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# Numerical examination of the operation modes of a weakly relativistic oversized backward wave oscillator

HOSHIYUKI YAMAZAKI, HIROKAZU TAKAGI, HIDEKAZU TANAKA and KAZUO OGURA
Graduate School of Science and Technology, Niigata University, 8050 Igarashi Ninocho, Niigata 950-2181, Japan

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**Abstract.** Backward wave oscillators (BWOs) have been studied as a candidate high-power microwave source. To increase the operation frequency, an oversized slow-wave structure (SWS) is used. The operation at reduced voltage is preferable for practical applications. This work is aimed at numerically examining the operation mode of a weakly relativistic oversized BWO. We examine not only the axisymmetric transverse magnetic mode but also the non-axisymmetric hybrid modes of the oversized SWS. Both of them are candidates for the operation modes. These modes are surface waves whose fields are concentrated near the SWS wall. They overlap in frequency and are not separated by stop-bands. For an efficient beam interaction, the injected electron beam needs to be controlled more accurately than in the non-oversized SWS case.

## 1. Introduction

Backward wave oscillators (BWOs) have been studied as a candidate high-power microwave source [1, 2]. In BWOs, a periodically corrugated slow-wave structure (SWS) is used and reduces the phase velocity of the electromagnetic mode close to the beam velocity, ensuring sufficient coupling of beam and electromagnetic modes. The physical shape and dimension of the SWS play major roles in controlling the wave–particle interaction as well as the nature of output such as frequency and power. Actually, in order to increase the power handling capability and/or the operating frequency, oversized SWSs have been used successfully [1, 3], in which the diameter of SWS is larger than the free-space wavelength of the output electromagnetic wave by several times or more. For practical applications and fundamental studies, devices operating at a reduced voltage are preferable [3]. However, efficient high-power operation in the weakly relativistic region becomes more difficult than in the relativistic case, as shown in this paper. The previous studies of BWOs have been restricted to operation in the fundamental axisymmetric transverse magnetic (TM) mode [1, 2]. However, non-axisymmetric operations of BWOs have been observed in experiments and their study has become important [3–5].

Periodic structures such as SWSs of BWOs and gratings are widely used in modern science and technology, for example in optics, acoustic design, infrared spectroscopy and microwave technology. Among the various methods to analyze

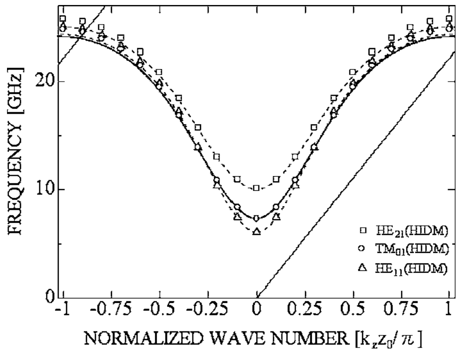


**Figure 1.** The dispersion relation for K-band oversized SWSs.

such periodic systems, the Rayleigh method based on the Rayleigh hypothesis is particularly simple and important in the study of the wave–particle interaction [4–6]. A self-consistent field theory considering three-dimensional perturbations of electrons became possible with the Rayleigh method, and the normal modes and their interaction with electron beams were analyzed [4, 5]. The Rayleigh hypothesis assumes that the fields inside and outside the corrugation can be expressed by the same function. There exists a theoretically proposed limit for its applicability which has been argued extensively. For 8 GHz (X-band) non-oversized cases, the numerical analyses based on the Rayleigh hypothesis can be used beyond the limit [2, 6, 7]. For oversized SWSs, the fields concentrate near the SWS wall [3] and are quite different from the volumetric waves of non-oversized SWSs with a strong field near the axis [2, 7]. The validity of the Rayleigh method should be examined for oversized SWSs.

This work is aimed at numerically examining the operation modes of weakly relativistic oversized BWOs, operating in a weakly relativistic region (less than 100 kV) and at high frequency (in the 20 GHz range: K-band). For the numerical examinations, we use two methods: the first is the Rayleigh method and the other is a numerical integration of Maxwell equations using the higher-order implicit difference method (HIDM) [6, 7]. The latter is free from the Rayleigh hypothesis. We examine the accuracy of the Rayleigh method for K-band SWSs by comparing it with HIDM. The Cherenkov instability of K-band oversized BWOs responsible for the microwave radiation is also discussed.

## 2. Numerical analysis for oversized BWO

We consider a cylindrical periodic SWS, having average radius $R_0$, corrugation amplitude $h$ and pitch length $z_0$. The wall radius varies along the axial direction $z$ as $R_0 + h\cos((2\pi/z_0)z)$. The dispersion characteristics of the structure are controlled by changing $R_0$, $h$ and $z_0$. The upper cutoff of the lowest mode is mainly determined by the combination of $h$ and $z_0$. The lower cutoff is determined by $R_0$. The K-band SWS parameters are chosen as $h = 0.17$ cm, $z_0 = 0.34$ cm and $R_0 = 1.57$ cm. The modulation depth $h$ is much larger than the proposed limit of Rayleigh hypothesis, about seven times larger than the limit.

