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Hereditary and Acquired Altruism in an Overlapping Generations Model with Heterogeneous Agents

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Abstract

This study examines the steady-state bequest behavior in hereditary or acquired altruism, using an overlapping generations model with both altruistic and non-altruistic agents. Unlike existing studies that assume agent types are hereditary, we consider a situation in which altruism is acquired and that altruistic parents are altruistic to heterogeneous children. We present the following results. First, when types are hereditary, the optimal bequest decreases according to the ratio of altruistic agents, whereas when types are acquired, the optimal bequest for both agent types increases according to the ratio of altruistic agents. Second, the altruistic agent's social welfare decreases (increases) with the ratio of altruistic agents in the hereditary (acquired) case. Third, the bequest amount is smaller when types are acquired. Finally, non-altruistic agents experience greater social welfare when types are acquired than when hereditary, whereas altruistic agents' social welfare decreases. These results suggest that the difference in the environment in which agent types are determined affects the altruistic type's bequest behavior and the resulting social welfare of both agent types.

Keywords: acquired altruism, bequest, heterogeneous agents, intergenerational altruism, overlapping generations model
JEL classifications: D64, E13, O41

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1 Introduction

Altruism of parents towards children is a natural and essential attitude for human beings, as well as for almost all higher animals. Intergenerational altruism has attracted the interest of many economic scholars during the past half century. Notably, since Barro (1974) and Becker (1980) explored an overlapping generations (OLG) model with altruistic preference, a considerable number of studies addressed various issues on bequest motives attributed to intergenerational altruism in a dynasty model of overlapping generations. From a theoretical aspect, by introducing intergenerational altruism into an OLG model, it bridged the gap between the OLG and Ramsey models, because the dynastic model substantially functions similar to the Ramsey model with infinitely-lived agents.

Recently, the introduction of agent heterogeneity into an OLG model has become an additional strand of research in growth theory. Since the late 1990s, several papers have examined the bequest behavior in an OLG model with agent heterogeneity on altruism, by combining heterogeneous agents with altruism. In addition, Michel and Pestieau (1998, 2005) explored an OLG model with heterogeneous agents on altruism and examined the effects of several fiscal policies. Mankiw (2000) also explored an OLG model with heterogeneous agents called spenders and savers and investigated the effectiveness of fiscal policies. However, the majority of existing studies assume that parents of each agent type have similar children and that their altruism is limited to the same type of children. In other words, they assume that types are hereditary.

The purpose of this study is to consider the different situations in which altruistic parents have both different types of children and are altruistic to both heterogeneous children, and to examine bequest motives. Specifically, we assume that children's types are not fixed but rather acquired. When agent types are hereditary, which is the situation assumed by existing studies, it can be regarded that a type of class society exists and that agents' offspring does not move to different classes. Only altruistic agents build dynasties for the future. By contrast, when agent types are acquired, society is fluid in that social class is not fixed or dynastic. It is not necessarily guaranteed that altruistic parents will have altruistic children. The more appropriate society to consider depends on the actual economic environment that we investigate. Our study aims to examine and compare steady-state bequest behavior when altruism is inherited or acquired, using an OLG model with both altruistic and non-altruistic agents.

We refer to existing research related to our study. Since the two seminal papers by Barro (1974) and Becker (1980) introduced intergenerational altruism into an OLG model, many studies addressed various problems in the dynastic model with altruistic preferences. In addition, Barro and Becker (1989) examined fertility choice, while Aiyagari (1989) specified the condition under which the equilibrium exists. Regarding the extension to agent heterogeneity, Nourry and Venditti (2001) characterized the equilibrium in an OLG with heterogeneous agents. Michel and Pestieau (1998) examined fiscal policies when both altruistic and non-altruistic agents exist. Furthermore, to extend and generalize their study, Michel and Pestieau (2005) considered agent heterogeneity in altruism and ability and explored the effects of fiscal policies. Mankiw (2000) investigated fiscal policies in a society consisting of spenders and savers, which are the same as non-altruistic and altruistic agents. Thibault (2005) also considered the savers-spenders theory developed by Mankiw (2000) to focus on the prevalence of rentiers. However, most existing studies assume that types of heterogeneous agents are fixed. In an attempt to explore a model with other forms of altruism, Falk and Stark (2001) considered a situation in which each individual is concerned with the welfare of multiple offspring generations ahead. Lambrecht *et al.* (2006) explored a model with family altruism in which parents only care about their children's income but not their expenditure. These articles, however, do not consider agent heterogeneity.

The study most closely related to our research is by Dutta and Michel (1998). To our knowledge, there are a few exceptions in this study of situations in which agent types are not inherited. They explored a model with finitely-lived individuals with imperfect altruism, which implies that each individual might or might not be altruistic, and succeeded in deriving the distribution of wealth between families. Furthermore, they comprehensively characterized the steady-state equilibrium in a growth model with agent heterogeneity on altruism. However, because they consider a situation in which each individual lives only for one period but without a genuine OLG model, parent generations do not know whether their children are altruistic or selfish. Therefore, parent generations cannot discriminate against the children's bequest amount depending on their types. In addition, to incorporate a stochastic process for parents' prediction of children types into the model, they need to assume that a type choice within a family follows a first-order Markov process. This assumption complicates the analysis to a certain extent.

