# Electronic states and metal-insulator transition in the triangular lattice $d-p$ model for layered cobaltates 

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#### Abstract

We investigate the electronic state of the 11 band $d-p$ model on the two-dimensional triangular lattice simulating a $\mathrm{CoO}_{2}$ plane in the layered cobalt oxides such as $\mathrm{Na}_{x} \mathrm{CoO}_{2}$ and $\mathrm{Na}_{x} \mathrm{CoO}_{2} \cdot y \mathrm{H}_{2} \mathrm{O}$. The tight-binding parameters are determined so as to fit the LDA band structure. Using the slave boson approach together with the $1 / N$ expansion method, we obtain the renormalized quasiparticle bands, where the $a_{1 g}$ band with a large hole Fermi surface is largely renormalized due to the strong correlation effect, while the $e_{g}^{\prime}$ bands with 6 hole pockets are almost unchanged. We also discuss the metal-insulator transition (MIT) with varying the $d-p$ charge transfer energy $\Delta$, and find that the MIT takes place at a critical value $\Delta_{c}=4.01 \mathrm{eV}$.


Key words: $\mathrm{Na}_{x} \mathrm{CoO}_{2}$, Triangular lattice, $d$ - $p$ model, Metal-insulator transition, Slave boson, $1 / N$ expansion PACS:

To investigate the electronic states of the $\mathrm{CoO}_{2}$ plane in the layered cobalt oxides such as $\mathrm{Na}_{x} \mathrm{CoO}_{2}$ and recently discovered superconductor $\mathrm{Na}_{x} \mathrm{CoO}_{2}$. $y \mathrm{H}_{2} \mathrm{O}$ [1], we employ the two-dimensional triangular lattice $d-p$ model with 11 orbitals: $t_{2 g}$ orbitals ( $d_{x y}$, $\left.d_{y z}, d_{z x}\right)$ and $e_{g}$ orbitals $\left(d_{x^{2}-y^{2}}, d_{3 z^{2}-r^{2}}\right)$ of a Co atom and $6 p$-orbitals of two O atoms on the upper and lower sides of a $\mathrm{CoO}_{2}$ plane. The noninteracting part of the Hamiltonian is given by

$$
\begin{align*}
H & =\epsilon_{p} \sum_{i \mu \sigma} p_{i \mu \sigma}^{+} p_{i \mu \sigma}+\sum_{i m m^{\prime} \sigma} \epsilon_{d}^{m m^{\prime}} d_{i m \sigma}^{+} d_{i m^{\prime} \sigma} \\
& +\sum_{i j \mu \mu^{\prime} \sigma} t_{i j \mu \mu^{\prime}}^{p p} p_{i \mu \sigma}^{+} p_{j \mu^{\prime} \sigma}+\sum_{i j m \sigma} t_{i j m}^{d d} d_{i m \sigma}^{+} d_{j m^{\prime} \sigma} \\
& +\sum_{i j \mu m \sigma}\left(t_{i j \mu m}^{p d} p_{i \mu \sigma}^{+} d_{j m \sigma}+\text { h.c. }\right), \tag{1}
\end{align*}
$$

where $p_{i \mu \sigma}^{+}$and $d_{i m \sigma}^{+}$are the creation operators for the $p$ and the $d$ electrons with site $i$, orbital $\mu(m)$ and $\operatorname{spin} \sigma$, respectively. In eq.(1), the atomic energy for the $d$ electron is explicitly given by: $\epsilon_{d}^{m m^{\prime}}=$
$\epsilon_{d} \delta_{m m^{\prime}}+\frac{\Delta_{t}}{3}\left(1-\delta_{m m^{\prime}}\right)$ for $m \in t_{2 g}$, and $\epsilon_{d}^{m m^{\prime}}=$ $\left(\epsilon_{d}+10 D q\right) \delta_{m m^{\prime}}$ for $m \in e_{g}$, where $\Delta_{t}$ and $10 D q$ are the trigonal and the tetrahedral crystal electric fields, respectively. The $d-p$ charge transfer energy is defined as $\Delta \equiv \epsilon_{p}-\epsilon_{d}$. The atomic energies together with the transfer integrals $t_{i j \mu \mu^{\prime}}^{p p}, t_{i j m}^{d d}$ and $t_{i j \mu m}^{p d}$, which are written by the Slater-Koster parameters, are determined so as to fit the tight-binding energy bands to the LDA bands for $\mathrm{Na}_{0.5} \mathrm{CoO}_{2}$ [2].

