# Sinusoidal phase modulating laser diode interferometer with a feedback control system to eliminate external disturbance

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**Abstract.** We propose a sinusoidal phase modulating laser diode interferometer that is insensitive to vibrations of the optical components and fluctuations in the optical wavelength of the laser diode. These external disturbances cause fluctuations in the phase of the interference signal. After we analyze the sinusoidal phase modulation in a laser diode interferometer, we describe the method of the feedback control of the injection current of the laser diode to eliminate the phase fluctuations of the interference signal. We construct two sinusoidal phase modulating interferometers for movement measurements and surface profile measurements, respectively. The experimental results make it clear that the interferometers work well in mechanically noisy circumstances.

*Subject terms: laser diodes; interferometry; feedback control; displacement metrology; surface metrology; surface profiles.*

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CONTENTS



## 1. INTRODUCTION

Heterodyne interferometry[1,2] and fringe scanning interferometry[3,4] have been widely used to measure surface profiles with high accuracy. Recently laser diodes (LDs) have been incorporated into heterodyne interferometers[5] and fringe scanning interferometers[6] as light sources and phase modulators. As another interferometric technique, we proposed sinusoidal phase modulating (SPM) interferometry, in which surface profiles are obtained with the Fourier transform method[7] and the integrating-bucket method.[8,9] We also reported the method of movement measurements in SPM interferometry.[10]

In this paper we describe a SPM laser diode interferometer that is insensitive to vibrations of optical components and fluctuations in the optical wavelength of the LD. The sinusoidal phase modulated interference signal is generated by modulating the injection current of the LD with a sinusoidal wave signal. The phase modulation in SPM interferometry is very simple compared with that in heterodyne interferometry and fringe scanning interferometry. The signal that is a trigonometric function of the phase difference between the object and reference waves can be easily obtained from the sinusoidal phase modulated interference signal. This signal is used as a feedback signal in controlling the injection current of the LD to reduce the fluctuations in the phase of the interference signal caused by the external disturbances. The special characteristics of the sinusoidal phase modulated interference signal allow us to construct easily an interferometer with the feedback control system to eliminate the external disturbances.

The sinusoidal phase modulation in a LD interferometer is analyzed theoretically in Sec. 2, and the principle of the feedback control of the injection current is described in Sec. 3. In Sec. 4 we describe a SPM interferometer for movement measurements in which we obtain a feedback signal from the interference signal generated with a stationary object. The movement measurements of a piezoelectric transducer did not suffer from external disturbances. In Sec. 5 we construct a SPM interferometer for surface profile measurements, measuring surface profiles of diamond-turned aluminum disks. The measurement repeatability is greatly improved by the feedback control of the injection current. The experimental results make it clear that the SPM interferometers presented here can be used in mechanically noisy circumstances.

## 2. SINUSOIDAL PHASE MODULATION IN A LASER DIODE INTERFEROMETER

Let us consider a Twyman-Green type interferometer, shown in Fig. 1. The injection current of a LD consists of a dc component $i_0$ and a time-varying component $\Delta i_c(t)$ as follows:

$$i(t) = i_0 + \Delta i_c(t) \ . \tag{1}$$

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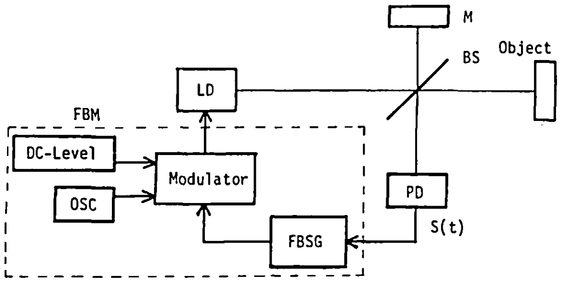


**Fig. 1. Feedback control system in sinusoidal phase modulating laser diode interferometer.**

The dc component determines a central wavelength of the light $\lambda_0$, and the $\Delta i_c$ produces a small change in the wavelength of the LD:

$$\Delta\lambda(t) = \beta\Delta i_c(t) . \tag{2}$$

Then the wavelength of the LD is given by

$$\lambda(t) = \lambda_0 + \Delta\lambda(t) . \tag{3}$$

The optical wave emitted from the LD is represented by

$$A(t) = \exp\left\{j2\pi c\int_0^t\left[\frac{1}{\lambda(t)}\right]dt\right\} = \exp[j\phi(t)] , \tag{4}$$

