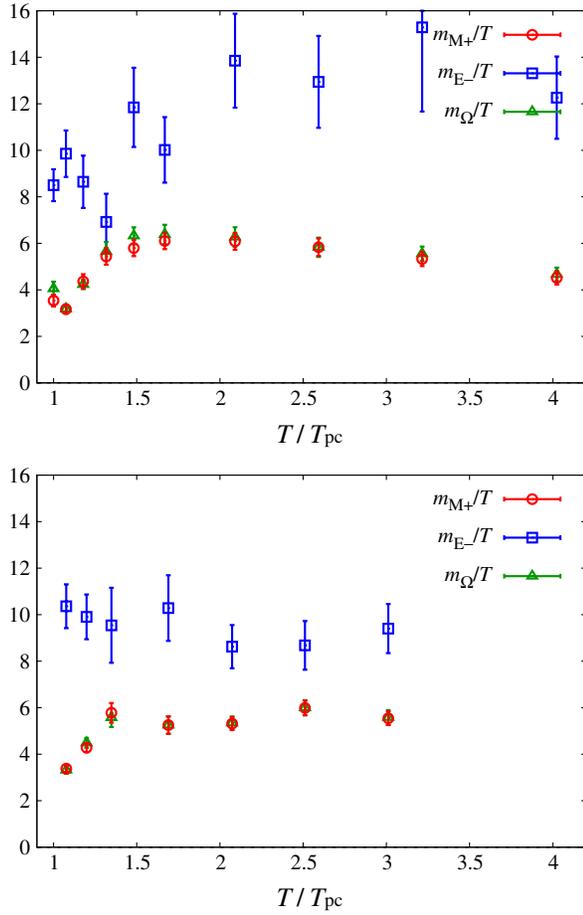


FIG. 3 (color online). Results of magnetic and electric screening mass m_{M+}/T and m_{E-}/T together with the screening mass obtained from the standard Polyakov-line correlation m_{Ω}/T as a function of temperature at $m_{PS}/m_V = 0.65$ (upper panel) and 0.80 (lower panel).

The screening masses of the Polyakov-line correlation functions in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory in the limit of large N_c and large 't Hooft coupling ($\lambda = g^2 N_c$) were calculated by AdS/CFT correspondence [12]. Under the same identification in the $(J, \mathcal{P}) = (0, +)$ channel as discussed above, Table 1 of Ref. [12] leads to

$\mathcal{N} = 4$ SYM:

$$\begin{aligned} m_{M+}/T &= 7.34, & m_{E-}/T &= 16.05, \\ m_{E-}/m_{M+} &= 2.19. \end{aligned} \quad (19)$$

Results of our screening masses and their ratio for $1.67 < T/T_{pc} < 3.22$ with $m_{PS}/m_V = 0.65$ in Table I indicate that

four-dimensional lattice QCD ($N_f = 2$):

$$\begin{aligned} m_{M+}/T &= 5.8(2), & m_{E-}/T &= 13.0(11), \\ m_{E-}/m_{M+} &= 2.3(3). \end{aligned} \quad (20)$$

TABLE I. Results of screening masses at $m_{PS}/m_V = 0.65$. First parentheses show statistical errors, while the second parentheses show systematic errors calculated from difference between screening masses in the range of $0.5 < rT < 1.0$ and $\sqrt{6}/4 < rT < 1.0$.

T/T_{pc}	m_{M+}/T	m_{E-}/T
1.00	3.5(2)(1)	8.5(6)(17)
1.07	3.2(1)(3)	9.9(10)(0)
1.18	4.4(3)(1)	8.6(11)(0)
1.32	5.4(3)(4)	6.9(12)(23)
1.48	5.8(3)(2)	11.8(17)(14)
1.67	6.1(3)(13)	10.0(14)(7)
2.09	6.1(3)(5)	13.9(20)(1)
2.59	5.8(3)(0)	12.9(19)(45)
3.22	5.3(3)(2)	15.3(36)(25)
4.02	4.5(2)(2)	12.3(17)(9)

TABLE II. Results of screening masses at $m_{PS}/m_V = 0.80$ with the errors calculated by the same procedure as Table I.

T/T_{pc}	m_{M+}/T	m_{E-}/T
1.08	3.4(1)(0)	10.4(9)(0)
1.20	4.3(1)(1)	9.9(9)(0)
1.35	5.8(4)(3)	9.5(16)(42)
1.69	5.3(3)(2)	10.3(14)(2)
2.07	5.3(2)(1)	8.6(9)(3)
2.51	6.0(3)(1)	8.7(10)(10)
3.01	5.5(2)(0)	9.4(10)(5)

In all three cases above, we have an inequality $m_{M+} < m_{E-}$ so that the magnetic sector dominates at large distances. Since we cannot compare the absolute magnitude of the screening masses in QCD and that in $\mathcal{N} = 4$ SYM due to different number of degrees of freedom, we take a ratio, m_{E-}/m_{M+} (Table III), and make a comparison for the three cases in Fig. 4. We find that the ratio agrees well with each other for the temperature range of $1.5 < T/T_{pc} < 3$.

TABLE III. Results of the screening ratio m_{E-}/m_{M+} in $N_f = 2$ lattice QCD simulations.

