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Slow cyclotron instability due to surface modulation of an annular beam

KAZUO OGURA[1], HOSHIYUKI YAMAZAKI[1], YUTAKA KIUCHI[1] and MD. RUHUL AMIN[2]

[1]Graduate School of Science and Technology, Niigata University, Niigata 950-2181, Japan

[2]Electrical and Electronic Engineering Department, Rajshahi University of Engineering and Technology, Rajshahi-6204, Bangladesh

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Abstract. The Cherenkov instability used in slow-wave devices has been well studied in the literature. However, in previous analyses, the beam motion is restricted to the longitudinal direction assuming an infinitely strong magnetic field. For the finite strength magnetic field, the transverse beam perturbation cannot be ignored and leads to the slow cyclotron instability. Recently, a new version of self-consistent field theory considering three-dimensional perturbation has been developed based on a solid beam, in which the effect of the transverse perturbation appears as a surface charge at a fixed boundary. In the case of a thin annular beam, the boundary is modulated and is essentially different from the solid beam case. We propose a self-consistent field theory considering the moving modified boundary surface. The slow cyclotron instability due to the modulation of an infinitesimally thin annular electron beam is presented.

1. Introduction

For an electron beam propagating along the direction of an axial magnetic field, there exist four beam modes: slow and fast space charge modes and slow and fast cyclotron modes. The microwave radiation can occur at frequencies approximately given by intersections between the slow beam modes and slow electromagnetic (EM) modes in slow-wave structures (SWSs) such as a dielectric loaded waveguide or a periodically modulated waveguide. In this paper, we refer to the instability at the intersection of slow space charge mode as the 'Cherenkov instability' and that at the intersection of the slow cyclotron mode as the 'slow cyclotron instability'. The latter occurs at the anomalous Doppler-shifted cyclotron frequency.

Slow-wave devices such as backward wave oscillators and traveling wave tubes have been well studied as a family of powerful slow-wave microwave devices [1]. In previous studies, only the Cherenkov instability due to the longitudinal electron motion has been considered with an assumption of infinitely strong magnetic field. For a finite strength magnetic field, not only the Cherenkov instability but also the slow cyclotron instability can be driven by a linearly streaming electron beam without initial perpendicular velocity [2–4]. Although the slow cyclotron instability is attributed to the transverse perturbation of beam, it is essentially different from the fast cyclotron instability due to the normal Doppler effect, which needs an initial perpendicular beam velocity. In [3], a new self-consistent field theory considering three-dimensional beam perturbations was developed based on a solid beam and the Cherenkov instability was analyzed. The effect of the transverse beam perturbation appears as a surface charge at a fixed beam surface. However, in the case of an infinitesimally thin annular beam, the boundary surface is modulated (modified and moving) due to the transverse perturbation. The treatment of the beam boundary is essentially different from the solid beam. A pioneering work can be seen in [5]. This is a classical linear theory based on distributed circuit elements. The transverse perturbation was taken into account as a change of beam coupling to the circuit. Since the EM mode should be given beforehand, normal modes of the system and their instabilities cannot be analyzed. Any self-consistent field theory considering the modulated boundary has not been presented, to the authors' knowledge.

In this work, we propose a new linear self-consistent field theory considering the modulated beam surface. A dielectric loaded cylindrical waveguide is used as the SWS. Normal modes of the system and their instabilities are numerically analyzed. The slow cyclotron instability driven by a thin annular electron beam is demonstrated and is studied.

2. Numerical Formula

We consider a dielectric SWS system composed of a metric cylinder with radius $R_{\rm W}$, which is partially loaded by a dielectric ($\varepsilon_{\rm r}$) from $R_{\rm d}$ to $R_{\rm W}$. The cylindrical coordinate system (r, θ, z) is used. A magnetic field $\mathbf{B}_0$ is applied uniformly in the positive z-direction. An infinitesimally thin annular electron beam with a radius $R_{\rm b}(0 < R_{\rm b} < R_{\rm d})$ is streaming axially. The beam has DC velocity $\mathbf{v}_0 = (0, 0, v_0)$, a DC surface density σ_0 and a DC current $\mathbf{I}_0 = (0, 0, 2\pi R_{\rm b}\sigma_0 v_0)$. The temporal and spatial phase factor of all perturbed quantities is assumed to be $\exp[i(kz+m\theta-\omega t)]$, where m is azimuthal mode number.