where $c$ is the velocity of the light. The light reflected from an object is an objective wave, and the light reflected from a mirror (M) is a reference wave. The optical path lengths of these waves are denoted by $l_o$ and $l_r$, respectively. The objective wave $U_o$ and reference wave $U_r$ on the photodiode (PD) are represented by

$$U_o = \exp[j\phi(t - \tau_o)] , \quad U_r = \exp[j\phi(t - \tau_r)] , \tag{5}$$

where $\tau_o = l_o/c$ and $\tau_r = l_r/c$. The time-varying component of the interference signal produced with the two waves is given by

$$S(t) = \cos[\phi(t - \tau_o) - \phi(t - \tau_r)] = \cos\Phi(t) . \tag{6}$$

Using the approximation

$$\frac{1}{\lambda(t)} \sim \frac{1}{\lambda_0}\left[1 - \frac{\Delta\lambda(t)}{\lambda_0}\right] \tag{7}$$

and the definition

$$\int \Delta\lambda(t)dt = \Delta\Lambda(t) , \tag{8}$$

the argument of Eq. (6) becomes

$$\Phi = \frac{2\pi}{\lambda_0} l - \frac{2\pi c}{\lambda_0^2}[\Delta\Lambda(t - \tau_o) - \Delta\Lambda(t - \tau_r)] , \tag{9}$$

where $l = l_r - l_o$. In the condition $\tau_r - \tau_o \ll 1$, we have the approximation

$$\Delta\Lambda(t - \tau_o) - \Delta\Lambda(t - \tau_r) \sim \frac{1}{c}\Delta\lambda(t) , \tag{10}$$

and Eq. (6) becomes

$$S(t) = \cos\left[\alpha - \frac{2\pi}{\lambda_0^2}\Delta\lambda(t)\right] , \tag{11}$$

where $\alpha = (2\pi/\lambda_0)l$.

For sinusoidal phase modulation,

$$\Delta i_c(t) = a\cos(\omega_c t + \theta) , \tag{12}$$

we obtain the interference signal

$$S(t) = \cos[z\cos(\omega_c t + \theta) + \alpha] , \tag{13}$$

where $z = -(2\pi/\lambda_0^2)\beta a l$.

## 3. ELIMINATION OF EXTERNAL DISTURBANCES WITH FEEDBACK CONTROL

The wavelength of the LD changes by $\Delta\lambda_T$ with temperature. Optical components in the interferometer vibrate in response to external mechanical vibrations. This causes the change $\Delta l$ in the optical path difference $l$ between the object and reference waves. These $\Delta\lambda_T$ and $\Delta l$ cause the fluctuation in the phase of the interference signal. The fluctuation is compensated by controlling the injection current to produce the change $\Delta\lambda_I$ in the wavelength of the LD. To consider the changes $\Delta\lambda_T$, $\Delta l$, and $\Delta\lambda_I$ in Eq. (11), $l$ and $\Delta\lambda$ are replaced with $l + \Delta l$ and $\Delta\lambda + \Delta\lambda_T + \Delta\lambda_I$, respectively. By neglecting the term of $(\Delta\lambda + \Delta\lambda_T + \Delta\lambda_I)\Delta l$, the interference signal for sinusoidal phase modulation is written as

$$S(t) = \cos[z\cos(\omega_c t + \theta) + \alpha + \delta(t)] , \tag{14}$$

where

$$\delta(t) = \frac{2\pi}{\lambda_0}\Delta l - \frac{2\pi l}{\lambda_0^2}(\Delta\lambda_T + \Delta\lambda_I) . \tag{15}$$

We try to reduce the phase $\delta(t)$ to zero by controlling the injection current or $\Delta\lambda_I$.

Let us explain how to generate the feedback signal for the injection current of the LD. The expansion of Eq. (14) is given by

$$\begin{aligned} S(t) = {} & \cos[\alpha + \delta(t)][J_0(z) - 2J_2(z)\cos(2\omega_c t + 2\theta) + \ldots] \\ & - \sin[\alpha + \delta(t)][2J_1(z)\cos(\omega_c t + \theta) \\ & - 2J_3(z)\cos(3\omega_c t + 3\theta) + \ldots] , \end{aligned} \tag{16}$$

where $J_n$ is the $n$th order Bessel function. Producing the signal $S(t)\cos(\omega_c t + \theta)$ and passing this signal through a low-pass filter, we obtain the feedback signal:

$$J_1(z)\sin[\alpha + \delta(t)] . \tag{17}$$

This feedback signal is available in the region of $z = 0.5 - 3.5$, where the value of the $J_1(z)$ is not so small. When the phase $\alpha$ is nearly multiples of $\pi$ rad, we can stabilize the phase $\delta(t)$ at zero with a proportional feedback control using the feedback signal given by Eq. (17). The phase $\alpha$ is adjusted with the dc component of the injection current. The feedback signal is generated in the feedback signal generator (FBSG) shown in

