The Correlation Between Hardness (Mean Contact Pressure by a Spherical Indenter) and Flow Stress*

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This paper deals with the correlation between hardness (mean contact pressure) P_m by a spherical indenter and flow stress Y under the uniaxial stress field. Firstly, the mean strain of the deformed material around the indentation ε_{ic}, which corresponds to the total strain under the uniaxial stress field, is defined following Tabor's idea. Secondly, hardness/flow stress ratio C is obtained experimentally for $C=9.8\,P_m/Y$, where Y is in MPa, and formulated for two conditions of material (1) elastic-plastic transient process: $C=1.1+(2/3)\ln(\varepsilon_{ic}E_s/Y)$, (2) ideal-plastic deformation: $C=2.9$; E_s is the Young's modulus of a material. Finally, by combining these relations, a method of estimating the flow stress-strain characteristic curve by spherical indentation is proposed. In addition, an example of estimation of the flow stress-strain characteristic curve of SUS 304 is shown over a wide range of strain by means of the proposed method.

Key Words: Material Testing, Hardness, Mean Contact Pressure, Total Mean Strain of an Indentation, Flow Stress, Hardness/Flow Stress Ratio, Stress-Strain Characteristic Curve

1. Introduction

When dealing with the relationship between hardness by a spherical indenter and flow stress, it is normal for the mean contact pressure defined as hardness to be correlated with the uniaxial stress defined as flow stress.

At the same time, it is necessary to correlate the mean strain of the deformed material around the indentation with the strain under the uniaxial stress field.

In fact, when permanent deformation of a material has been produced, the ratio between hardness multiplied by 9.8 and flow stress [MPa] (hardness/flow stress ratio C) varies from about 1.1 to about 3, this fact being confirmed theoretically and experimentally in previous studies[(1)~(3)].

For the case where the value of C is about 3, which means that the material deformed by the indenting sphere has reached approximately the state of ideal-plastic deformation, a quantitative relation of the hardness/flow stress ratio has been already established by Tabor and other researchers[(1)(4)].

On the other hand, the range where the value of C varies from about 1.1 to about 3 is called the elastic-plastic transient indenting process. Although this range was treated in Johnson's study[(5)], in which he extended Hill's theoretical solution[(6)] of expanding a spherical cavity to this problem, his approach was not satisfactory in determining the mean strain of an indentation when correlating hardness with flow stress. Therefore, the quantitative relationship between hardness and flow stress was incomplete.

This paper mainly considers the above points and proposes a new profile ratio of an indentation at the end of the plastic flow of a material; the profile ratio of total indentation is introduced to follow a previous paper[(7)].

Firstly, by assuming the following two conditions, i. e., (1) purely elastic contact between an indenting sphere and a material, and (2) idealplastic deformation of material in spherical indent-ation; an equivalent coefficient of total indentation strain can be obtained, and the mean strain of total indentation

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corresponding to the total strain under the uniaxial stress field is defined as the value of the equivalent coefficient of total indentation strain multiplied by the profile ratio of total indentation.

Secondly, the experimental results on hardness and flow sress are correlated on the basis of the mean strain of total indentation. Consequently, it is clarified that the relationship between both values in the elastic-plastic transient indenting process is formulated with a structure similar to Johnson's expression[5], which has been obtained by extending Hill's theoretical solution[6] with some modification of constants.

Furthermore, using this formulated equation, together with the formerly reported formula[7] for the elastic-plastic transient indenting process, it is shown with SUS 304 that the flow stress-strain characteristic curve can be estimated over a wide range of strain by using indenting hardness test results with a spherical indenter.

2. Correlation Between Hardness and Flow Stress Under the Condition of Ideal-plastic Deformation of a Material

The correlation between hardness and flow stress in the process of spherical indentation has been already clarified by Tabor[1] under the condition of ideal-plastic deformation. The relationships obtained by Tabor[1] using a steel ball indenter are summarized as follows:

P_m : hardness (mean contact pressure)
Y : flow stress [uniaxial stress; in MPa]
C : hardness/flow stress ratio
L : load
d : diameter of a permanent indentation
R, D : radius and diameter of an indenting sphere
(d/D) : apparent profile ratio of permanent indentation
$C_{\varepsilon a}$: equivalent coefficient of apparent indentation strain
ε_{ia} : mean strain of apparent indentation

$$C=9.8P_m/Y \quad (1)$$
$$C \fallingdotseq 2.8 \quad (2)$$
$$\varepsilon_{ia}=C_{\varepsilon a}(d/D) \quad (3)$$
$$C_{\varepsilon a} \fallingdotseq 0.2 \quad (4)$$

Here, hardness P_m is defined by Eq. (5).

$$P_m=0.102\times 4L/(\pi d^2) \quad (5)$$

In the case of work-hardening materials, Tabor thought that the flow stress at a strain under the uniaxial stress field, which corresponds to the mean strain of apparent indentation given by Eq. (3), should be correlated to the hardness.

3. Correlation Between Hardness and Flow Stress in the Elastic-Plastic Transient Process of a Material

Marsh[8] showed that the experimental value of hardness/flow stress ratio C follows Eq. (6),

$$C=A+B\ln(E_s/Y_y) \quad (6)$$

where E_s is Young's modulus and Y_y is the yield stress, and further that Eq. (6) coincides with the expression of a solution theoretically derived by Hill [6]. However, constants A and B were different form those of Hill's theoretical solution.