The ground state of the cobalt ion in $\mathrm{Na}_{x} \mathrm{CoO}_{2}$ is known to be a low spin state: $\mathrm{Co}^{4+}\left(S=\frac{1}{2}, t_{2 g}^{5} e_{g}^{0}\right)$ for $x=0$ and $\mathrm{Co}^{3+}\left(S=0, t_{2 g}^{6} e_{g}^{0}\right)$ for $x=1$. Then, we assume the occupied states $t_{2 g}^{n}$ with $n \leq 4$ are excluded; which is reproduced by taking into account of the infinite Coulomb repulsion $U$ between $t_{2 g}$ holes. The model can be described by the Hamiltonian eq.(1) with replacing $d_{i m \sigma}^{+} \rightarrow f_{i m \sigma}^{+} b_{i}$ for $m \in$ $t_{2 g}$, under the local constraints: $\sum_{m \sigma} f_{i m \sigma}^{+} f_{i m \sigma}+$ $b_{i}^{+} b_{i}=1$ for all $i$, where $b_{i}^{+}$is the creation operator for the slave boson representing the hole empty state


Fig. 1. The energies for the tight-binding bands (dashed lines) and those for the renormalized quasiparticle bands (solid lines) at $x=0.5$ of $\mathrm{Na}_{x} \mathrm{CoO}_{2}$.
$t_{2 g}^{6}$ and $f_{i m \sigma}^{+}$is that for the pseudo fermion representing the single hole state $t_{2 q}^{5}$. As the $e_{g}$ orbitals are almost empty due to a large value of $10 D q \sim$ 1.14 eV , the Coulomb interaction for $e_{g}$ electrons together with that for $p$ electrons is neglected.

Within the leading order w.r.t. $1 / N$, where $N$ is the spin-orbital degeneracy of $t_{2 g}$ and $N=\sum_{m \sigma}=$ 6 in the present case, the Dyson equations for the single particle Green's functions are obtained and solved analytically at $T=0$ as previously done for the periodic Anderson model [3] and the $d-p$ model [4]. Then, the energies for the renormalized quasiparticle bands $E_{k s \sigma}$ are obtained by diagonalizing the renormalized Hamiltonian, $\tilde{H}=$ $\sum_{s=1}^{11} \sum_{k \sigma} E_{k s} c_{k s \sigma}^{+} c_{k s \sigma}$, which is given by eq.(1) with replacing $t_{i j m}^{d d} \rightarrow Z t_{i j m}^{d d}, t_{i j \mu m}^{p d} \rightarrow \sqrt{Z} t_{i j \mu m}^{p d}$ and $\epsilon_{d} \rightarrow E_{d}$. The renormalization factor $Z$ and the renormalized atomic energy $E_{d}$ are obtained by solving the following self-consistent equations:

$$
\begin{align*}
1-Z & =\sum_{k s m \sigma}\left|v_{k s m}\right|^{2} f\left(E_{k s}\right)  \tag{2}\\
\epsilon_{d}-E_{d} & =\frac{1}{2} \sum_{k s \mu m \sigma}\left(\frac{t_{k \mu m}^{p d}}{\sqrt{Z}} u_{k s \mu} v_{k s m}^{*}+c . c .\right) f\left(E_{k s}\right) \\
& +\sum_{k s m \sigma} t_{k m}^{d d}\left|v_{k s m}\right|^{2} f\left(E_{k s}\right), \tag{3}
\end{align*}
$$

where $u_{k s \mu}$ and $v_{k s m}$ are coefficients for the unitary transformation diagonalizing $\tilde{H}: c_{k s \sigma}=$ $\sum_{\mu} u_{k s \mu} p_{k \mu \sigma}+\sum_{m} v_{k s m} d_{k m \sigma}$.
Figure 1 shows the energies for the renormalized quasiparticle bands together with those for


Fig. 2. The renormalization factor $Z$ (a) and the chemical potential $\mu(\mathrm{b})$ as functions of $x$ for several values of the $d-p$ charge transfer energy $\Delta$.
the noninteracting tight binding bands at $x=0.5$ of $\mathrm{Na}_{x} \mathrm{CoO}_{2}$, where $t_{2 g}$ bands with $s=7,8,9$ are shown in the figure, while $p$ bands with $s=1-6$ and the $e_{g}$ bands with $s=10,11$ are not shown. Due to the strong correlation effect, the $a_{1 g}$ band with a large hole Fermi surface around the $\Gamma$ point is largely renormalized near the Fermi level, while the $e_{g}^{\prime}$ bands with 6 hole pockets near the $K$ point are almost unchanged.

In Fig. 2, the renormalization factor $Z$ and the chemical potential $\mu$ are plotted as functions of $x$, where the number of holes in the $t_{2 g}$ bands is given by $n_{\text {hole }}=1-x$. When $\Delta$ is varied, the MIT takes place at a critical value $\Delta_{c}=4.01 \mathrm{eV}$ : for $\Delta>\Delta_{c}$ the system is insulator at $x=0$, while for $\Delta<\Delta_{c}$ the system is metallic even at $x=0$. The critical value is larger than the LDA value $\Delta_{\mathrm{LDA}}=1.8 \mathrm{eV}$ [2] and is comparable to the experimental value from PES $\Delta_{\text {PES }}=4 \mathrm{eV}[5]$.

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