In this study, we consider a typical OLG model in which each individual lives during two periods and in which altruistic parents can differentiate between their children's types to examine how the optimal bequest is affected by agent heterogeneity. Unlike existing research, we assume that parents can discriminate against the children's bequest amount depending on their types. Comparing the two different situations in agent heterogeneity, we present the following results. First, when agent types are hereditary, the optimal bequest decreases with the ratio of altruistic agents, whereas when types are acquired, the optimal bequests to both agent types increase with the ratio of altruistic agents. Second, the altruistic agent's social welfare decreases (increases) with the ratio of altruistic agents in the hereditary (acquired) case. Third, the bequest amount is less when types are acquired than when they are hereditary. Finally, the non-altruistic agents experience greater social welfare when types are acquired than when hereditary, whereas the altruistic agents' social welfare decreases. These results suggest that the difference in the environment in which agent types are determined affects the altruistic type's bequest behavior and the resulting social welfare for both agent types.

The remainder of this paper proceeds as follows. Section 2 presents an OLG model with heterogeneous agents on altruism when agent types are hereditary or acquired. Section 3 summarizes the existing results from the steady-state equilibrium when types are hereditary. Section 4 derives the results from the steady-state equilibrium when types are acquired. Section 5 compares the characteristics of the optimal bequest and social welfare in both cases, while Section 6 concludes with final remarks.

2 The model

We consider a simple two-period OLG model. By denoting the population size of generation t by N_t, we define $n \equiv (N_{t+1}/N_t) - 1 \geq 0$ as the net population growth rate, assuming it is constant over time. In the total population, there are two types of agents: altruistic agents and non-altruistic (egoistic) agents, that are indexed by type $i = A$ and E, respectively. We denote the population ratio of type A and type E by $p \in (0,1)$ and $1-p$, respectively. We assume that p is constant over time. Thus, pN_t and $(1-p)N_t$ are the populations of type A and type E in generation t, respectively.

2.1 Individuals

Each individual lives during two periods irrespective of the type, and two generations — young and old — populate each period. Individuals consume private goods in both the young and old periods. The younger individuals supply one unit of their labor inelastically and earn wages. In addition, the younger individuals receive the bequest from their parents, only if the parents are altruistic. Wages and the bequest amount are either spent or saved. Senior individuals retire and spend using their savings and the bequest, if any. We refer to the young individuals in period t generation t. Only the altruistic types are motivated to leave bequests to their children.

We denote consumption levels of type i's generation t in the young and the old periods by c_t^i and d_{t+1}^i, respectively. s_t^i denotes the savings of type i's generation t in the young period and x_t^i denotes the bequest received from their parents in period t. The budget constraints in the respective young and old periods are as follows:

$$c_t^i + s_t^i = w_t + x_t^i, \tag{2.1}$$

$$d_{t+1}^i + (1+n)x_{t+1}^i = R_{t+1}s_t^i, \tag{2.2}$$

where w_t and R_t denote the wage rate and the gross interest rate in period t, respectively. By integrating (2.1) and (2.2), the international budget constraint of type t's generation t is obtained as follows:

$$c_t^i + \frac{d_{t+1}^i}{R_{t+1}} = w_t + x_t^i - \frac{1+n}{R_{t+1}}x_{t+1}^i. \tag{2.3}$$

The utility functions of individuals differ depending on whether the type is altruistic or not. Both types have a standard additively separable life-cycle utility, which is assumed to be log-linear for brevity. The life-cycle utility function of type i's generation t is as follows:

$$U_t^i \equiv U(c_t^i, d_{t+1}^i) = \ln(c_t^i) + \beta \ln(d_{t+1}^i), \tag{2.4}$$

where $\beta \in (0,1)$ is the time preference rate and identical irrespective of agent types. The non-altruistic type, which is denoted by type E, maximizes their life-cycle utility function. Thus, type E considers the following maximization problem:

$$\max_{\{c_t^E, d_{t+1}^E\}} U(c_t^E, d_{t+1}^E) = \ln(c_t^E) + \beta \ln(d_{t+1}^E). \tag{2.5}$$

Because type E has no bequest motive, it is clear that $x_{t+1}^E = 0$.

Contrastingly, altruistic type A, that express altruism toward children, is concerned about the children's utility. In this study, we consider the two different cases depending on whether types are hereditary or acquired. First, in Section 3, we examine bequest behavior when types are hereditary. Then, in Section 4, we examine bequest behavior when types are acquired. The hereditary case is the benchmark to compare results to the acquired case because previous studies explore the results in the hereditary case, in which the altruistic nature is inherited from the parent generation to the children generation. Thus, type A agents consistently have type A children and likewise, type E agents also have type E children. There is no mutation across types. In contrast, in the acquired case, altruistic parents have both types of children following an exogenously given constant proportion and express altruism towards both types of children. For brevity, we assume that type E without intergenerational altruism has no children.[1] [2]

In the hereditary case, because type A is only concerned about their altruistic children, type A's indirect utility function is given as follows:

$$V_t^A = V(x_t^A) \equiv \max_{\{c_t^A, d_{t+1}^A, x_{t+1}^A\}} \left\{ U(c_t^A, d_{t+1}^A) + \gamma V(x_{t+1}^A) \right\}, \tag{2.6}$$

where $\gamma \in (0,1)$ denotes the degree of intergenerational altruism. We assume that $\gamma < \beta$. (2.2) is arranged as follows:

$$V_t^A = \max_{\{c_t^A, d_{t+1}^A, x_{t+1}^A\}} \sum_{t=0}^{\infty} \gamma^t U(c_t^A, d_{t+1}^A). \tag{2.7}$$