In later years, on the basis of the above studies, Johnson[5] extended Hill's solution to a from which included the influence of the apex angle of an indenter in it, by assuming that the volumetric expansion of the cavity was equal to the volume of the material displaced by the indenter, and gave an expression of hardness/flow stress ratio C [Eq. (7)] for the elastic-plastic transient indenting process using a pyramidal or a spherical indenter [Fig. 8(b)].

$$C=\frac{2}{3}\left\{1+\ln\left(\frac{1}{3}\frac{E_s}{Y}\tan\beta\right)\right\} \quad (7)$$

In addition, he thought that when the material behaved as work-hardening material [Fig. 3(b)], the mean strain of permanent indentation ε_{ip} might be given by Eq. (8),

$$\varepsilon_{ip}=0.2\tan\beta \quad (8)$$

which had a form similar to that of Tabor's expression. Further he assumed that the flow stress at permanent strain ε_p under the uniaxial stress field, which corresponded to the foregoing mean strain of permanent indentation, should be correlated to the hardness.

The value of $\tan\beta$ in Johnson's equation is given by Eq. (9) geometrically from Fig. 1(a).

$$\tan\beta=a/(R_c-\delta_c)\fallingdotseq(a/R_c)=(d/D_c) \quad (9)$$

Although $\tan\beta$ is equal to the profile ratio of total indentation (d/D_c), which will be defined later by authors, the value of (d/D_c) essentially includes the elastic part and the plastic part of an indentation, and therefore Johnson's idea of calculating the mean strain of permanent indentation ε_{ip} using the value of $\tan\beta$ is not valid. Furthermore, although it is necessary to consider the plastic and the elastic strain around the indentation, the discussion about this point is not clear, and the idea regarding the mean strain of an indentation is not sufficient in Johnson's paper[5]. For these reasons, these problems will be discussed in the following chapters.

4. Relationship Between Hardness and Profile Ratio of an Indentation

Before the mean strain of an indentation is defined, the relationship between hardness and the profile ratios of an indentation, which is the basis of the mean strain of an indentation, will be investigated.

4.1 Profile ratio of total indentation and other profile ratios

When an indenting sphere is indented into a flat material and the plastic deformation of the material is completed, the condition of this contact and the condition when the load was just removed are idealized as in Fig. 1(a) and (b), respectively.

We defined that $I(\mathrm{E})$ is the elastic parameter of the indenting sphere, $S(E)$ is the elastic parameter of the material and $f(E)$ is their sum. They are given in Eqs. (10) to (12) inclusive.

$$I(E)=(1-\mu_i^2)/E_i \quad (10)$$

$$S(E)=(1-\mu_s^2)/E_s \quad (11)$$

$$f(E)=I(E)+S(E) \quad (12)$$

where, E_i, E_s, μ_i and μ_s are defined as Young's modulus and the Poisson ratio of the indenting sphere and the material, respectively.

Furthermore. load ratio C_{LR} is defined as the ratio of a load to the squared diameter of an indenting sphere as in Eq.(13).

$$C_{LR}=0.102L/D^2 \quad (13)$$

The relation of the contact is given by Eq. (14) when the load L was applied from the state shown in Fig. 1(b) to the state shown in Fig. 1(a).

$$d=[3Df(E)L/\{1-(D/D_p)\}]^{1/3} \quad (14)$$

At the same time, the common contact diameter $D_c(=2R_c)$ of a spherical cup between the indenting sphere and the material is given by Hertz's law of elastic contact.

$$D_c=D\Big/\left[1-\frac{I(E)}{f(E)}\left\{1-\left(\frac{D}{D_p}\right)\right\}\right] \quad (15)$$

Accordingly, the profile ratio of the indentation (d/D_c) at the end of the plastic flow of the material is derived as follows, using Eqs. (10) to (15) inclusive.

$$(d/D_c)=(d/D)[1-\{3\times 9.8C_{LR}I(E)/(d/D)^3\}] \quad (16)$$

$$=\left(\frac{d}{D}\right)\left\{1-\frac{3\times 9.8C_{LR}f(E)}{(d/D)^3}\right\}+\frac{3\times 9.8C_{LR}S(E)}{(d/D)^2} \quad (17)$$

The first term on the right side of Eq. (17) is the true profile ratio of permanent indentation (d/D_p) [$D_p=2R_p$: diameter of the spherical cup of a permanent indentation], as shown in Eq. (18), and this is nothing but the profile ratio of the permanent deformation of the indentation.

$$(d/D_p)=(d/D)[1-\{3\times 9.8C_{LR}f(E)/(d/D)^3\}] \quad (18)$$

Only the second term remains under the condition of elastic contact where the term (d/D_p) becomes zero, so this is nothing but the profile ratio of the elastic deformation (recovery) of the indentation. Therefore, this second term may be defined as the profile ratio of elastic indentation $(d/D)_r$ as shown in Eq. (19).

$$(d/D)_r=3\times 9.8C_{LR}S(E)/(d/D)^2 \quad (19)$$

Accordingly, the profile ratio of the indentation at the end of the plastic flow of the material is given by Eq. (20).

$$(d/D_c)=(d/D_p)+(d/D)_r \quad (20)$$

This expression is expressed as the sum of the plastic and the elastic deformations of the profile ratio of the indentation.

Consequently, in this paper, this profile ratio of the indentation is defined as the profile ratio of total indentation.

