Step-profile measurement by backpropagation of multiple-wavelength optical fields

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Multiple-wavelength optical fields on a detecting plane of an interferometer are generated from the interference signals detected for an object surface. The generated optical fields are backpropagated along the optical axis. An optical field along the optical axis is reconstructed by summing the backpropagated fields over the multiple wavelengths. The intensity and phase distributions of the reconstructed optical field provide the position of the object surface with an accuracy of a few nanometers.
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Multiple-wavelength interferometers have been used to measure an optical path difference (OPD) longer than the optical wavelength [1–3]. In the measurements, the phase distribution of the interference signal obtained for the multiple wavelengths is utilized to determine the OPD. Generally, a gradient of the phase distribution with respect to the wavenumbers is calculated to obtain the value of the OPD. This method using the phase gradient is referred to as the "phase gradient method" hereafter. When the phase distribution contains measurement error, the accuracy of the measurement of the OPD becomes low. In this paper, a new method with higher measurement accuracy is proposed, which is referred to as the "backpropagation method."

A sinusoidal phase-modulating interferometer using multiple wavelengths for step-profile measurement is shown in Fig. 1. The object has an optical surface with a step shape whose surface position is expressed by the OPD of L_o. An image of the object is made by the lens on the CCD image sensor. The reference mirror is vibrated sinusoidally with a form of $a\cos \omega_c t$ by the piezoelectric transducer. The wavelength of the light source is scanned as

$$\lambda_m = \lambda_0 + m\Delta\lambda \quad m = 0,1,\ldots,M-1, \tag{1}$$

with a scanning width of $B_\lambda = M\Delta\lambda$. The interference signal detected with the CCD image sensor is given by

$$S(t,m) = A_m + B_m \cos(Z\cos\omega_c t + \alpha_m), \tag{2}$$

where A_m and B_m are constant with time, $\alpha_m = (2\pi/\lambda_m)L_o$, and $Z = 4\pi a/\lambda_m$, which is regarded as almost a constant because B_λ is much smaller than λ_0. By extracting B_m and α_m from $S(t,m)$ with sinusoidal phase-modulating interferometry [4], the following detected optical field is generated:

$$D(m) = B_m \exp(j\alpha_m) \quad m = 0,1,\ldots,M-1. \tag{3}$$

The optical field $D(m)$ is backpropagated to a position specified by the OPD of L to obtain an optical field

$$U_m(L) = D(m)\exp[-j(2\pi/\lambda_m)L]. \tag{4}$$

The optical field reconstructed using the multiple wavelengths is given by

$$U_R(L) = \sum_{m=0}^{M-1} U_m(L) = |U_R|\exp(j\Phi_R). \tag{5}$$

When only one reflecting surface exists at $L = L_o$, it can be considered that $B_m = 1$. By substituting $D(m) = \exp(j\alpha_m)$ into Eq. (5) and letting $L_D = L - L_o$, Eq. (5) is reduced to

$$U_R = \sin[\pi(B_\lambda/\lambda_0^2)L_D]/\sin[\pi(\Delta\lambda/\lambda_0^2)L_D], \tag{6}$$

$$\Phi_R = -(2\pi/\lambda_C)L_D, \tag{7}$$

where

$$\lambda_C = \lambda_0 + [(M-1)\Delta\lambda/2]. \tag{8}$$

Equations (6) and (7) show that the intensity $I_R = |U_R|^2$ has a maximum value and the phase Φ_R is zero at $L = L_o$. The phase has a linear distribution whose period is the central wavelength λ_C given by Eq. (8). These characteristics enable us to measure the position of L_o with a measurement error of less than a few micrometers. The condition that a phase

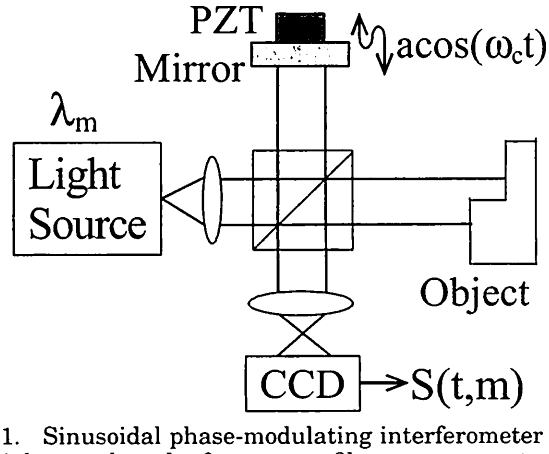

Fig. 1. Sinusoidal phase-modulating interferometer using multiple wavelengths for step-profile measurement.