In contrast, type A cares about both types of children in the acquired case irrespective of whether they are altruistic or not. Type A's indirect utility function is given as follows:

$$V_t^A = \max_{\{c_t^A, d_{t+1}^A, x_{t+1}^{Aa}, x_{t+1}^{Ae}\}} \left\{ U(c_t^A, d_{t+1}^A) + \gamma \big(p V_{t+1}^A + (1-p) V_{t+1}^E \big) \right\}, \tag{2.8}$$

where x_{t+1}^{Aa} and x_{t+1}^{Ae} denote the bequest amounts from parents to altruistic and non-altruistic children,

[1] Although we can consider a general situation in which type E has children, the analysis becomes very complicated because it is required to change the population growth rates of both types to keep the population ratio constant in the steady state. Moreover, the quantitative results do not change even if type E agents have children, because they do not indicate any altruism towards their children.

[2] Even when types are hereditary, a philanthropic situation might occur in that altruist agents express altruism towards outsiders with no altruism. In another extreme cases, even when types are acquired, it could assume the situation in which altruistic parents favour altruistic children only. However, because both situations are unrealistic and artificial to a certain extent, we label the former the hereditary case and the latter the acquired case.

respectively.

2.2 Firms

Firms produce output by using labor and capital under perfect competition. The aggregate production function is constant returns to scale and independent of time. The production function follows a Cobb-Douglas form, that is, $Y_t = AK_t^{\alpha}L_t^{1-\alpha}$, where Y_t, K_t, and L_t refer to total supply of goods, total labor force, and total capital in period t, respectively. The total labor is equal to the population of generation t, that is, $L_t = N_t$. The per capita production function is provided as follows:

$$y_t = Ak_t^{\alpha}, \tag{2.9}$$

where $y_t \equiv Y_t/N_t$ and $k_t \equiv K_t/N_t$ represent per capita supply and capital, respectively. Firms maximize profit in per capita terms, $\pi_t = y_t - r_t k_t - w_t$. Capital depreciates completely at the end of each period. The first-order conditions for profit maximization are given as follows:

$$R_t = \alpha A k_t^{\alpha-1}, \tag{2.10}$$

$$w_t = (1-\alpha)Ak_t^{\alpha}. \tag{2.11}$$

Throughout the study, we assume dynamic efficiency, that is, $R_t \geq 1+n$, $\forall t$.

2.3 Capital market equilibrium

In the economy, the capital market must be cleared. The capital market equilibrium in period t satisfies the following condition:

$$(1+n)k_{t+1} = ps_t^A + (1-p)s_t^E. \tag{2.12}$$

3 Hereditary case

In this section, we examine the bequest behavior when types are hereditary. This case is the benchmark to compare results to the case in which types are acquired, according to previous study results.

First, we consider the utility maximization problem of a non-altruistic agent. Because type E noticeably possesses no bequest motive, both given and giving bequests are zero, that is, $x_t^E = x_{t+1}^E = 0$. By solving type E's utility maximization problem (2.5) subject to the intertemporal budget constraint (2.3), we obtain the optimal levels of consumption and savings as follows:

$$c_t^E = \frac{w_t}{1+\beta}, \tag{3.1}$$

$$d_{t+1}^E = \frac{\beta R_{t+1} w_t}{1+\beta}, \tag{3.2}$$

$$s_t^E = \frac{\beta w_t}{1+\beta}. \tag{3.3}$$

Second, we consider the utility maximization problem of an altruistic agent. Type A has a bequest motive. However, as shown below, if the degree of intergenerational altruism γ is not large enough, type A does not leave the bequest to their children. We assume that γ is sufficiently large so that $x_{t+1}^A > 0$. The condition that $x_{t+1}^A > 0$ holds is presented in Subsection 3.1. Type A maximizes the utility presented in (2.6) or (2.7) subject to the intertemporal budget constraint (2.3). We solve this maximization problem in the following two steps.[3]

In the first step, considering the given and the giving bequest amounts, x_t^A and x_{t+1}^A, type A chooses the optimal consumption levels. The optimal levels of consumption and savings are obtained as follows:

$$c_t^A = \frac{1}{1+\beta}(w_t + x_t^A - \frac{1+n}{R_{t+1}} x_{t+1}^A), \tag{3.4}$$

$$d_{t+1}^A = \beta R_{t+1} c_t^A, \tag{3.5}$$

$$s_t^A = \frac{\beta}{1+\beta}(w_t + x_t^A) - \frac{1}{1+\beta}\frac{1+n}{R_{t+1}} x_{t+1}^A. \tag{3.6}$$

Note that $\mathrm{d}c_t^A/\mathrm{d}x_t^A = 1/(1+\beta) > 0$, $\mathrm{d}d_{t+1}^A/\mathrm{d}x_t^A = \beta R_{t+1}/(1+\beta) > 0$, $\mathrm{d}c_t^A/\mathrm{d}x_{t+1}^A = -(1+n)/(1+\beta)R_{t+1} < 0$, and $\mathrm{d}d_{t+1}^A/\mathrm{d}x_{t+1}^A = -\beta(1+n)/(1+\beta) < 0$ for subsequent calculations. The transversality condition is required as follows:

$$\lim_{t\to+\infty} \gamma^{t+1} U_d(c_{t-1}^A, d_t^A) x_t^A = \lim_{t\to+\infty} \gamma^{t+1} \beta \frac{x_t^A}{d_t^A} = 0. \tag{3.7}$$

By substituting the optimal consumption levels (c_t^A, d_{t+1}^A), which are shown in (3.4) and (3.5), into the utility function, type A's utility is expressed as a function of only the giving bequest amount x_{t+1}^A.