The lowest modes for K-band SWSs are shown in Fig. 1. The solid curve is $TM_{01}$ and dashed curves hybrid $HE_{11}$ and $HE_{21}$ obtained by the Rayleigh method. The

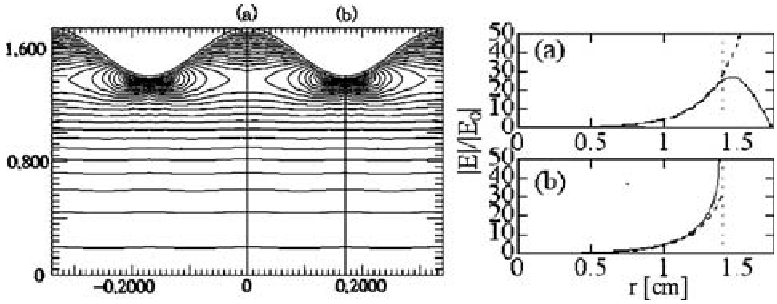


**Figure 2.** The distribution of electric field intensity for K-band SWS at $3/4\pi$.

numerically obtained data using HIDM are plotted by the circles ($TM_{01}$), triangles ($HE_{11}$) and squares ($HE_{21}$). The straight line is a beam line with 100 kV. The lowest non-axisymmetric HE modes exist very close to the lowest TM mode and can become candidates for the non-axisymmetric operations.

The modulation depth of K-band SWSs is about seven times that of theoretically established limit of the Rayleigh hypothesis. However, the dispersion relations obtained by the Rayleigh method and HIDM are in fairly good agreement. In particular, in the region up to $k_z z_0 = 3/4\pi$, results show very good coincidence. Figure 2 is a field distribution of $TM_{01}$ at $k_z z_0 = 3/4\pi$. The $TM_{01}$ mode is a surface wave whose fields are concentrated near the SWS wall. The field is strongest at the top of SWS wall, and is weakest inside the corrugation. The radial distribution of electric field intensity is shown in the right-hand side of Fig. 2 at (a) $z = 0$ and (b) $z = z_0/2$. The numerical results are plotted by solid curves (HIDM) and broken curves (Rayleigh method). The dotted lines show the position of the SWS wall. The electric field intensity obtained by the Rayleigh method is diverging inside the corrugation, because singular points appear. The field profiles from HIDM and the Rayleigh method show fairly good agreement outside the corrugation even for a modulation depth much larger than the proposed limit of Rayleigh hypothesis, about seven times larger than the limit. This is larger compared with the X-band SWS, for which the Rayleigh method can be used up to five times larger than the limit [7]. Note that singular points and diverging properties of the field disappear if the criterion of the Rayleigh hypothesis is fulfilled. The Rayleigh method is available to examine K-band SWS's field properties outside corrugation and gives valid results in the range from 0 to $3/4\pi$.

For the Cherenkov interaction, the TM component is essential [5]. We examine the component by using the Rayleigh method. Figure 3(a) shows the dispersion relation and 80 keV beam line. The dispersion curves are overlapped and are not separated by stop-bands in the vicinity of wave number $k_z z_0 = \pi$. Figure 3(b) shows the ratio of transverse electric (TE) and TM components of the zero spatial harmonic. For non-axisymmetric HE and EH modes, the ratio changes depending on wave number. The lowest HE modes are surface waves similar to the $TM_{01}$ mode, and their TM component becomes dominant near the operation point above $1/2\pi$. The non-axisymmetric HE modes existing close to the $TM_{01}$ mode are candidates for the operation modes. On the other hand, EH modes are not surface waves and their TM component becomes small near the operation point. The EH modes might not contribute to the oscillation.

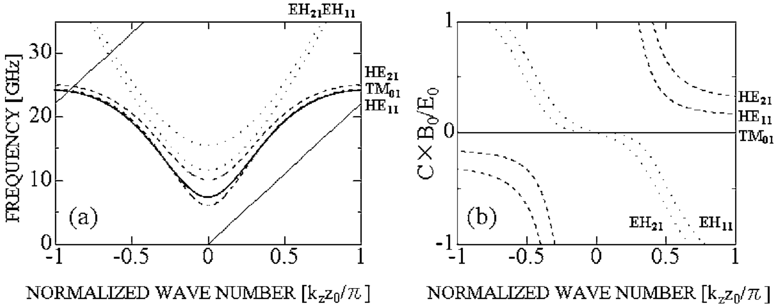


**Figure 3.** The ratio of TE and TM components of the zero spatial harmonic.

## 3. Discussion and conclusion

The candidates for oversized BWO operation modes are surface waves whose fields are concentrated near the SWS wall. The beam interactions can be examined by using the field theory based on the Rayleigh method. We examine the dependence of the temporal growth rate of $TM_{01}$ on the beam radius for non-oversized X-band and for oversized K-band SWSs. The temporal growth rate becomes the maximum in the vicinity of the inner corrugation wall for both cases. If the electron beam is away from corrugation wall, the temporal growth rates decrease. The radial dependence of growth rate corresponds to that of electric field intensity. The growth rate with $R_b/R_w \cong 1$ becomes one half by decreasing the beam radius to $R_b/R_w \cong 0.9$ for the K-band SWS and to $R_b/R_w \cong 0.1$ for the X-band SWS, much smaller than the K-band case. This indicates that the injected electron beam in the oversized SWS needs to be controlled more accurately than in the non-oversized SWS case.

In conclusion, the influence of the singularity due to the limitation of the Rayleigh hypothesis should be carefully examined when the Rayleigh method is used beyond the proposed limit. The Rayleigh method is still useful for the oversized K-band SWS. Non-axisymmetric HE and axisymmetric $TM_{01}$ modes are the candidates for the operation and exist close to each other. They are surface waves, and hence the beam interactions are strongly influenced by the beam radius.

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