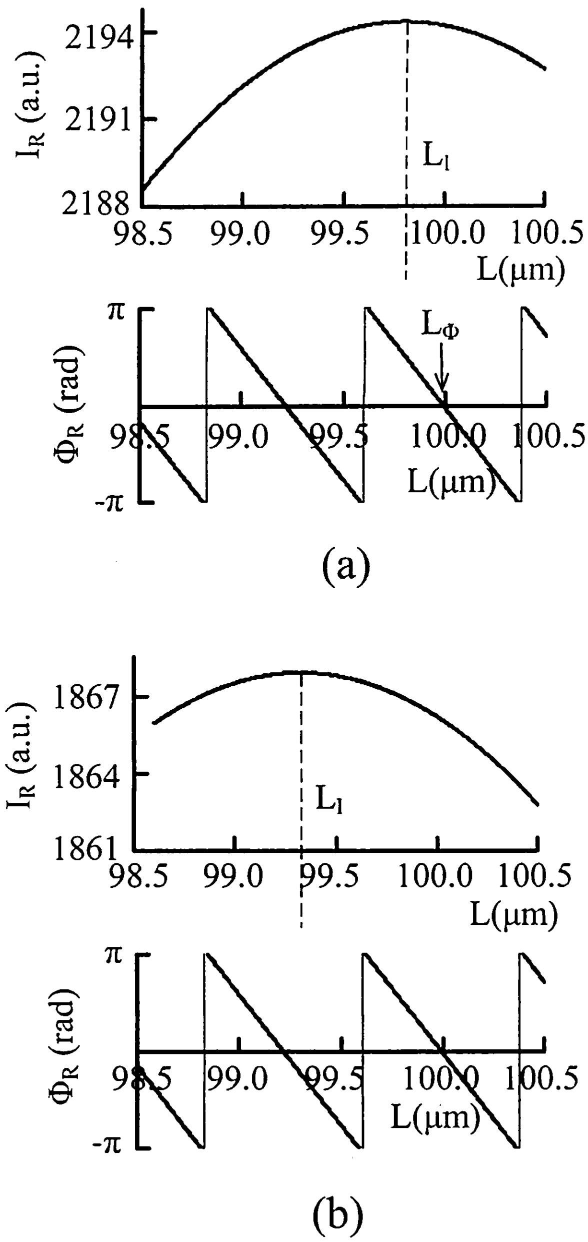

Fig. 2. (Color online) Simulation results of the backpropagation method at (a) $\sigma=0.2$ rad and (b) $\sigma=0.4$ rad.

change in α_m caused by the wavelength change $\Delta\lambda$ must be less than 2π leads to the measurement range of the position

$$L_{\max} = \lambda_0{}^2/\Delta\lambda, \tag{9}$$

and the distribution of $U_R(L)$ is repeated with a period of $L_{\max}$ over all values of L. When L_o is larger than $L_{\max}$, the intensity I_R has a maximum value at the position of $L_{oq}=L_o-qL_{\max}$ between zero and $L_{\max}$, where q is a positive integer. If an approximate value of L_o is known beforehand with an error of less than $L_{\max}/2$, the value of q can be determined.