In the second step, type A determines the optimal level of the giving bequest to maximize its indirect utility as follows: $\max_{\{x_{t+1}^A\}} V_t^A = U(c_t^A, d_{t+1}^A) + \gamma V(x_{t+1}^A)$. The first-order condition is given as follows:

$$\frac{1}{c_t^A}\frac{\mathrm{d}c_t^A}{\mathrm{d}x_{t+1}^A} + \beta \frac{1}{d_{t+1}^A}\frac{\mathrm{d}d_{t+1}^A}{\mathrm{d}x_{t+1}^A} + \gamma V'(x_{t+1}^A) \leq 0 \Leftrightarrow \gamma V'(x_{t+1}^A) \leq \frac{1+n}{R_{t+1} c_t^A}. \tag{3.8}$$

In the case of corner solution, that is, when $x_{t+1}^A = 0$ holds, (3.8) is satisfied with strict inequality. Because

[3] This concise procedure is implemented as per Das *et al.* (2015).

our analysis focuses on the interior solution of the optimal bequest, we assume that (3.8) is satisfied with equality throughout the study. Now we consider the value function of type A's generation t as follows:

$$V(x_t^A) = \max\{U(c_t^A, d_{t+1}^A) + \gamma V(x_{t+1}^A)\}. \tag{3.9}$$

According to the envelope theorem of this Bellman equation, the differentiation of the value function satisfies

$$V'(x_t^A) = \frac{\mathrm{d}U(c_t^A, d_{t+1}^A)}{\mathrm{d}x_t^A} = \frac{1}{c_t^A}\frac{\mathrm{d}c_t^A}{\mathrm{d}x_t^A} + \frac{1}{R_{t+1}c_t^A}\frac{\mathrm{d}d_{t+1}^A}{\mathrm{d}x_t^A} = \frac{1}{c_t^A}. \tag{3.10}$$

Thus, the first-order condition (3.8) satisfies the following equation:

$$c_{t+1}^A = \frac{\gamma R_{t+1}}{1+n} c_t^A. \tag{3.11}$$

However, it is challenging to derive the optimal levels of consumption and bequest concisely for altruistic agents on the transition path.

3.1 The steady-state analysis

Now, we consider the steady-state equilibrium. The time subscript t is dropped as such that for example, $R_t \equiv R$ and $w_t \equiv w$.

First, type E's steady-state consumption and savings are immediately derived from (3.1)–(3.3).

$$c^E = \frac{w}{1+\beta}, \tag{3.12}$$

$$d^E = \frac{\beta R w}{1+\beta}, \tag{3.13}$$

$$s^E = \frac{\beta w}{1+\beta}. \tag{3.14}$$

Second, we derive type A's steady-state consumption and bequest. In (3.11), when the bequest level is positive, $x^A > 0$, the modified golden rule is satisfied in the steady state as follows:

$$\gamma R = 1+n. \tag{3.15}$$

By substituting the firm's profit maximization condition, (2.10) with (3.15), we obtain the steady-state per capita capital as follows:

$$k = \left(\frac{\gamma \alpha A}{1+n}\right)^{\frac{1}{1-\alpha}}. \tag{3.16}$$

As clearly shown in (3.16), the steady-state capital level k is independent of the population ratio of types

p, while k increases with the degree of intergenerational altruism γ.

As per Weil (1987), for the optimal bequest to be positive, the per capita capital k that satisfies (3.16) must be larger than the capital when there is only a non-altruistic agent in the economy. Because the market clearing condition is $(1+n)\widehat{k} = s^E = \beta w/(1+\beta)$ when all agents are non-altruistic, and substituting this with the firm's profit maximization condition (2.11), we obtain the steady-state per capita capital when all agents are non-altruistic as follows:

$$\widehat{k} \equiv \Big(\frac{\beta(1-\alpha)A}{(1+\beta)(1+n)}\Big)^{\frac{1}{1-\alpha}}. \tag{3.17}$$

Thus, we obtain the sufficient condition for a positive bequest as follows:

$$k > \widehat{k} \Leftrightarrow \gamma > \frac{\beta(1-\alpha)}{\alpha(1+\beta)}. \tag{3.18}$$

To guarantee the positive optimal bequest, we assume that (3.18) is satisfied throughout the study.