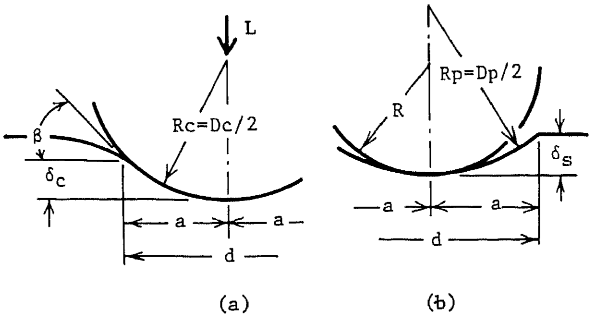


Fig. 1 Idealized contact between an indenting sphere and a material

4.2 Relationship between hardness and profile ratios under the condition of elastic contact

Let the notation with a suffix e mean the value under the condition of elastic contact with $D_p=\infty$. The relation between hardness P_{me} and the apparent profile ratio of indentation (d_e/D) is obtained from Eqs. (5) and (14).

$$P_{me}=0.102[4/\{3\pi f(E)\}](d_e/D) \quad (21)$$

In addition, the next equation is derived from Eqs. (12) and (15).

$$D_c\cdot S(E)=D\cdot f(E) \quad (22)$$

If Eq. (22) is substituted into Eq. (21), the relation between hardness P_{me} and the profile ratio of total indentation (d_e/D_c) under the condition of elastic contact is given by Eq. (23).

$$P_{me}=0.102[4/\{3\pi S(E)\}](d_e/D_c) \quad (23)$$

4.3 Relationship between hardness and the profile ratio of total indentation when a permanent indentation is produced

The relations between hardness P_m and the true profile ratio of a permanent indentation in the elastic-plastic transient indenting process reported in a previous paper[7] are shown in Eqs. (24) and (25).

$$P_m=P_{up}(d/D_p)^{x_p} \quad (24)$$

$$P_m=P_{up}\left[\left(\frac{d}{D}\right)\left\{1-\frac{3\times 9.8\pi f(E)}{4(d/D)}P_m\right\}\right]^{x_p} \quad (25)$$

From Eqs. (13) and (16), Eq (26) is derived.

$$(d/D_c)=(d/D)[1-\{3D\cdot I(E)\cdot L/d^3\}] \quad (26)$$

When Eqs. (26), (12), (5) and (25) are used together, finally, the relationship between hardness P_m and the profile ratio of total indentation is obtained as:

$$P_m = P_{up}\left[\left(\frac{d}{D_c}\right)\left\{1-\frac{3\times 9.8\pi S(E)}{4(d/D_c)}P_m\right\}\right]^{x_p} \quad (27)$$

where P_{up} and x_p are the ultimate plastic hardness and the plastic hardening index by a spherical indenter, respectively.

4.4 Relationships between hardness and profile ratio of an indentation and between flow stress and strain

Relationships between hardness P_m and the various profile ratios of an indentation (d/D), (d/D_p), and (d/D_c) are shown in Fig. 2 on logarithmic coordinates.

The relationship between P_m and (d/D) cannot be shown with a single line in Fig. 2, since this relationship is dependent on the mechanical properties of the indenting sphere. For example, the curve of P_m-(d/D) using a tungsten carbide ball indenter (W ball) is idealized with a broken line, and the curve using a steel ball indenter (S ball) is idealized with a two-dot chained line in Fig. 2.

On the other hand, it can be seen that the P_m-(d/D_c) relation expressed by Eqs. (23) and (27) is independent of the mechanical properties of the indenting sphere, so that it may be shown by a single line. The relation between the flow stress, multiplied by 0.102, and the strain is also shown in Fig. 2, for comparison.

Consequently, referring to the above relationships, it may be understood that the basic correlation should be made between the flow stress Y- total strain ε relation and the hardness P_m-profile ratio of total indentation (d/D_c) relation.

5. Mean Strain of an Indentation by a Spherical Indenter

On the basis of the above discussions, the method which defines the mean strain of an indentation from

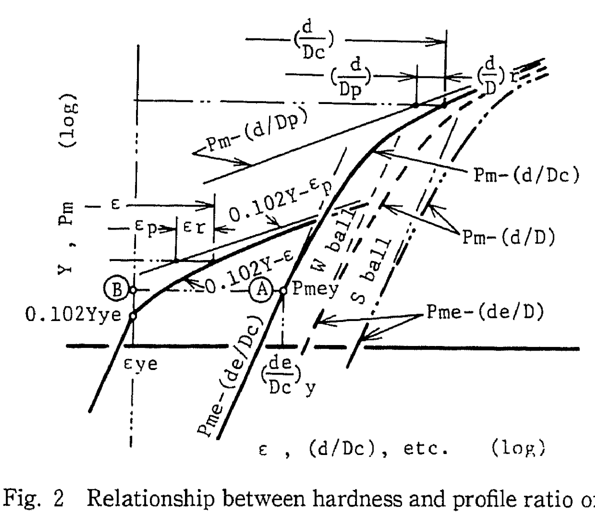


Fig. 2 Relationship between hardness and profile ratio of an indentation, and relationship between flow stress and strain

the profile ratio of that indentation will be mentioned in this chapter.

Firstly, as described in a previous paper[7], for many materials, the relationship between flow stress Y and total strain ε [Eq. (30)] can be illustrated as in Fig. 2 under uniaxial compression.