To verify the characteristics of the backpropagation method regarding measurement error in α_m, we performed numerical simulations under the following conditions: $\lambda_0=767$ nm, $\Delta\lambda=1$ nm, $M=15$, and $L_o=100$ μm. It was assumed that the measured phase distribution α_m contains a measurement error that has a normal distribution function with a mean of zero and a standard deviation σ. Figures 2(a) and 2(b) show the simulation results of the backpropagation method at $\sigma=0.2$ rad and $\sigma=0.4$ rad, respectively. In Fig. 2(a), the peak position of the intensity I_R is $L_I=99.808$ μm, and the zero position L_Φ of the phase Φ_R that is the closest to the position of L_I is regarded as the position of the object. Because the deviation of $|L_I-L_o|$ is less than $\lambda_C/2$ in Fig. 2(a), an accurate measured value of $L_\Phi=99.990$ μm can be obtained, where $\lambda_C=774$ nm. On the other hand, an accurate measured value cannot be obtained for the case shown in Fig. 2(b) because the deviation of $|L_I-L_o|$ is larger than $\lambda_C/2$ for $L_I=99.324$ μm, although the zero position of the phase Φ_R around $L=100$ μm is 99.993 μm. These results make it clear that the phase distribution is scarcely affected by the measurement error of α_m although the intensity distribution is greatly affected by it.

The phase distribution α_m used in Fig. 2(a) is unwrapped with respect to $1/\lambda_m$ as shown in Fig. 3, where the unwrapped phase distribution is denoted by β_m. The values of β_m shown in Fig. 3 do not lie on a straight line because of the noise of $\sigma=0.2$ rad. A straight line is fitted for the values of β_m with the least-squares method, and a phase gradient of $g=\partial\beta_m/\partial(1/\lambda_m)$ is obtained, which provides a measured value L_g of L_o. Other simulations were also carried out for the noises with different values of σ. Ten trials at a fixed value of σ were carried out to calculate a standard deviation of $\varepsilon_L=|L_\Phi-L_o|$, which is denoted by $S\{\varepsilon_L\}$. The simulation results are shown in Table 1, where the values of L_I, L_Φ, and L_g are values obtained in one trial. It was made clear from the simulation that L_g and L_I have the same characteristics, and L_Φ provides the exact position of the object when the value σ of the noise is less than 0.3 rad.

The setup shown in Fig. 1 was constructed where a tunable laser diode with an external cavity of Littman–Metcalf configuration was used as the wavelength-scanning light source. The characteristics of the light source were the same as those in the

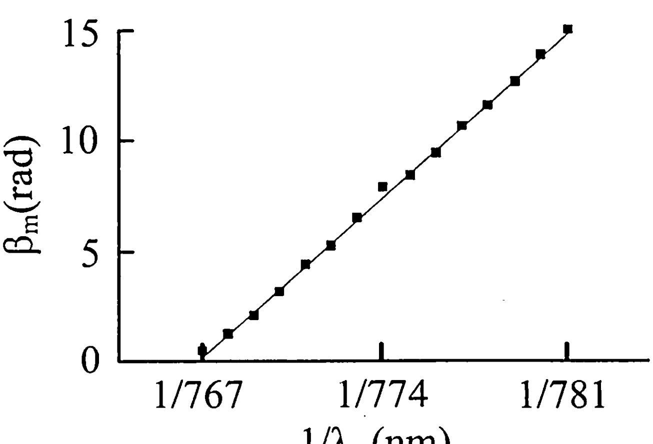

Fig. 3. (Color online) Simulation results of the unwrapped phase distribution at $\sigma=0.2$ rad that is used in the phase gradient method.

Table 1. Simulation Results of the Backpropagation Method and the Phase Gradient Method

σ (rad)	L_I (μm)	L_g (μm)	L_Φ (μm)	$S\{\varepsilon_L\}$ (nm)
0.20	99.808	99.805	99.990	4
0.25	99.913	99.923	100.009	10
0.30	99.716	99.707	99.985	15
0.40	99.324	99.344	99.219	631

