Methods of Information in Medicine

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Band-Suppressed Restoration of X-Ray Images Blurred by Body Movement

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Abstract. The restoration of X-ray images that have been blurred due to body movement are discussed. The observation system for these images is described using a mathematical model, and several restoration filters composed of a series of such models are proposed. These filters restore band-suppressed approximations of the original images. In addition, redundancy is introduced into these restoration filters in order to suppress additive noise. These filters are expanded to be applicable not only to parallel translations, but also to rotations by coordinate transformation. The proposed methods are applied to blurred X-ray images of a bone model of the elbow joint. The parameters of the restoration filter are estimated using a marker attached to the subject as a reference signal.

Keywords: X-Ray Images, Restoration, Band Suppression, Blur, Body Movement

1. Introduction

X-ray images of the human body are often used in clinical analysis or diagnosis. However, these images are sometimes blurred due to body movement. For example, blood vessel images obtained by thoracic X-rays are often blurred due to the movement of the diaphragm in respiration. Children especially have a difficult time remaining still during an X-ray examination. The extent of the blur can be reduced by decreasing the exposure time, but even this does not ensure that a clear X-ray image can be obtained. Therefore, images must be restored to their original state. Most traditional restoration methods attempt to completely restore the original signal, which theoretically requires an infinite Nuemann series. Moreover, when the observed image is obscured by noise, the original image can not be obtained. Several restoration filters composed of a series of fundamental filters have been proposed for biomedical signals such as blood pressure signals, electrocardiograms, and blood temperature signals [1-6]. These filters have successfully restored band-suppressed approximations of the signals observed in real-time using biomedical instruments. Moreover, the noise included in the observed signals is suppressed by redundant models of the filters [7]. The present authors have expanded this restoration filter to be applicable to two spatial dimensions, and have constructed a restoration filter for images blurred by parallel translations [8]. In the present paper we apply the proposed restoration not only to parallel translations, but also to rotations. The blurs are estimated by attaching a marker, which is used as a reference signal, to the subject. In addition, the theory underlying band-suppressed restoration of images and possible applications are also discussed.

2. Definition of the Problem

Let f be an original image, n be the additive noise, and A be the observation system operator. The observed image, a_0, is defined by

$$a_0 = A\,f + n. \tag{1}$$

The observed image a_0 is distorted by A and n. We attempt to restore the band-suppressed approximation of the original image rather than the complete original image. In the present study, band-suppression refers to the gradual decay in frequency that is caused by the linear filter. Let B be the restoration filter and P be the filter that causes band-suppression. A restored band-suppressed image, Pf, is then defined as

$$Pf = B\,a_0. \tag{2}$$

We consider the following limitations on the restoration process as described in (2):

(i) Pf is approximately equivalent to f.
(ii) The additive noise, n, must be minimized.

Let A in (1) be

$$\Gamma(s)f(x,y) = \frac{1}{2s}\int_{-\infty}^{\infty} e^{-\frac{|\tau|}{s}} f(x-\tau,y)\,d\tau,\ s > 0 \tag{3}$$

where (x,y) represents the coordinates of a two-dimensional plane, and s is a parameter that represents the degree of distortion. $\Gamma(s)$ corresponds to the first-order low-pass filter having time constant s for the x-directions. The transfer functions of parallel translation for blurs along the x-axis are expressed by the product of $\Gamma(s)$. Moreover, not only parallel translations, but also rotations are expressed by applying coordinate transformations. In the following section, the proposed restoration method is applied to the images obtained by the observation system represented by the product of $\Gamma(s)$ in (3).

3. Band-Suppressed Restoration

In this section, we consider the band-suppressed restoration filtering of the images.

3.1 Restoration Filter for an *L*th-Order Observation System (*M* = *L*)

The observed systems can be represented not only by the first-order system in (3), but also by higher-order systems. We define the Lth-order observation operator as

$$A = \prod_{l=1}^{L} \Gamma(s_l),\ \mathrm{Re}\{s_l\} > 0 \tag{4}$$

and the Lth-order band-suppressed operator as

$$P = \Gamma(s_0)^L,\ s_0 < \mathrm{Re}\{s_l\}. \tag{5}$$

where $\{s_l\}$ are any complex values. To satisfy limitation (i), mentioned previously in Section 2, s_0 must be set to a small value. Therefore, a restoration filter composed of the band-suppressed operator, $\Gamma(s_0)$ is proposed. The fundamental observation images $\{a_m\}$ are recursively defined as

$$a_m = \{I - \Gamma(s_0)\}\, a_{m-1},\ m = 1, 2, 3, \ldots \tag{6}$$

where I is the identity operator. $I - \Gamma(s_0)$ corresponds to the high-pass filter having time constant s_0 for the x-directions. By using $\{a_m\}$, the band-suppressed image may be expressed as