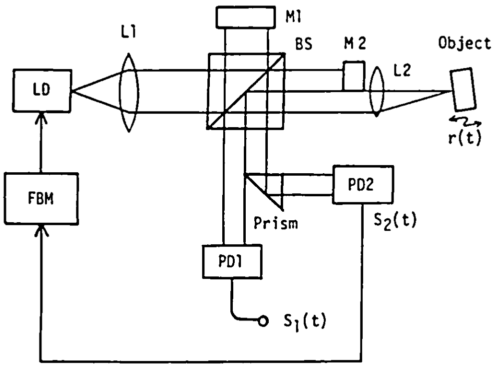


Fig. 2. SPM laser diode interferometer with the feedback control system for movement measurements.

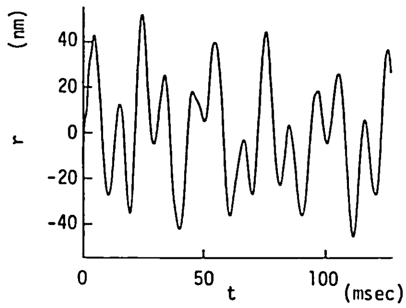


Fig. 3. Movement $r(t)$ measured when the feedback control did not operate.

Fig. 1. The portion within the dotted lines in Fig. 1 is called the feedback modulator (FBM). This modulator produces the injection current of the LD, which is controlled so that the phase $\delta(t)$ is reduced to zero.

## 4. MOVEMENT MEASUREMENTS

### 4.1. Interferometer

Figure 2 shows a SPM interferometer with the feedback control system for movement measurements. The light emitted from a LD is collimated with lens 1 (L1). The light reflected from mirror 1 (M1) is a reference wave. The light passed through a beam splitter (BS) is an object wave. A portion of the object wave is illuminated onto an object through lens 2 (L2). The movement of the object is represented by $r(t)$. The reflected light from the object and the reference light are superimposed on photodiode 1 (PD1). The optical path difference is $l_1$, and it changes by $\Delta l_1$ due to mechanical vibrations of the optical components. The interference signal detected with PD1 is written as

$$S_1(t) = \cos[z_1\cos(\omega_c t + \theta) + \alpha_{10} + \alpha_1(t) + \delta_1(t)] \ , \qquad (18)$$

where

$$\delta_1(t) = \frac{2\pi}{\lambda_0}\Delta l_1 - \frac{2\pi l_1}{\lambda_0^2}(\Delta\lambda_T + \Delta\lambda_I) \ , \qquad (19)$$

$\alpha_1(t) = (4\pi/\lambda_0)r(t)$, and $\alpha_{10} = (2\pi/\lambda_0)l_1$. On the other hand, the rest of the object wave is illuminated onto mirror 2 (M2). The reflected light and the reference light are deflected with a prism, and travel to photodiode 2 (PD2). The optical path difference is $l_2$ and its change is represented by $\Delta l_2$. The interference signal detected with PD2 is written as

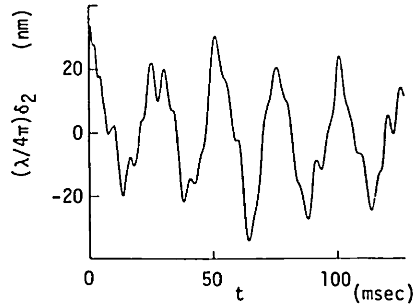


Fig. 4. Phase $\delta_2(t)$ measured when the feedback control did not operate.

$$S_2(t) = \cos[z_2\cos(\omega_c t + \theta) + \alpha_{20} + \delta_2(t)] \ , \qquad (20)$$

where $\delta_2(t) = (2\pi/\lambda_0)\Delta l_2 - (2\pi l_2/\lambda_0^2)(\Delta\lambda_T + \Delta\lambda_I)$ and $\alpha_{20} = (2\pi/\lambda_0)l_2$. The feedback signal is generated from this interference signal in the FBM. It is assumed that the phase $\delta_2(t)$ is reduced to a small value $\Delta\delta$ by the feedback control as follows:

$$\delta_2(t) = \frac{2\pi}{\lambda_0}\Delta l_2 - \frac{2\pi l_2}{\lambda_0^2}(\Delta\lambda_T + \Delta\lambda_I) = \Delta\delta \ . \qquad (21)$$

By substituting Eq. (21) into Eq. (19), we get the phase $\delta_1(t)$:

$$\delta_1(t) = \frac{2\pi}{\lambda_0}\left(\Delta l_1 - \frac{l_1}{l_2}\Delta l_2\right) + \frac{l_1}{l_2}\Delta\delta \ . \qquad (22)$$

Since the optical path difference $l_1$ is longer than the optical path difference $l_2$ and the change $\Delta l_1$ is not completely equal to the change $\Delta l_2$ in this interferometer, the phase fluctuation $\delta_1(t)$ cannot always be reduced to the amount $\Delta\delta$.