Type A's steady-state consumption and savings are derived from (3.4)–(3.6).

$$c^A = \frac{w+(1-\gamma)x^A}{1+\beta}, \tag{3.19}$$

$$d^A = \beta R c^A, \tag{3.20}$$

$$s^A = \frac{\beta w+(\beta-\gamma)x^A}{1+\beta}. \tag{3.21}$$

The market clearing condition in the steady state is $(1+n)k = ps^A + (1-p)s^E$. w and also s^E do not depend on p because k is independent of p. Thus, as p grows, s^A decreases, that is, $\mathrm{d}s^A/\mathrm{d}p = -s^A/p < 0$. In other words, as the population ratio of the altruistic type decreases, type A's savings increase. The present value of lifetime consumption of the altruistic agent is given as follows:

$$c^A + \frac{d^A}{R} = w+(1-\gamma)x^A. \tag{3.22}$$

Because $1-\gamma = \big(R-(1+n)\big)/R$ under the modified golden rule, type A's present value of lifetime consumption is equal to the wage plus the return of the bequest.

By substituting s^A and s^E in the market clearing condition and by solving x^A, we obtain the steady-state bequest as follows:

$$x^A = \frac{(1+n)(1+\beta)k-\beta w}{p(\beta-\gamma)}. \tag{3.23}$$

As the population ratio of the altruistic agents increases, the optimal bequest amount decreases. Furthermore, whether the bequest is positive does not depend on the population ratio p. The per capita capital is independent of p in (3.16), whereas the bequest amount decreases with p. Thus, as the ratio of the non-altruistic agents increases, the altruistic agents increase the per capita bequest to countervail the lack of savings by the non-altruistic agents. In addition, because x^A strictly increases with k and k increases with γ, the bequest increases as the degree of intergenerational altruism of agent A increases.

Finally, we derive the social welfare of both types in the steady state. Type A's social welfare is as follows:

$$V^A = \frac{\ln(c^A) + \beta \ln(d^A)}{1-\gamma}, \tag{3.24}$$

where $c^A = \big(w + (1-\gamma)x^A\big)/(1+\beta)$ and $d^A = \beta R c^A$. Because c^A and d^A increase with x^A and x^A decreases with p, the increase in the population ratio of altruistic agent p decreases type A's social welfare. In other words, the increase in non-altruistic agents paradoxically increases type A's social welfare.

Contrastingly, by substituting $c^E = w/(1+\beta)$ and $d^E = \beta R w/(1+\beta)$ with type E's utility function and arranging the equation, type E's social welfare is derived as follows:

$$\begin{aligned} U^E &= \ln(c^E) + \beta \ln(d^E) \\ &= (1+\beta)\ln(w) + \beta \ln(R) + \beta \ln(\beta) - (1+\beta)\ln(1+\beta) \\ &= \big[(1+\beta)\alpha - \beta(1-\alpha)\big]\ln(k) + X, \end{aligned} \tag{3.25}$$

where $X \equiv \ln(\frac{1-\alpha}{1+\beta})^{1+\beta} + \ln(\alpha\beta)^{\beta} + (1+2\beta)\ln(A)$. Note that $(1+\beta)\alpha - \beta(1-\alpha) > 0$ because we assume that (3.18) is guaranteed.

Because k does not depend on p, type E's social welfare is independent of p. However, from (3.25), type E's social welfare increases with k. Thus, the increase in the degree of intergenerational altruism of type A increases type E's social welfare.

We summarize the obtained results in the following proposition. These results have already been revealed by Michel and Pestieau (1998, 2005).

Proposition 1. *Consider the steady state when types are hereditary, assuming that the altruistic agents make the positive bequest, that is, assuming that* $\gamma > \beta(1-\alpha)/\alpha(1+\beta)$ *holds.*
(i) *The steady state follows the modified golden rule. That is,* $\gamma R = 1+n$.
(ii) *The per capita capital* k *is independent of the population ratio of the altruistic types* p *and increases with the degree of intergenerational altruism* γ.
(iii) *The bequest* x^A *decreases with* p *and increases with* γ.
(iv) *The altruistic agent's social welfare decreases with* p. *The non-altruistic agent's social welfare does not depend on* p *but increases with* γ.

Proposition 1(iv) suggests that the increase in non-altruistic agents leads to Pareto improvement.

4 Acquired case

In contrast to the previous section, we examine the bequest behavior when types are acquired in this section. The altruistic agents express altruism towards their children irrespective of whether their children are altruistic or not. Similar to the analysis in the previous section, we assume that γ is sufficiently large so that type A leaves a positive bequest to their children. We denote a bequest from type A's parents to type A's children in period t by x_t^{Aa} and a bequest from type A's parents to type E's children in period t by x_t^{Ae}, respectively.

First, we consider the utility maximization problem of a non-altruistic agent. Because type E evidently has no bequest motive, they do not leave any bequest to their children, that is, $x_{t+1}^E = 0$. However, they receive the bequest from the altruistic parents. Thus, type E's utility maximization problem is similar to that in Subsection 3.1, except for that the bequest x_t^{Ae} is added. By solving (2.5) subject to the intertemporal budget constraint, we obtain the optimal levels of consumption and savings as follows:

$$c_t^E = \frac{w_t + x_t^{Ae}}{1+\beta}, \tag{4.1}$$

$$d_{t+1}^E = \frac{\beta R_{t+1}(w_t + x_t^{Ae})}{1+\beta}, \tag{4.2}$$

$$s_t^E = \frac{\beta(w_t + x_t^{Ae})}{1+\beta}. \tag{4.3}$$

It is immediately found that $\mathrm{d}c_t^E/\mathrm{d}x_t^{Ae} = 1/(1+\beta) > 0$ and $\mathrm{d}d_{t+1}^E/\mathrm{d}x_t^{Ae} = \beta R_{t+1}/(1+\beta) > 0$.