$$Pf = \sum_{m=0}^{M} b_m a_m \tag{7}$$

where $\{b_m\}$ are the linear combination coefficients. We derive the order, M, and the coefficients, $\{b_m\}$, which satisfy (7). To satisfy (2), the order of the restoration filter, M, must be higher than the order of the observation system, L. The restoration filter for (4) is derived using

$$B = \sum_{m=0}^{L} b_m \{I - \Gamma(s_0)\}^m \tag{8}$$

where

$$b_m = \sum_{i=0}^{\binom{L}{m}} \prod_{j=1}^{m} \frac{s_{d(i,j)}^2 - s_0^2}{s_0^2}. \tag{9}$$

$\binom{L}{m} = 0$ if $m < 0$ or $L < m$. Otherwise, $\binom{L}{m}$ is a binomial coefficient. The quantity $d(i,j)$ is the natural number that satisfies $1 \leqq d(i,1) < \ldots < d(i,m) \leqq L$ and $\{d(i',1), \ldots, d(i',m)\} \neq \{d(i'',1), \ldots, d(i'',m)\}$ while $i' \neq i''$. The influence of noise was not considered in this system.

3.2 Redundant Restoration Filter (*M* > *L*)

In this section, we attempt to suppress the noise by introducing redundancy into the order of the restoration filter. We propose an $M(> L)$th-order restoration filter that restores the signals observed by the Lth-order observation system. For the Lth-order observation operator described in (4), we define the Mth-order band-suppressed operator as follows:

$$P = \Gamma(s_0)^M\ (M < L). \tag{10}$$

The redundant restoration filter for (4) is derived by

$$B = \sum_{m=0}^{M} b_m \{I - \Gamma(s_0)\}^m \tag{11}$$

where

$$b_m = (-1)^m \sum_{i=0}^{L} (-1)^l \binom{M-L}{m-l} \sum_{i=0}^{\binom{L}{l}} \prod_{j=1}^{l} \frac{s_{d(i,j)}^2 - s_0^2}{s_0^2}. \tag{12}$$

Figure 1 shows a block diagram of the Mth-order restoration filter for an Lth-order observation system.

3.3 Coordinate Transformations

The observation system in (3) represented the blur in the direction of the x-axis. However, blurs in X-ray images due to body movement may occur in any direction. Moreover, not all blurs are represented by parallel translations. Here we assume that the blurs of images can be represented by Affine transformation, i.e., parallel translation and rotation, because the exposure time is relatively short for X-rays. We estimate the blur using a previously established marker attached to the subject as a reference signal. A dot image, i.e., a pulse signal, is chosen as a marker. The blurred markers were estimated by searching the neighboring points for which the brightness was similar to the original points. First of all, we assume that the image is blurred by rotation around some point (x_0, y_0). Assuming (x_n, y_n), $n = 1, 2, \ldots, N$, to be the original coordinates of the markers and (x_n', y_n') to be blurred points, (x_0, y_0) satisfies following equations:

$$(x_n' - x_n)x + (y_n' - y_n)y = \frac{x_n'^2 - x_n^2 + y_n'^2 - y_n^2}{2}$$

$$(n = 1, 2, \ldots, N). \tag{13}$$

The origin (x_0, y_0) can be estimated using more than one marker. After estimating (x_0, y_0), the blurred image caused by rotation is converted to a parallel blurred image by Cartesian-polar coordinate transformation as follows:

$$\begin{cases} r = \sqrt{(x - x_0)^2 + (y - y_0)^2} \\ \theta = \tan^{-1}\left(\dfrac{y - y_0}{x - x_0}\right) \end{cases} \tag{14}$$

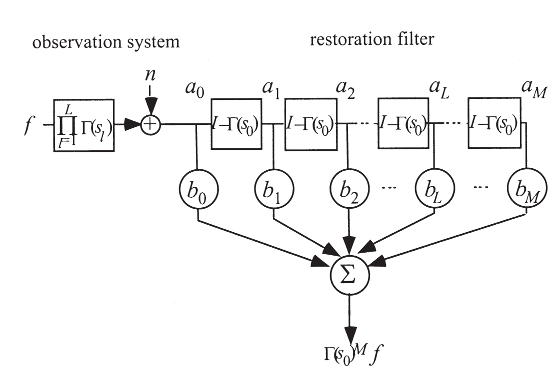

Fig. 1 *M*th-order restoration filter for an *L*th-order observation system.