### 4.2. Experimental results

We measured movements of the piezoelectric transducer vibrating sinusoidally with a frequency of 100 Hz. The frequency of the sinusoidal phase modulation was 1 kHz and the cutoff frequency of the low-pass filter employed in the FBM was 200 Hz. The movement $r(t)$ and the phase $\delta_2(t)$ were obtained using the method described in Ref. 10. Figures 3 and 4 show the movement $r(t)$ and the phase $\delta_2(t)$ measured when the feedback control did not operate. The measured movement contains the phase fluctuation $\delta_1(t)$, which is almost equal to the measured phase $\delta_2(t)$. Figures 5 and 6 show the movement $r(t)$ and the phase $\delta_2(t)$ measured when the feedback control operated well. The measured phase $\delta_2(t)$ corresponds to the $\Delta\delta$ in Eq. (21). The optical path differences $l_1$ and $l_2$ were 20 mm and 15 mm, respectively. The phase $\delta_1(t)$ is reduced to be a small value, and the measured movement can be regarded to be a sinusoidal wave.

## 5. SURFACE PROFILE MEASUREMENTS

### 5.1. Interferometer

Figure 7 shows a SPM interferometer with feedback control system for surface profile measurements. Lens 2 (L2) makes an image of an object on a linear CCD image sensor. The surface profile of the object is represented by $r(x)$. The light reflected

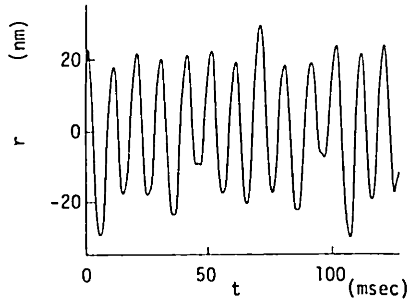


**Fig. 5. Movement $r(t)$ measured when the feedback control operated.**

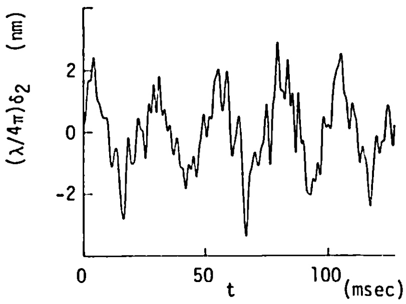


**Fig. 6. Phase $\delta_2(t)$ measured when the feedback control operated.**

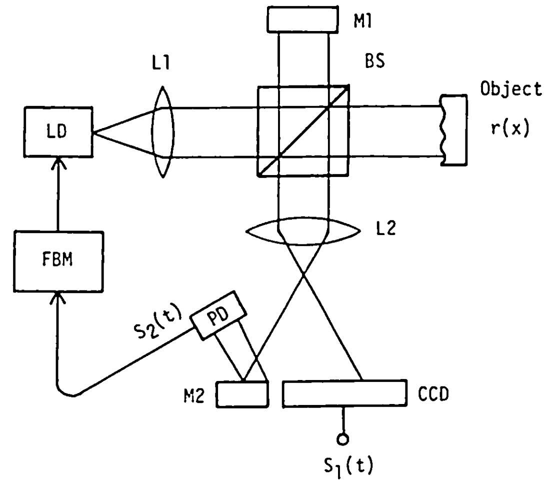


**Fig. 7. SPM laser diode interferometer with feedback control system for surface profile measurements.**

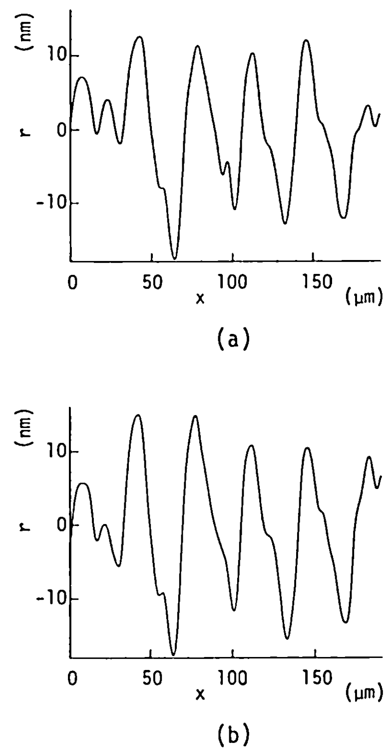


**Fig. 8. Two surface profiles measured at an interval of a few minutes when the feedback control did not operate.**

from M1 is a reference wave. The light near the CCD image sensor is reflected by M2 and reaches to the photodiode. The optical path difference for a point $x$ on the surface is given by $l + 2r(t)$. The interference signal detected with the CCD image sensor is written as