Here, permanent strain ε_p of n-th power hardening material and elastic strain ε_r are given by Eqs. (28) and (29), respectively.

$$Y = C_p \cdot \varepsilon_p^n \quad (28)$$

$$\varepsilon_r = Y/E_s \quad (29)$$

$$\varepsilon = \varepsilon_p + \varepsilon_r \quad (30)$$

On the basis of these results, if Eq. (20), which shows the profile ratio of total indentation, is compared with Eq. (30), which shows the total strain, the mean strain of an indentation may be defined as the value of the profile ratio of an indentation multiplied by the equivalent coefficient of indentation strain, following Tabor's Eq. (3).

Provided that the mean strain of an indentation is assumed to give the strain under the uniaxial stress field, the following correspondences may be obtained.

$$\varepsilon_{ic} = C_{\varepsilon c}(d/D_c) \rightarrow \varepsilon \quad (31)$$

$$\varepsilon_{ip} = C_{\varepsilon p}(d/D_p) \rightarrow \varepsilon_p \quad (32)$$

$$\varepsilon_{ir} = C_{\varepsilon r}(d/D)_r \rightarrow \varepsilon_r \quad (33)$$

where,

ε_{ic}, ε_{ip} and ε_{ir} : mean strain of total, permanent and elastic indentation using a spherical indenter

$C_{\varepsilon c}$, $C_{\varepsilon p}$ and $C_{\varepsilon r}$: equivalent coefficient of total, permanent and elastic indentation strain using a spherical indenter

5.1 Equivalent coefficient of elastic indentation strain $C_{\varepsilon r}$

In order to determine this coefficient, the hardness value at which the deformation of a material subjected to spherical indentaion changes from the elastic deformation to the elastic-plastic deformation should be related with the stress at which the deformation under the uniaxial stress field changes from the elastic deformation to the elastic-plastic deformation.

Firstly, it is assumed that the onset of the plastic deformation of a material under an indenting sphere begins when the mean contact pressure has reached the value of $0.102\times1.1\ Y_{ye}$ (Y_{ye}: yield stress), regardless of work-hardening or non-work-hardening materials [Figs. 3(a) and (b)]. Therefore, Eq. (34) may be used.

$$P_{mey} = 0.102\times1.1\ Y_{ye} \quad (34)$$

Namely, hardness/flow stress ratio C has a value of 1.1 at this position. In order to perform this correlation, position Ⓐ $[P_{mey}, (d_e/D_c)_y]$ must be moved, by an unknown quantity, to Ⓑ $[0.102\times1.1\ Y_{ye}, \varepsilon_{ye}]$ as shown in Fig. 2. This unknown quantity may be taken as the equivalent coefficient of elastic indentation strain to be obtained.

The position where the plastic deformation of the material begins is equal to that where the material still satisfies the elastic equations Eqs. (29) and (23), so $Y_{ye}=E_s\ \varepsilon_{ye}$ is obtained from Eq. (29) with $\varepsilon_r \to \varepsilon_{ye}$, $Y \to Y_{ye}$. If this equation and Eq. (23) are substituted into the right and left sides of Eq. (34), respectively, and if rearranged [the values suffixed with y in the equations mean the values at the yield point (for example, ε_y: yield strain) and $(1-\mu_s^2)=0.9$ is assumed], Eq. (35) is obtained as follows:

$$\varepsilon_{ye}=[4/\{3.3\pi(1-\mu_s^2)\}](d_e/D_c)_y \qquad (35)$$

Equation (35) gives the relative position on the abscissa between positions Ⓑ of ε_{ye} and Ⓐ of $(d_e/D_c)_y$, if two curves are described on the same graph, as shown in Fig. 2.

At the same time, since this position is equal to the last position of the elastic deformation (the position where the onset of permanent deformation begins), so that $(d_e/D_c)_y$ is equal to $(d/D)_r$ at this position, the equivalent coefficient of elastic indentation strain $C_{\varepsilon r}$ is obtained as in Eq. (36), from Eqs. (33) and (35).

$$C_{\varepsilon r}=4/\{3.3\pi(1-\mu_s^2)\}\fallingdotseq 0.43 \qquad (36)$$

5.2 Equivalent coefficient of permanent indentation strain $C_{\varepsilon p}$

Under conditions of ideal-plastic deformation, the value of the profile ratio of elastic indentation $(d/D)_r$ is considerably smaller than that of permanent indentation, so that approximate relationship $(d/D)\fallingdotseq(d/D_c)\fallingdotseq(d/D_p)$ is obtained. From Tabor's Eq. (3) and Eq. (32), it is seen that the approximate relationship $C_{\varepsilon p}\fallingdotseq C_{\varepsilon a}$ is obtained. Consequently, the equivalent coefficient of permanent indentation $C_{\varepsilon p}$ is given as in Eq. (37).

$$C_{\varepsilon p}=2/\{3.3\pi(1-\mu_s^2)\}\fallingdotseq 0.21 \qquad (37)$$

5.3 Equivalent coefficient of total indentation strain $C_{\varepsilon c}$

From the above discussion, the equivalent coefficient of total indentation strain $C_{\varepsilon c}$ which converts the profile ratio of total indentation (d/D_c) to total strain ε under the uniaxial stress field is obtained as shown in Eq. (38), using the previously given equations.

$$C_{\varepsilon c}=\frac{2}{3.3\pi(1-\mu_s^2)}\left[1+\frac{S(E)}{\{(d/D)^3/(3\times 9.8C_{LR})\}-I(E)}\right] \qquad (38)$$

This equation coincides with Eq. (36) when the material and the indenting sphere are under the condition of elastic contact, and corresponds to Eq. (37) when the material has approached the condition of ideal-plastic deformation.