From the linearized relativistic equation of electron motion under small signal conditions and Maxwell's equations, the first-order velocity $\mathbf{v}_1 = (v_{1\rm r}, v_{1\theta}, v_{1z})$ of an electron can be obtained. The radial velocity $v_{1\rm r}$ causes the radial displacement of the annular. The azimuthal and axial velocities ($v_{1\theta}$ and v_{1z}) cause the perturbed surface current density $\mathbf{K}_1$ and charge σ_1, which are correlated by the continuity equation on the surface.

At the annular surface, discontinuity conditions for the normal components of electric $\mathbf{E}$ and magnetic $\mathbf{B}$ fields are obtained by applying Gauss's laws and are given by

$$\varepsilon_0(\mathbf{E}_{\rm out} - \mathbf{E}_{\rm in}) \cdot \mathbf{n} = \sigma \quad \text{and} \quad (\mathbf{B}_{\rm out} - \mathbf{B}_{\rm in}) \cdot \mathbf{n} = 0. \tag{2.1}$$

Here, $\mathbf{n}$ is the unit normal vector on the surface and suffix 'in(out)' means the inside(outside) of the beam. The surface is modified from $R_{\rm b}$ to $R_{\rm b} + r_1 (R_{\rm b} \geqslant r_1)$ and, hence, $\mathbf{n}$ inclines from $(1, 0, 0)$ to $(1, \delta_\theta, \delta_z)$, to the first order. Here, δ_θ and δ_z are the first-order inclination angles in the r–θ and r–z planes. For the tangential components, Faraday's and Ampere's laws are applied to the small rectangular closed path C (Stokesian loop) around the boundary surface. The boundary conditions for a constant speed $\mathbf{v}$ of boundary surface and an arbitrary surface current density $\mathbf{K}$ were discussed and were given by [6]:

$$\mathbf{n} \times (\mathbf{E}_{\rm out} - \mathbf{E}_{\rm in} + \mathbf{v} \times (\mathbf{B}_{\rm out} - \mathbf{B}_{in})) = 0 \tag{2.2}$$

$$\mathbf{n} \times \left(\mathbf{B}_{\rm out} - \mathbf{B}_{\rm in} - \frac{1}{c^2}\mathbf{v} \times (\mathbf{E}_{\rm out} - \mathbf{E}_{\rm in})\right) = \mu_0 \mathbf{K} \tag{2.3}$$

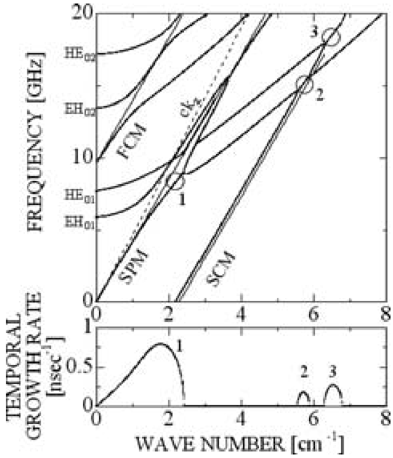

Figure 1. Dispersion curves of axisymmetric mode for the dielectric SWS system with $R_W = 1.445$ cm, $R_d = 0.855$ cm, $R_b = 0.8$ cm, $\varepsilon_r = 4$, $B_0 = 0.8$ T, $V_b = 660$ kV and $I_b = 2.3$ kA.

We consider a linearized case. It can be proved that these relationships are valid for the non-constant boundary velocity $\mathbf{v}_1 = (v_{1r}, v_{1\theta}, v_{1z})$ which depends on time and space. Although six equations are obtained for six components of **E** and **B** from the boundary conditions (2.1), (2.2) and (2.3), there are four independent equations. For example, the equation for B_r can be derived from E_θ and E_z, and the equation for B_θ from E_r and B_z.