Second, we consider the utility maximization problem of an altruistic agent. Assuming that γ is sufficiently large, type A leaves both their children positive bequests, x_{t+1}^{Aa} and x_{t+1}^{A}. Type A's intertemporal budget constraint is given as follows:

$$c_t^A + \frac{d_{t+1}^A}{R_{t+1}} = w_t + x_t^{Aa} - \frac{1+n}{R_{t+1}}\left(px_{t+1}^{Aa} + (1-p)x_{t+1}^{Ae}\right). \tag{4.4}$$

Type A maximizes the utility (2.8) subject to (4.3). We solve this maximization problem by implementing the following two steps.

In the first step, considering the given and the giving bequest amounts, x_t^{Aa}, x_{t+1}^{Aa}, and x_{t+1}^{Ae}, type A chooses the optimal consumption levels. The optimal levels of consumption and savings are obtained as follows:

$$c_t^A = \frac{1}{1+\beta}\left(w_t + x_t^{Aa} - \frac{1+n}{R_{t+1}}\left(px_{t+1}^{Aa} + (1-p)x_{t+1}^{Ae}\right)\right), \tag{4.5}$$

$$d_{t+1}^A = \beta R_{t+1} c_t^A, \tag{4.6}$$

$$s_t^A = \frac{\beta}{1+\beta}(w_t + x_t^{Aa}) - \frac{1}{1+\beta}\frac{1+n}{R_{t+1}}\left(px_{t+1}^{Aa} + (1-p)x_{t+1}^{Ae}\right). \tag{4.7}$$

For subsequent calculations, we show the first-order partial derivatives as follows: $\mathrm{d}c_t^A/\mathrm{d}x_t^{Aa} = 1/(1+\beta) > 0$, $\mathrm{d}c_t^A/\mathrm{d}x_{t+1}^{Aa} = -p(1+n)/(1+\beta)R_{t+1} < 0$, and $\mathrm{d}c_t^A/\mathrm{d}x_{t+1}^{Ae} = -(1-p)(1+n)/(1+\beta)R_{t+1} < 0$. Similar to the previous section, the transversality condition (3.19) is required by defining $x_t^A \equiv x_t^{Aa} + x_t^{Ae}$ in this section. By substituting the optimal consumption levels (c_t^A, d_{t+1}^A), which are shown in (4.5) and (4.6), with the utility function, type A's utility is expressed by the giving bequest amounts for both types' agents $(x_{t+1}^{Aa}, x_{t+1}^{Ae})$.

In the second step, type A determines the optimal levels of $(x_{t+1}^{Aa}, x_{t+1}^{Ae})$ to maximize the indirect utility function as follows: $\max_{\{x_{t+1}^{Aa}, x_{t+1}^{Ae}\}} V_t^A = U(c_t^A, d_{t+1}^A) + \gamma\big(pV^A(x_{t+1}^{Aa}, x_{t+1}^{Ae}) + (1-p)V^E(x_{t+1}^{Ae})\big)$. Under the assumption of (3.18), we focus only on the interior solution, that is, $x_{t+1}^{Aa} > 0$ and $x_{t+1}^{Ae} > 0$. The first-order conditions with regards to x_{t+1}^{Aa} and x_{t+1}^{Ae} are given as follows:

$$\frac{1}{c_t^A}\frac{\mathrm{d}c_t^A}{\mathrm{d}x_{t+1}^{Aa}} + \frac{\beta}{d_{t+1}^A}\frac{\mathrm{d}d_{t+1}^A}{\mathrm{d}x_{t+1}^{Aa}} + \gamma p\frac{\partial V^A}{\partial x_{t+1}^{Aa}} = 0 \Leftrightarrow \gamma\frac{\partial V^A}{\partial x_{t+1}^{Aa}} = \frac{(1+n)}{R_{t+1}c_t^A}, \tag{4.8}$$

$$\frac{1}{c_t^A}\frac{\mathrm{d}c_t^A}{\mathrm{d}x_{t+1}^{Ae}} + \frac{\beta}{d_{t+1}^A}\frac{\mathrm{d}d_{t+1}^A}{\mathrm{d}x_{t+1}^{Ae}} + \gamma\Big(p\frac{\partial V^A}{\partial x_{t+1}^{Ae}} + (1-p)\frac{\mathrm{d}V^E}{\mathrm{d}x_{t+1}^{Ae}}\Big) = 0 \Leftrightarrow \gamma\Big(p\frac{\partial V^A}{\partial x_{t+1}^{Ae}} + \frac{1-p}{c_{t+1}^E}\Big) = \frac{(1-p)(1+n)}{R_{t+1}c_t^A}. \tag{4.9}$$

Thereafter, we consider the value function of type A's generation t as follows:

$$V^A(x_t^{Aa}, x_t^{Ae}) = \max\big\{U(c_t^A, d_{t+1}^A) + \gamma\big(pV^A(x_{t+1}^{Aa}, x_{t+1}^{Ae}) + (1-p)V^E(x_{t+1}^{Ae})\big)\big\}. \tag{4.10}$$

According to the envelope theorem of this Bellman equation, the partial differentiations of the value function satisfy the following equations.