5.4 Mean strain of total indentation ε_{ic}

The mean strain of the deformed material around the indentation (the mean strain of total indentation ε_{ic}) is obtained ultimately as Eq. (39), from Eqs. (10), (11), (12), (16), (31) and (38).

$$\varepsilon_{ic}=0.21\left[1+\frac{3\times 9.8C_{LR}}{(d/D)^3}\{S(E)-I(E)\}\right]\left(\frac{d}{D}\right) \qquad (39)$$

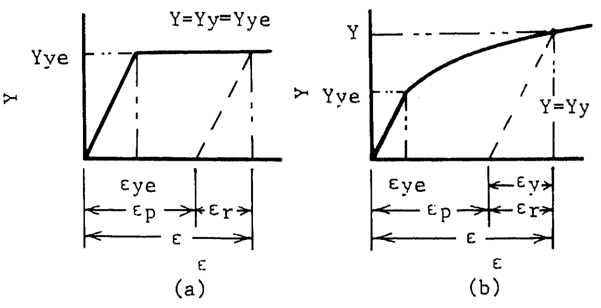


Fig. 3 Non-work-hardening and work-hardening characteristic behavior of materials

6. Experimental Method

6.1 Indenters and loads

Tungsten carbide ball indenters, 2 mm and 5 mm in diameter (W2 indenter and W5 indenter), were used as shown in Table 1. A load ranging from 9.8 N to 490 N was applied using a Vickers hardness tester. A small compression tester was used for loads over 490 N.

6.2 Materials

For the purpose of measuring flow stress, various commercial rod materials, 50 mm in diameter, were used. In order to provide materials with various mechanical and work-hardening properties, heat treatments and prestrains were given as shown in Table 2.

SS41(A) and AA(A) were annealed for 5 hr at temperatures shown in Table 2, then were machined into rods of 48 mm in diameter and 70-75 mm long. Prestrains (shown in Table 2 in parentheses) were given by using a compression tester with a maximum load capacity of 200 tons. To examine inhomogeneity in the radial direction of the rods, specimens for measuring flow stress, 12 mm in diameter and about 30 mm long, were cut out of the rods in the longitudinal direction at several radial positions where indenting tests were to be performed. After large strain foil

Table 1 Specifications of indenters

Indenter	Ball Material	D mm	Ei GPa	μi	I(E) 1/{10^3GPa}
W 2 W 5	Tungsten carbide	2 5	608	0.21	1.57

1 GPa = 102 Kgf/mm^2

gauges were applied on the surface of these specimens, compression tests were carried out using a compression tester with a maximum load capacity of 30 tons.

All specimens for indenting test by a spherical indenter had diameters of about 50 mm and about 9-15 mm long. The Vickers hardness tests at various loads were carried out in order to investigate homogeneity of the indenting test surface of the specimen.

6.3 Measurement of the profile of a permanent indentation

The dimensions of a permanent indentation were measured using a toolmaker's microscope in the same way as in the previous paper.

7. Experimental Results and Discussion

7.1 Flow stress Y-total strain ε

The experimental results of flow stress ($0.102\times Y$)-total strain (ε) of each material are shown in Figs. 4 to 7 and 9 on logarithmic coordinates. It is seen from the figures that the prestrained materials such as SS41(A) and AA(A), may be dealt with as elastic-perfectly plastic materials, and that the materials such as SUS630H900, AA and SUS304 exhibit work-hardening behavior; in particular, SUS 304 especially shows remarkably this character.

7.2 Hardness P_m-mean strain of total indentation ε_{ic}

In order to assure the homogeneity of the testing surface, the measurements of P_m at small load ratios were restricted to materials of SUS 630 H 900 and AA.

The relationships between hardness P_m and the mean strain of total indentation ε_{ic} calculated from Eq. (39), when the indenter W2 (●) and the indenter W5 (○) were indented at each testing load, are shown in Figs. 4 to 7 and 9.

In the case of SS 41(A) and AA(A), which may be dealt with as nearly elastic-perfectly plastic mate-

Table 2 Specifications of the materials studied

Metal {JIS}	Hardness (HV) 9.8N-490N	Es GPa	S(E) 1/{10^3GPa}
SS41(A) {SS41} (38.5%) 650 ℃	185 - 175	206	4.37
AA(A) {A2017} (20%) 415 ℃	82 - 85	71	12.7
SUS630 {SUS630} H900	430	196	4.50
AA {A2017} Commercial	125 - 135	72	12.5
SUS304 {SUS304} Commercial	210 - 170	190	4.70

1 N= 0.102 Kgf, 1 GPa = 102 Kgf/mm^2

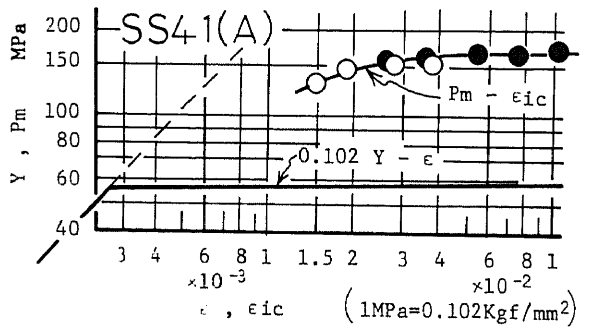


Fig. 4 Relationship between $0.102Y$ and ε and that between P_m and ε_{ic} for SS 41(A)