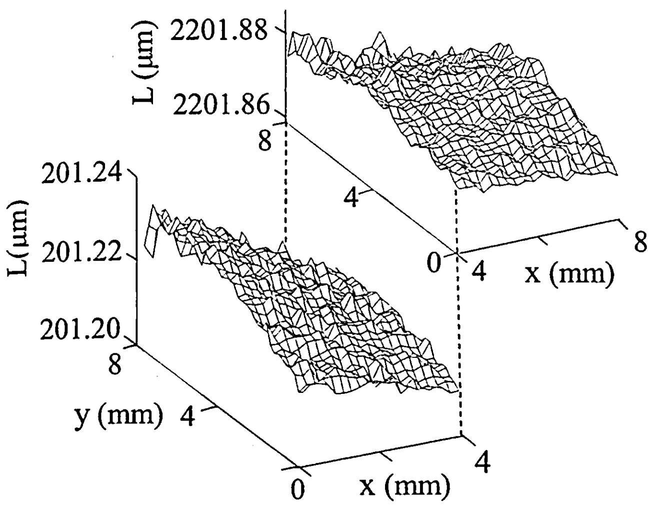

Fig. 4. Measurement result of an optical surface with a step shape of about 1000 μm height.

numerical simulations. An optical surface with a step shape was made by stacking a gauge block of 1000 μm thickness on top of another gauge block. The measurement results for this surface profile are shown in Fig. 4, where the position distributions of the upper and lower surfaces forming the step profile are drawn with the two separate coordinates of x, y, and L. Figures 5(a) and 5(b) show the distributions of I_R and Φ_R at points A and B, whose x-coordinates are 2 and 6 mm, respectively, on the line of $y=4$ mm. The values of L_I and L_Φ at point A were $L_{IA}=201.153$ μm and $L_{\Phi A}=201.215$ μm, the values being less than $L_{\max}=588.289$ μm. The values of L_I and L_Φ at point B were $L_{IB}=2201.831$ μm and $L_{\Phi B}=2201.857$ μm, where $L_{oq}=L_o-qL_{\max}=2201.857-1764.867=436.99$ μm with $q=3$ for $L_o=L_{\Phi B}$. The value of L_g at point A was 201.148 μm, which is almost equal to L_{IA}. The measurements were repeated at intervals of a few minutes. A value of measurement repeatability of about 2 nm was obtained from the rms value of the differences between the two measured surface profiles. These results indicate that the standard deviation σ of the noise is less than 0.2 rad.

In conclusion, it has been made clear from the numerical simulations and the experiments that the backpropagation method enables measurement of a large OPD with an accuracy of a few nanometers even when the measured phase of the interference signal contains an error of about 0.2 rad. The backpropagation method provides a measurement accuracy higher than the phase gradient method, although it requires greater processing power. The backpropagation method will be useful for measuring front and rear surface profiles of glass plates and thin films.

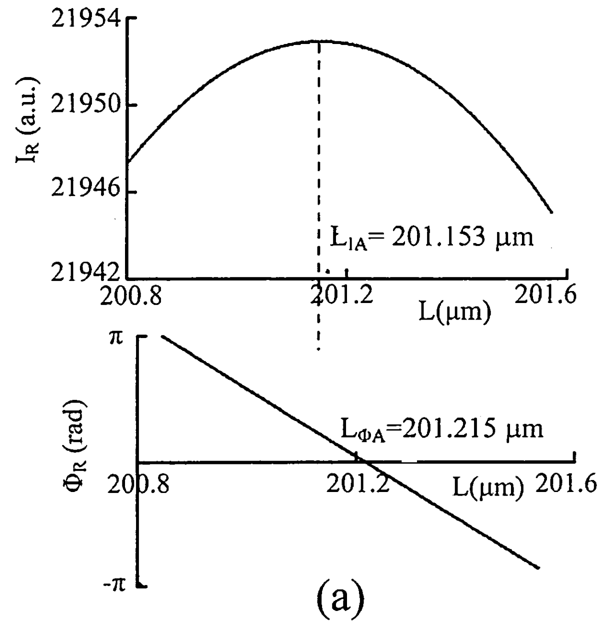

Fig. 5. (Color online) Distributions of I_R and Φ_R at points A and B, whose x-coordinates are (a) 2 mm and (b) 6 mm, respectively, on the line of $y=4$ mm in Fig. 4.

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