$$\frac{\partial V^A}{\partial x_t^{Aa}} = \frac{\partial U(c_t^A, d_{t+1}^A)}{\partial x_t^{Aa}} = \frac{1}{c_t^A}\frac{\mathrm{d}c_t^A}{\mathrm{d}x_t^{Aa}} + \frac{1}{R_{t+1}c_t^A}\frac{\mathrm{d}d_{t+1}^A}{\mathrm{d}x_t^{Aa}} = \frac{1}{c_t^A}, \tag{4.11}$$

$$\frac{\partial V^A}{\partial x_t^{Ae}} = \frac{\partial U(c_t^A, d_{t+1}^A)}{\partial x_t^{Ae}} = \frac{1}{c_t^A}\frac{\mathrm{d}c_t^A}{\mathrm{d}x_t^{Ae}} + \frac{1}{R_{t+1}c_t^A}\frac{\mathrm{d}d_{t+1}^A}{\mathrm{d}x_t^{Ae}} = 0. \tag{4.12}$$

Thus, the first-order conditions (4.8) and (4.9) satisfy the following equation.

$$c_{t+1}^A = c_{t+1}^E = \frac{\gamma R_{t+1}}{1+n}c_t^A. \tag{4.13}$$

Since $c_{t+1}^A = c_{t+1}^E$, the parent generation leaves the bequests to both their children so that altruistic and non-altruistic agents consume at the same level.

However, because it is challenging to derive the optimal levels of consumption and bequest on the transition path, we consider the steady state in the following subsection.

4.1 The steady-state analysis

First, type E's steady-state consumption and savings are immediately derived from (4.1)–(4.3).

$$c^E = \frac{w + x^{Ae}}{1+\beta}, \tag{4.14}$$

$$d^E = \frac{\beta R(w + x^{Ae})}{1+\beta}, \tag{4.15}$$

$$s^E = \frac{\beta(w + x^{Ae})}{1+\beta}. \tag{4.16}$$

Second, we derive type A's consumption and bequest levels in the steady state. Similar to the hereditary case presented in the section, it is evident from (4.13) that the modified golden rule, $\gamma R = 1+n$, holds when bequest levels are positive. Furthermore, similar to the hereditary case, because the steady-state per capita capital satisfies (3.16), k is independent of p and increases with γ. Type A's steady-state consumption and savings are derived from (4.5)–(4.7).

$$c^A = \frac{w + (1-\gamma p)x^{Aa} - \gamma(1-p)x^{Ae}}{1+\beta}, \tag{4.17}$$

$$d^A = \beta R c^A, \tag{4.18}$$

$$s^A = \frac{\beta w + (\beta - \gamma p)x^{Aa} - \gamma(1-p)x^{Ae}}{1+\beta}. \tag{4.19}$$

Because $c^A = c^E$ holds from (4.13), x^{Aa} and x^{Ae} satisfy the following equation:

$$c^A = c^E \Leftrightarrow (1-\gamma p)x^{Aa} = \big(1+\gamma(1-p)\big)x^{Ae}. \tag{4.20}$$

By substituting s^A and s^E with the marketing clearing condition in the steady state, the following equation is derived:

$$(1+n)k = ps^A + (1-p)s^E \Leftrightarrow (1+n)(1+\beta)k = \beta w + (\beta - \gamma p)\big(px^{Aa} + (1-p)x^{Ae}\big). \tag{4.21}$$

By solving x^{Aa} and x^{Ae} in the simultaneous equations (4.20) and (4.21), we obtain the steady-state bequest levels as follows:

$$x^{Aa} = \frac{\big((1+n)(1+\beta)k - \beta w\big)\big(1+\gamma(1-p)\big)}{\beta - \gamma p}, \tag{4.22}$$

$$x^{Ae} = \frac{\big((1+n)(1+\beta)k - \beta w\big)(1-\gamma p)}{\beta - \gamma p}. \tag{4.23}$$

Because $x^{Aa} > x^{Ae}$ holds, the altruistic parents leave a larger bequest to the altruistic children than to the non-altruistic children. Although k and w are independent of p, both x^{Aa} and x^{Ae} depend on p.

The first-order partial derivatives of x^{Aa} and x^{Ae} with respect to p are derived as follows:

$$\frac{\partial x^{Aa}}{\partial p} = \frac{\gamma(1-\beta+\gamma)\big((1+n)(1+\beta)k-\beta w\big)}{(\beta-\gamma p)^2} > 0, \tag{4.24}$$

$$\frac{\partial x^{Ae}}{\partial p} = \frac{\gamma(1-\beta)\big((1+n)(1+\beta)k-\beta w\big)}{(\beta-\gamma p)^2} > 0. \tag{4.25}$$

Thus, as the population ratio of the altruistic agents increases, both the optimal bequest levels increase. This characteristic varies from the hereditary case, in which the increase in p decreases the bequest.

Moreover, it is also shown that both x^{Aa} and x^{Ae} increase with γ. The first-order partial derivatives of (4.22) and (4.23) with respect to γ are derived as follows:

$$\frac{\partial x^{Aa}}{\partial \gamma} = Y'\frac{1+\gamma(1-p)}{\beta-\gamma p} + Y\frac{p+\beta(1-p)}{(\beta-\gamma p)^2} > 0, \tag{4.26}$$

$$\frac{\partial x^{Ae}}{\partial \gamma} = Y'\frac{1-\gamma p}{\beta-\gamma p} + Y\frac{p(1-\beta)}{(\beta-\gamma p)^2} > 0, \tag{4.27}$$

where $Y \equiv (1+n)(1+\beta)k-\beta w > 0$ and $Y' \equiv \partial Y/\partial\gamma > 0$.[4] In (4.26) and (4.27), as the degree of intergenerational altruism γ increases, the bequest levels for both agents increase.