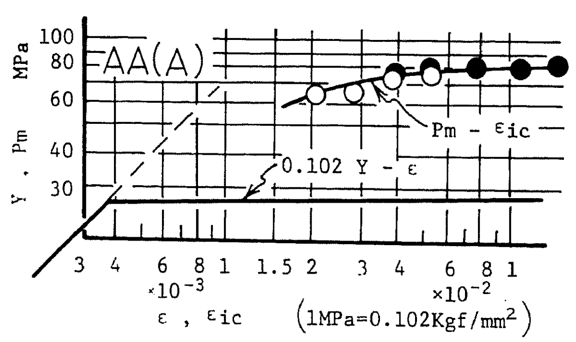


Fig. 5 Relationship between $0.102Y$ and ε and that between P_m and ε_{ic} for AA(A)

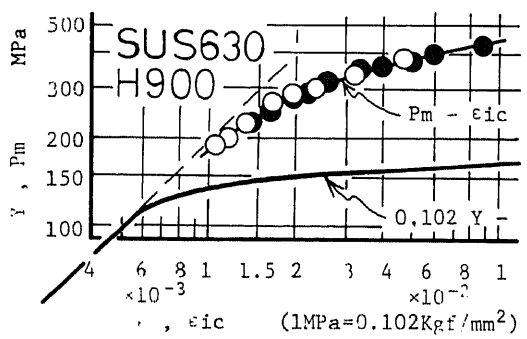


Fig. 6 Relationship between $0.102Y$ and ε and that between P_m and ε_{ic} for SUS 630 H 900

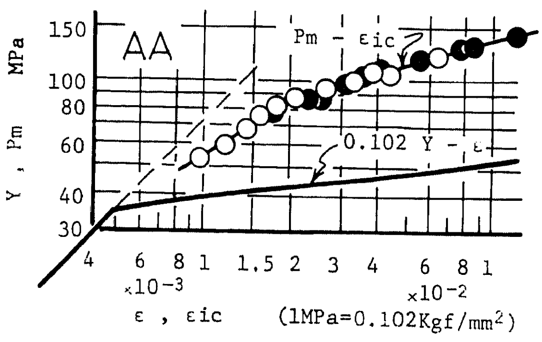


Fig. 7 Relationship between $0.102Y$ and ε and that between P_m and ε_{ic} for AA

rials, it is seen that the value of P_m becomes almost constant when the value of ε_{ic} becomes larger than 3×10^{-2}, which corresponds to the value about 0.15 of (d/D), and 4×10^{-2}, which corresponds to the value about 0.2 of (d/D), respectively. This may indicate that the yielded portion of these materials has reached nearly the perfect-plastic state at this range of strain.

On the other hand, in the case of materials such as SUS 630 H 900, AA and SUS 304 which exhibit work-hardening behavior, the value of P_m still continues to increase, whereas the value of ε_{ic} becomes larger than 8×10^{-2}, which corresponds to the value about 0.5 of (d/D).

7.3 Hardness/flow stress ratio C-mean strain of total indentation ε_{ic}, mean strain of total indentation/elastic recovery strain ratio $(\varepsilon_{ic}/\varepsilon_r)$

The value of C obtained from the relationships between P_m-ε_{ic} and $0.102\times Y-\varepsilon$ is shown in Fig. 8(a) against ε_{ic} on the logarithmic abscissa. The C-ε_{ic} relationship of each material is different, presumably because these relationships are influenced by the difference of the yield stress or Young's modulus of each material. Thus, it is impossible to formulate the $C-\varepsilon_{ic}$ relationship under this condition.

Then, referring to Hill's Eq. (6) and Johnson's Eq. (7), it is expected that the value of C may be related to E_s/Y, or elastic recovery strain $\varepsilon_r=Y/E_s$ $(=\varepsilon_{ir}$: in the case of elastic contact) [Figs. 3(a) and (b)], which appears in Eqs. (6) and (7).

From this point of view, the experimental values of C were plotted in Fig. 8(b) against the value of $(\varepsilon_{ic}/\varepsilon_r)$ on the logarithmic scale. Different from Fig. 8(a), a unique relation is observed between the experimental values of C and $(\varepsilon_{ic}/\varepsilon_r)$ in Fig. 8(b), irrespective of difference in work-hardening characteristics, Young's moduli, yield stresses, etc.

Figure 8 (b) shows that the values of C start at about 1, increase nearly straight with a slightly "convex upwords" tendency near the end of the straight line and finally approach a constant value at values of $(\varepsilon_{ic}/\varepsilon_r)$ larger than 15.