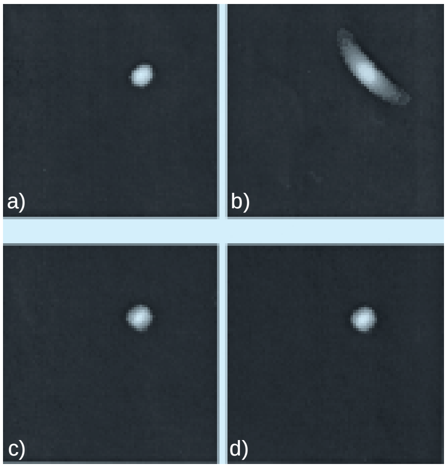

Fig. 2 Restoration of an image of the marker. (a) Original image, *f*. (b) Blurred image, a_0. (c) Band-suppressed image, *Pf*. (d) Restored image, Ba_0.

where r and θ are the axes in polar coordinate system. Replacing x and y in (3) with θ and r, respectively, the restoration filter in (11) can be applied to blurs caused by rotation.

If the image is blurred in parallel translation, the origin can not be obtained using the above method. In this case, we must estimate the direction of the blur. The angle between the x-axis and the direction of blur, ϕ, is obtained by

$$Ø = \tan^{-1}\left(\frac{y'_n - y_n}{x'_n - x_n}\right). \tag{15}$$

The blurred image at angle, ϕ, is restored by converting the coordinates as follows:

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \tag{16}$$

where u and v are converted axes in Cartesian coordinate system. Replacing x and y in (3) with u and v, respectively, the restoration filter in (11) can be applied to parallel translations in any direction. Finally, the inverse transformation of coordinates is applied to the restored image.

3.4 Parameter Estimation

In order to realize the restoration filter, B, parameters such as the order M, the time constant s_0, and the restoration coefficients $\{b_m\}$ must be determined in

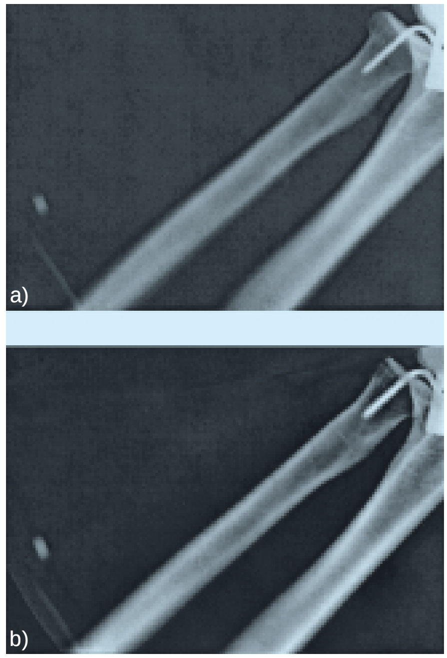

Fig. 3 Restoration of a blurred X-ray image of an elbow joint. (a) Blurred X-ray image. (b) Restored X-ray image of (a)

advance. The order M ($> L$) and the time constant s_0 ($> \mathrm{Re}(s_l)$) can be estimated by considering the frequency band-width of the original image and the noise associated with the observed image. We must estimate the degree of distortion in the blur, because the coefficients $\{b_m\}$ of the restoration filters are determined from the parameters $\{s_l\}$ of the observation operator. The restoration parameters $\{b_m\}$ are estimated using the response of the reference signal as the image for the a_0 that approximates the band-suppressed signal Pf. The restoration filter can be constructed using the estimated $\{b_m\}$ without actually calculating $\{s_l\}$.

4. Simulation

We applied the above described method to an X-ray image of an elbow joint and confirmed the restorative capabilities of the proposed filter. Figure 2 (a) shows the original image of the marker, a lead ball 3 mm in diameter, that is fixed on the body. This image was measured in advance and used as a reference image. The observed image (b) was obtained by rotating the subject. The exposure time was set to 0.6 seconds. We determined the restoration parameters $\{b_m\}$ by comparing the observed image (b) to the band-suppressed image (c) and constructed the restoration filter. The restored image (d) was roughly identical to the band-suppressed image described in (c). Using the restoration filter constructed from the reference signal in Figure 2, we restored an X-ray image of an elbow. The restored image shown in Figure 3 (b) is significantly clearer than the observed image shown in Figure 3 (a).

5. Conclusions

We proposed a filter for restoring X-ray images that have been blurred due to body movements. First, we described the observation of these images using a mathematical model. Next, we proposed the restoration filters, which were composed of redundant linear combinations of fundamental filters, and which restored band-suppressed approximations to their original signals. These filters were successfully applied not only in parallel translation blurs, but also blurs caused rotation. Finally, we applied these methods to blurred X-ray images of a bone model of an elbow joint and were able to obtain clear images. We have determined that the proposed method will be useful in a broad range of practical applications.

Acknowledgements

The authors wish to thank N. Suzuki for her experimental assistance. The present study was supported in part by a grant from the Telecommunications Advancement Organization of Japan.

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