$$S_1(t,x) = \cos[z\cos(\omega_c t + \theta) + \alpha_{10} + \alpha_1(x) + \delta(t)] \ , \qquad (23)$$

where $\alpha_1(x) = (4\pi/\lambda_0)\,r(x)$, $\alpha_{10} = (2\pi/\lambda_0)l$, and the phase $\delta(t)$ is given by Eq. (15). The interference signal detected with the PD is written as

$$S_2(t) = \cos[z\cos(\omega_c t + \theta) + \alpha_{20} + \delta(t)] \ , \qquad (24)$$

where the phase $\alpha_{20}$ is a constant. An object produces the signals $S_1(t)$ and $S_2(t)$, and the distance between the measuring points with the CCD image sensor and a detecting point with the PD is very short. Therefore, the phase fluctuations in the signals $S_1(t)$ and $S_2(t)$ can be considered to be identical. In other words, the conditions of $l_1 = l_2$ and $\Delta l_1 = \Delta l_2$ hold in this interferometer for the parameters in Eq. (22). The signal $S_2(t)$ is fed to the FBM to generate the feedback signal. This feedback control system reduces the phase fluctuation $\delta(t)$ to $\Delta\delta$ in both of the signals $S_1(t)$ and $S_2(t)$.

## 5.2. Experimental results

We measured surface profiles of diamond-turned aluminum disks. The surface profile was obtained from the CCD output using the Fourier transform method described in Ref. 7. First, the feedback control was not operating. A surface profile was measured and the result is shown in Fig. 8(a). The same surface was measured after a few minutes, and the result is shown in Fig. 8(b). There are slight differences between the two measured surface profiles. The measurement repeatability for Figs. 8(a) and 8(b) was between about 3.5 and 7.0 nm. Next, the feedback control was operating well. The same surface profile was measured at an interval of a few minutes. The results are shown in Fig. 9. The two measured surface profiles are almost identical. The measurement repeatability for Figs. 9(a) and 9(b) was between about 0.5 and 1.0 nm. Thus, the measurement repeatability was greatly improved by the feedback control to eliminate external disturbances.

## 6. CONCLUSIONS

We constructed SPM LD interferometers that were very insensitive to external disturbances such as mechanical vibrations and fluctuations in temperature. In the movement measurements, the optical path difference and its fluctuation in one interference

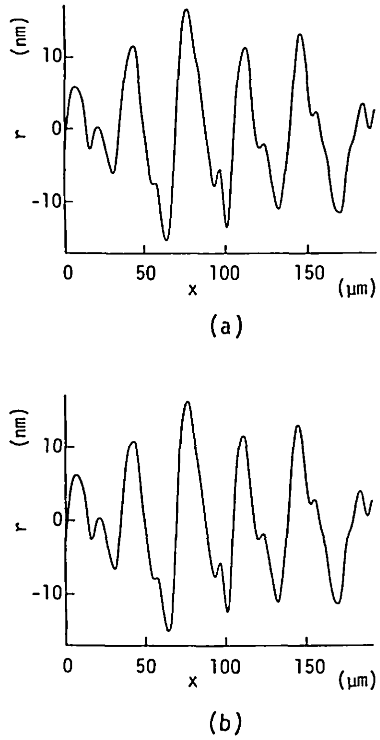


**Fig. 9. Two surface profiles measured at an interval of a few minutes when the feedback control operated.**

signal are not equal to those in the other interference signal. Therefore, the measured movements contain the phase fluctuations that cannot be eliminated by the feedback control. However, since the remaining phase fluctuations are small, the measured movements approach the real movements. On the other hand, since the optical path differences and their fluctuations in the two interference signals are identical for the surface profile measurement, the phase fluctuations are almost eliminated and the measurement repeatability is greatly improved by the feedback control. The experimental results show clearly that the SPM LD interferometers presented here can be used in mechanically noisy circumstances.

## 7. ACKNOWLEDGMENT

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