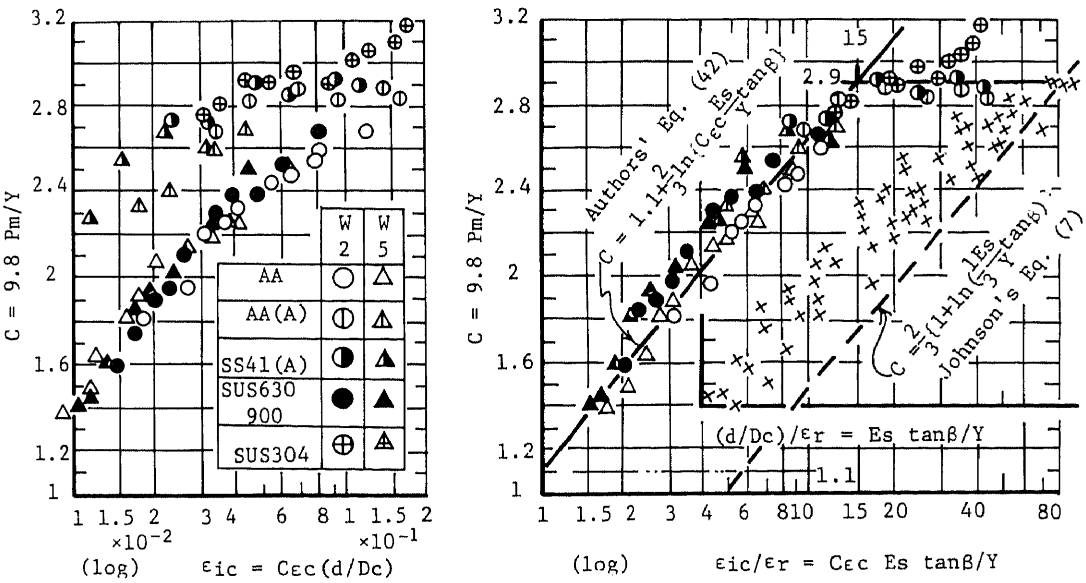
The experimental results show that hardness/flow stress ratio C may be formulated as a function of the mean strain of total indentation/elastic recovery strain ratio $(\varepsilon_{ic}/\varepsilon_r)$ in the elastic-plastic transient spherical indenting process.

Accordingly, the $C-(\varepsilon_{ic}/\varepsilon_r)$ relationships are formulated as shown in Eqs. (40) and (41), corresponding to two conditions of (1) elastic-plastic transient process and (2) ideal-plastic deformation.

$$\text{(1)}\quad \left.\begin{array}{l} 1<(\varepsilon_{ic}/\varepsilon_r)<15,\ 1.1<C<2.9 \\ C=1.1+(2/3)\ln(\varepsilon_{ic}/\varepsilon_r) \end{array}\right\} \qquad (40)$$

$$\text{(2)}\quad 15<(\varepsilon_{ic}/\varepsilon_r)\quad C=2.9 \qquad (41)$$

Following Johnson[5], the experimental relationship between C and $E_s\tan\beta/Y=(d/D_c)/\varepsilon_r$ [indicated with symbols X for all materials] and the calculated values from Johnson's Eq. (7) are shown in Fig. 8(b). It is understood that they are quantitatively different from each other. Furthermore, since the profile ratio of total indentation (d/D_c) divided by ε_r has been used in the abscissa, even though the mean strain of total indentation ε_{ic} divided by ε_r should be used, the experimental values and the calculated



(a) Relationship between hardness/flow stress ratio C and ε_{ic}

(b) Relationship between hardness/flow stress ratio C and $(\varepsilon_{ic}/\varepsilon_r)$

Fig. 8

values do not coincide with the values calculated from Hertz's elastic contact theory at the transient position of the elastic contact. (Please refer to Fig. 4 of Johnson's paper)[5]

Equation (40) may now be rewritten as shown in Eq. (42).

$$C=1.1+\frac{2}{3}\ln\left\{C_{\varepsilon c}\frac{E_s}{Y}\tan\beta\right\} \tag{42}$$

It can be seen that Eq. (42) expresses the same form as Johnson's Eq. (7), which is given by extending Hill's theoretical solution to expanding of a spherical cavity, with different constants.

8. Method to Estimate the Flow Stress-Strain Characteristic Curve from the Indenting Hardness Test by a Spherical Indenter

The Vickers pyramidal indenter has the important characteristic of having a constant mean strain around the indentation [about 8% strain, which corresponds to the strain when the spherical indenter has been indented to the value of $(d/D)=0.375$]; therefore, it shows a constant hardness value even though the load is changed, because of its geometrical form. It should be noted, however, that Vickers hardness test can only estimate the flow stress at a strain of about 8%.

On the other hand, although it is disadvantageous that the hardness value by a spherical indenter changes as the applied load is changed, it is expected that the flow stress-strain characteristics can be estimated over a wide range of strain by means of utilizing Eqs. (40) and (41), which express the hardness/flow stress ratio C of various materials from high strain to very small strain.

Therefore, the method of estimating the flow stress - strain characteristics by spherical indentation should be briefly mentioned;