$m_{PS}/m_V = 0.65$		0.80	
T/T_{pc}	m_{E-}/m_{M+}	T/T_{pc}	m_{E-}/m_{M+}
1.00	2.4(2)	1.08	3.1(2)
1.07	3.1(3)	1.20	2.3(2)
1.18	2.0(3)	1.35	1.7(3)
1.32	1.3(2)	1.69	2.0(3)
1.48	2.0(3)	2.07	1.6(1)
1.67	1.6(2)	2.51	1.4(1)
2.09	2.3(3)	3.01	1.7(2)
2.59	2.2(3)		
3.22	2.9(6)		
4.02	2.7(4)		

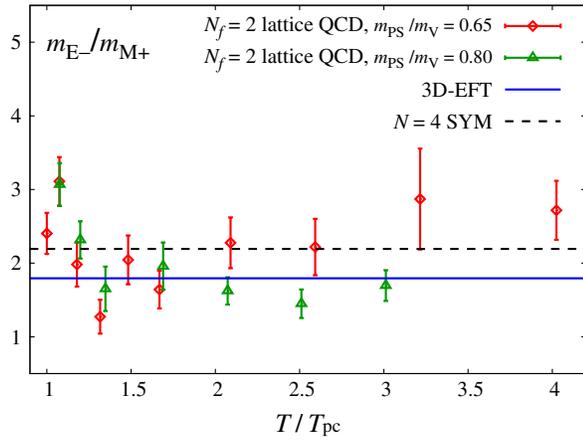


FIG. 4 (color online). Comparison of the screening ratio, m_{E-}/m_{M+} , with predictions in the dimensionally-reduced effective field theory (3D-EFT) [9] and $\mathcal{N} = 4$ supersymmetric Yang-Mills theory (SYM) [12].

IV. SUMMARY

We investigated the screening properties of the quark-gluon plasma from the Polyakov-line correlation functions classified under the Euclidean-time reflection (\mathcal{R}) symmetry and the charge conjugation (\mathcal{C}) symmetry. The Polyakov-line correlators with the quantum numbers of $(\mathcal{R}, \mathcal{C}) = (\text{even}, +)$ and $(\text{odd}, -)$ are defined in gauge-invariant form.

From the lattice simulations of $N_f = 2$ QCD above T_{pc} with the renormalization group improved gluon action and the clover-improved Wilson quark action, we extracted the magnetic screening mass m_{M+} and the electric screening mass m_{E-} , and found that $m_{E-}/m_{M+} = 2.1(2)$ for $1 < T_{pc} < 3$. Therefore, the standard Polyakov-line correlation

functions at long distance is dictated by the magnetic screening. Our electric-magnetic ratio is consistent with those obtained from the dimensionally-reduced effective field theory (3D-EFT) and the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory (SYM).

In the present work, we have not attempted to make projections of the Polyakov-line operator Ω to the operators with definite J (two-dimensional angular momentum) and \mathcal{P} (two-dimensional parity). It would be a future task to make such projections, so that we can compare the four-dimensional lattice results with 3D-EFT and SYM in more details. In Ref. [13], we extracted four screening masses \tilde{m}_X ($X = M+, M-, E+, E-$) from gauge-fixed correlation functions $\tilde{C}_X(r, T) = \langle \text{Tr}[\Omega_X(\mathbf{x})\Omega_X(\mathbf{y})] \rangle$; we found that the long distance behavior of the color-singlet Polyakov-line correlation functions is dictated by $\tilde{m}_{E\pm}$ which is smaller than m_{M+} and m_{E-} for $T > 1.5T_{pc}$. Further studies are needed to have physical interpretation of this result.

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- [1] Reviewed in U. Kraemmer and A. Rebhan, *Rep. Prog. Phys.* **67**, 351 (2004).
- [2] T. Matsui and H. Satz, *Phys. Lett. B* **178**, 416 (1986); T. Hashimoto, K. Hirose, T. Kanki, and O. Miyamura, *Phys. Rev. Lett.* **57**, 2123 (1986).
- [3] Reviewed in H. Satz, *Nucl. Phys.* **A783**, 249 (2007).
- [4] S. Nadkarni, *Phys. Rev. D* **33**, 3738 (1986); **34**, 3904 (1986).
- [5] O. Kaczmarek, F. Karsch, F. Zantow, and P. Petreczky, *Phys. Rev. D* **70**, 074505 (2004); **72**, 059903(E) (2005); O. Kaczmarek and F. Zantow, *Phys. Rev. D* **71**, 114510 (2005); M. Döring, S. Ejiri, O. Kaczmarek, F. Karsch, and E. Laermann, *Eur. Phys. J. C* **46**, 179 (2006).
- [6] A. Nakamura and T. Saito, *Prog. Theor. Phys.* **111**, 733 (2004).
- [7] Y. Maezawa, N. Ukita, S. Aoki, S. Ejiri, T. Hatsuda, N. Ishii, and K. Kanaya (WHOT-QCD Collaboration), *Phys. Rev. D* **75**, 074501 (2007).
- [8] P. Arnold and L. G. Yaffe, *Phys. Rev. D* **52**, 7208 (1995).
- [9] A. Hart, M. Laine, and O. Philipsen, *Nucl. Phys.* **B586**, 443 (2000).
- [10] M. Laine and M. Vepsäläinen, *J. High Energy Phys.* **09** (2009) 023.
- [11] A. Nakamura, T. Saito, and S. Sakai, *Phys. Rev. D* **69**, 014506 (2004).
- [12] D. Bak, A. Karch, and L. G. Yaffe, *J. High Energy Phys.* **08** (2007) 049.
- [13] Yu. Maezawa *et al.* (WHOT-QCD Collaboration), *Proc. Sci., LATTICE2008* (2008) 194.
- [14] A. Ali Khan *et al.* (CP-PACS Collaboration), *Phys. Rev. D* **63**, 034502 (2000); **64**, 074510 (2001).