論文名：
Title of Doctoral Thesis:

Inheritance properties on cone continuity for set-valued maps via scalarization
（スカラー化による集合値写像の錐連続性に対する遺伝的性質）（要約）

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（以下要約を記入する）
A composite function is a function which is the nesting of two or more functions to form a single new function. Such operation frequently preserves several mathematical properties of each nested function. This research focuses on cone semicontinuity of set-valued maps and several types of scalarization functions. Several inherited properties are proved based on binary relations called “set relations” used in set-to-set comparisons in set optimization. They are six types and well-known as important one of basic relations and are targeted in this thesis. Let X be a topological space and Y a real topological vector space. For $A, B \in P(Y)$, and a binary relations on $P(Y)$,

(i) $A \leq_C^{(1)} B \overset{\text{def}}{\iff} \forall a \in A,\ \forall b \in B,\ a \leq_C b \iff A \subset \bigcap_{b\in B}(b - C)$;

(ii) $A \leq_C^{(2L)} B \overset{\text{def}}{\iff} \exists a \in A \text{ s.t. } \forall b \in B,\ a \leq_C b \iff A \cap \left(\bigcap_{b\in B}(b - C)\right) \neq \emptyset$;

(iii) $A \leq_C^{(3L)} B \overset{\text{def}}{\iff} \forall b \in B,\ \exists a \in A \text{ s.t. } a \leq_C b \iff B \subset A + C$;

(iv) $A \leq_C^{(2U)} B \overset{\text{def}}{\iff} \exists b \in B \text{ s.t. } \forall a \in A,\ a \leq_C b \iff \left(\bigcap_{a\in A}(a + C)\right) \cap B \neq \emptyset$;

(v) $A \leq_C^{(3U)} B \overset{\text{def}}{\iff} \forall a \in A,\ \exists b \in B \text{ s.t. } a \leq_C b \iff A \subset B - C$;

(vi) $A \leq_C^{(4)} B \overset{\text{def}}{\iff} \exists a \in A,\ \exists b \in B \text{ s.t. } a \leq_C b \iff A \cap (B - C) \neq \emptyset$.

A composite function is a function which is the nesting of two or more functions to form a single new function. Such operation frequently preserves several mathematical properties of each nested function. For set-valued cases, there are similar properties which are inherited by scalarization function defined on a set (a set function); that is, if a set-valued function is lower (or upper) continuous then a composite function of its set-valued function and some scalarization function has similar semicontinuity. There are many researchers who have studied composite functions and their properties. One of famous compositions is a map of a set-valued map and a scalarizing function motivated by Tammer in 1983. Let Y be a topological vector space. Let C be a convex cone. For each j=1, $2L$, $3L$, $2U$, $3U$, 4, we define the following scalarizing functions

$$I_C^{(j)}(A;V,d) := \inf\left\{t \in \mathbb{R} \,\middle|\, A \leq_C^{(j)} (V+td)\right\},$$

$$S_C^{(j)}(A;V,d) := \sup\left\{t \in \mathbb{R} \,\middle|\, (V+td) \leq_C^{(j)} A\right\},$$

for any A, V in $P(Y)$ and d in Y. These types of scalarization measure how far a given reference set V needs to be moved towards a specific direction d to fulfill each set relation between a target set A and its moved reference set $V+td$. V and d are index parameters for scalarization. The challenging properties are related to a continuity property and its generalizations. In 2019, K.Ike, M.Liu, Y.Ogata, and T.Tanaka have examined some of such compositions and they have proved their preservation of generalized continuity using a convex cone which induces a preorder among sets. The aim of this dissertation is to unravel the mechanism by which composite functions of a set-valued map and a scalarization function transmit semicontinuity of parent set-valued maps through several scalarization for sets. Finally, we find that some of such compositions can preserve our generalized continuity, called, cone continuity, properties of both a set-valued map and a scalarization function naturally.

Theorem 3.2. *Let $F : X \to \mathcal{P}(Y)$, $\varphi : \mathcal{P}(Y) \to \mathbb{R} \cup \{\pm\infty\}$, $x_0 \in X$, $\preccurlyeq$ a binary relation on $\mathcal{P}(Y)$, and $C \subset Y$ a convex cone. If F is $(\preccurlyeq, C)$-continuous at x_0 and φ is $(\preccurlyeq, C)$-lower semicontinuous at $F(x_0)$, then $\varphi \circ F$ is lower semicontinuous at x_0.*

Theorem 3.3. *Let $F : X \to \mathcal{P}(Y)$, $\varphi : \mathcal{P}(Y) \to \mathbb{R} \cup \{\pm\infty\}$, $x_0 \in X$, $\preccurlyeq$ a binary relation on $\mathcal{P}(Y)$, and $C \subset Y$ a convex cone. If F is $(\preccurlyeq, C)$-continuous at x_0 and φ is $(\preccurlyeq, C)$-upper semicontinuous at $F(x_0)$, then $\varphi \circ F$ is upper semicontinuous at x_0.*

However, the left compositions require compactness property of some sets so that the compositions can do so. Moreover, we illustrate many examples which show the reason why some scalarization functions cannot be cone continuous without such assumptions. We summarize inheritance properties on cone continuity for set-valued maps via scalarization in tables in the last chapter.