8.1 Materials showing a constant hardness value when the applied load is changed

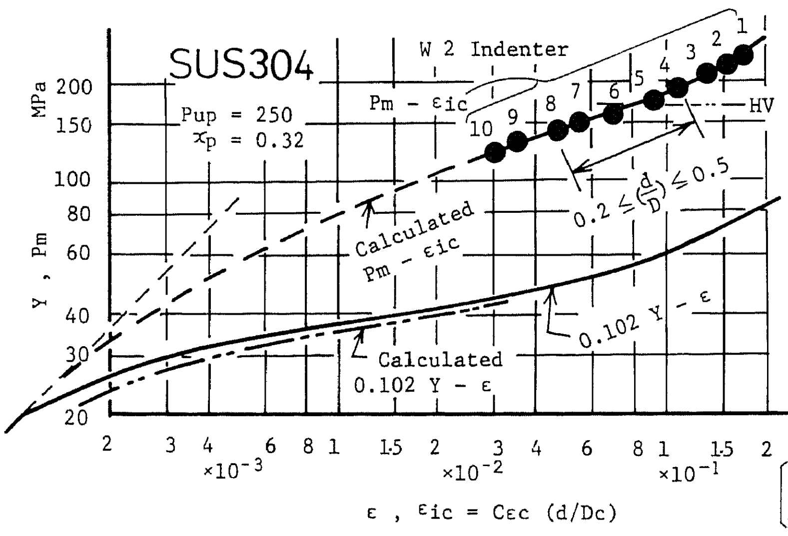
Since materials which reach the state of a constant hardness value for (d/D) larger than 0.2, which corresponds to the value about 4×10^{-2} of ε_{ic}, may be dealt with as elastic-perfectly plastic materials (see Figs. 4 and 5), flow stress may be estimated from Eq. (41), and the flow stress-strain characteristic curve may be estimated as shown in Figs. 4 and 5.

8.2 Materials having a variable hardness value as the applied load is changed

Materials whose hardness values changes, whereas the value of (d/D) becomes larger than 0.4 or 0.5, are considered work-hardening materials. Since materials which exhibit a marked hardening behavior are lacking in homogeneity along the direction of the depth from the surface, it is expected that the hardness value under the condition of a lower load ratio will not coincide with the hardness value of the base material.

For this reason, the method of estimating the flow stress-strain characteristics at low strains will be mentioned, using SUS 304 as an example, by means of calculating from the equations given in this and the previous papers[7].

Firstly, the values P_{up} and x_p shown in Eq. (24) must be obtained by hardness tests within a (d/D) value range of about 0.2-0.5. Next, the P_m-(d/D)-C_{LR}-ε_{ic} relationship must be obtained using Eqs. (25) and (39) (see Fig. 9). If this relationship is obtained, each pair of P_m-ε_{ic} is substituted into Eq. (40) [from the



Experimental conditions

Number	L (N)	$(\frac{d}{D})$	Number	L (N)	$(\frac{d}{D})$
1	4900	0.80	6	490	0.32
2	3430	0.70	7	294	0.25
3	2450	0.61	8	196	0.21
4	1470	0.50	9	98	0.16
5	980	0.43	10	68.6	0.14

$$\begin{bmatrix} 1\ \text{MPa} = 0.102\ \text{Kgf/mm}^2 \\ 1\ \text{N} = 0.102\ \text{Kgf} \end{bmatrix}$$

Fig. 9 Experimental and calculated values of $0.102Y-\varepsilon$ and $P_m-\varepsilon_{ic}$ for SUS 304 relationship of $\varepsilon_r = Y/E_s$], and the solution of flow stress Y which satisfies Eq.(40) may then be obtained.

The line of alternate long and two short dashes in Fig. 9 shows the $0.102 \times Y - \varepsilon$ relationship obtained by using the above calculating method. It can be seen that the calculated values agree well with the experimental values indicated by the continuous line. However, Fig. 9 shows that the values of P_{up} and x_p given within a (d/D) value range of about 0.2-0.5 cannot be applied to marked work-hardening materials under a larger load (ratio).

Therefore, in the case of marked work-hardening materials, it is understood from Fig. 8(b) that flow stress may be estimated as the hardness value multiplied by 3.3, which is measured under the condition of a (d/D) value larger than about 0.5. The strain at this point, however, may be obtained by using Eq.(39).

Consequently, by the method mentioned above, the flow stress-strain characteristic curve may be estimated within a low-to-middle strain range (smaller than about 20% strain).

9. Conclusions

The results obtained are summarized as follows.

(1) In order to obtain the quantitative relationship of hardness/flow sress ratio C, firstly, the ratio of the diameter of a permanent indentation d to the common contact diameter D_c at the end of the plastic flow of a material, namely, the profile ratio of total indentation, is introduced, and it is shown that the relationship between hardness P_m and the profile ratio of total indentation (d/D_c) may be correlated to that between flow stress Y and total strain ε.

(2) In order to obtain the mean strain of an indentation corresponding to the total strain under the uniaxial stress field, firstly, by means of carefully observing two positions, i. e., the position where the material deformed by an indenting sphere changes its behavior from elastic to elastic-plastic and the position where the material reaches the ideal-plastic state, the equivalent coefficient of total indentation strain $C_{\varepsilon c}$ is obtained.

Next, following Tabor this value multiplied by the total profile ratio of indentation (d/D_c) is defined as the mean strain of total indentation ε_{ic}.

(3) By correlating the P_m-ε_{ic} relationship with that of $0.102 \times Y - \varepsilon$, obtained experimentally for several materials, hardness/flow stress ratio C is given, and it is clarified that hardness/flow stress ratio C may be formulated as a function of the mean strain of total indentation/elastic recovery strain ratio $(\varepsilon_{ic}/\varepsilon_r)$.

(4) By means of using the equation for hardness/flow stress ratio C and the equations for physical quantities obtained in a previous paper appearing in the elastic-plastic transient indenting process by a spherical indenter, a method of estimating the flow stress-strain characteristic curve is proposed. And, it is shown for SUS 304, as an example, that the flow stress-strain characteristics can be estimated in a range from very low to moderate strains.

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