The EM fields inside ($r < R_b$) and outside ($R_b < r < R_d$) the beam are correlated by the boundary conditions at the annular surface. The vacuum and dielectric regions are correlated by the boundary conditions at the vacuum–dielectric boundary, which does not move. The dispersion relation is obtained using these boundary conditions and the conditions at the waveguide wall ($r = R_w$), i.e. two electric field components tangential to the wall (E_{1z} and $E_{1\theta}$) should be zero.

3. Numerical results

The dispersion characteristics of the dielectric loaded SWS driven by an annular beam are analyzed numerically using the proposed field theory. The normal modes of the system are hybrids of the transverse magnetic (TM) and transverse electric (TE) modes even for the axisymmetric case. To designate the hybrid modes, two letters of EH and HE in the field of plasma physics are used. Qualitatively, TM is dominant in EH mode and TE is dominant in HE mode. In Fig. 1, axisymmetric hybrid EH_{0n} and HE_{0n} modes are shown. Here, n is any positive integer. The slow space charge (SPM) and slow cyclotron (SCM) modes can excite both EH_{0n} and HE_{0n} modes, resulting in the Cherenkov and slow cyclotron instabilities. Compared with the solid beam case with the same parameters, the growth rate is about 1.2 times that for the Cherenkov instability and is about twice that of the slow cyclotron instability. The fast cyclotron mode (FCM) due to the normal Doppler effect cannot cause any instability, since there is no initial perpendicular velocity of beam.

Non-axisymmetric slow cyclotron instabilities are also excited by the modulated annular beam. Figure 2 shows the growth rate of the slow cyclotron instability for $m = \pm 1$. Since the perturbations are assumed to be $\exp[i(kz + m\theta - \omega t)]$, EM fields

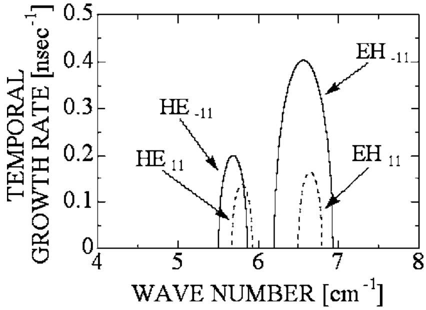

Figure 2. The temporal growth rates of nonaxisymmetric slow cyclotron instability with $R_W = 1.445\,\mathrm{cm}$, $R_\mathrm{d} = 0.855\,\mathrm{cm}$, $R_\mathrm{b} = 0.8\,\mathrm{cm}$, $\varepsilon_\mathrm{r} = 4$, $B_0 = 0.8\,\mathrm{T}$, $V_\mathrm{b} = 660\,\mathrm{kV}$ and $I_\mathrm{b} = 2.3\,\mathrm{kA}$.

with $m = 1 (m = -1)$ rotate clockwise (anticlockwise) in the laboratory frame of reference. However, the electron passes the EM wave at the slow cyclotron interaction. In the beam frame of reference, the rotational direction of the perturbed cyclotron motion is the same as EM wave with $m = -1$, and the growth rate of the slow cyclotron instability becomes larger than that for the $m = 1$ case. The instability is influenced by the rotational direction of perturbed cyclotron montion and EM field polarization, i.e. the sign of m.

4. Summary

A self-consistent field theory considering the modulated (modified and moving) boundary of an annular beam has been proposed based on the dielectric loaded SWS system. By using the proposed field theory, the slow cyclotron instabilities driven by a modulated annular beam have been analyzed. Normal EM modes are hybrid modes having all field components, even in the axisymmetric cases. The slow cyclotron mode is able to couple with both modes, resulting in the slow cyclotron instability. With the same parameters, instabilities for the annular beam are stronger than those for a solid beam. For non-axisymmetric cases, the instability is influenced by the rotational direction of perturbed cyclotron motion and EM field.

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