Finally, we derive the social welfare of both types in the steady state. Because $c^A = c^E$ and $d^A = d^E$, both types' life-cycle utilities are equal, that is, $U^A = U^E$. Type E's social welfare is derived as follows:

$$U^E = \ln(c^E) + \beta\ln(d^E), \tag{4.28}$$

where $c^E = (w+x^{Ae})/(1+\beta)$ and $d^E = \beta R(w+x^{Ae})/(1+\beta)$. As mentioned previously, because k is independent of p, w and R are also independent of p, whereas x^{Ae} increases with p. Therefore, the increase in p increases type E's social welfare. Contrastingly, k is a strictly increasing function of γ. Because w increases with k but R decreases with k, the increase in γ increases the bequest x^{Ae} and wage rate w, whereas the interest rate R decreases. However, when we express type E's indirect utility as the value function of x^{Ae} as such that $V^E = V^E(x^{Ae})$, $\mathrm{d}V^E/\mathrm{d}x^{Ae} = 1/c^E > 0$ is satisfied according to the envelope theorem. Thus, as γ increases, type E's social welfare increases.

Next, we derive type A's social welfare. Because $U^A = U^E$, the following equation is satisfied.

$$V^A = \frac{U^A}{1-\gamma p} + \gamma(1-p)U^E = \frac{(1+\gamma(1-p)(1-\gamma p))U^A}{1-\gamma p}. \tag{4.29}$$

The same logic as non-altruistic agents applies for altruistic agents. The increases in p and γ improves type A's social welfare. In other words, as the population ratio of altruistic agents and the degree of

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[4] $Y' = (1+n)(1+\beta)(\partial k/\partial\gamma) - \beta(\partial w/\partial\gamma) = [(1+n)(1+\beta) - \beta\alpha(1-\alpha)Ak^{\alpha-1}](\partial k/\partial\gamma)$. The term in the brackets of this equation is essentially positive under the assumption of (3.18). Partial differentiation of (3.16) with respect to γ, yields $\frac{\partial k}{\partial\gamma} = \frac{\alpha A}{(1-\alpha)(1+n)}\left(\frac{\gamma\alpha A}{1+n}\right)^{\frac{\alpha}{1-\alpha}} > 0$. Therefore, $Y' > 0$ holds.

intergenerational altruism increase, altruistic agents enhance their social welfare.

We summarize the above results in the following proposition.

Proposition 2. *Consider the steady state when types are acquired, assuming that altruistic types leave the positive bequest, that is, assuming that $\gamma > \beta(1-\alpha)/\alpha(1+\beta)$ holds.*
(i) *The steady state follows the modified golden rule, that is, $\gamma R = 1+n$.*
(ii) *The per capita capital k is independent of p and increases with γ.*
(iii) *The bequest levels left to both altruistic and non-altruistic agents, x^{Aa} and x^{Ae}, increase with p and γ.*
(iv) *Both agent's social welfare increases with p and γ.*

Propositions 2(i) and 2(ii) match the claims in Propositions 1(i) and 1(ii). Irrespective of whether agent types are hereditary or acquired, the modified golden rule applies and the steady-state per capita capital level is determined by both firms' production technology and the capital market condition, but not by the demand side. The results of Propositions 2(iii) and 2(iv) are in stark contrast to those in Propositions 1(iii) and 1(iv). When types are hereditary, the bequest level from altruistic parents to altruistic children decreases as the population ratio of altruistic agents increases. In contrast, when types are acquired, the bequest level from altruistic parents to both of altruistic and non-altruistic children increases as the population ratio of altruistic agents increases. The difference is attributed to whether the parent generation is able to properly control the savings of the generation of children by leaving the bequest. When agent types are hereditary, altruistic parents cannot control the savings of non-altruistic agents because they are not their children and are not left with any bequests. By variation, when agent types are acquired, altruistic parents are able to control the levels of savings of both altruistic and non-altruistic agents through the bequest. Since the capital level required for the economy is completely determined only by a firm's technology, namely, the supply side, the parent generation is required to adjust the total savings to properly encourage capital accumulation while balancing the capital market equilibrium condition, $(1+n)k = ps_A + (1-p)s_E$. As the parent generation cannot change s_E when agent types are hereditary, the increase in altruistic types requires the reduction of s_A and leads to decrease in bequest x^A. Contrastingly, when agent types are acquired, the parent generation is able to change both s_A and s_E via x^{Aa} and x^{Ae}. Since $s^E > s^A$ must be held in order that $c^A = c^E$ holds in the steady state, the increase in altruistic types reduces total savings. Thus, when types are acquired, as p increases, parents can increase the bequest levels to both agents.

5 Comparison

By using the results from the steady-state bequest behavior in the two previous sections, we compare the bequest levels and social welfare when agent types are hereditary and acquired.

First, we compare the bequest levels. The steady-state bequest level when types are hereditary, x^A, is shown in (3.23). The steady-state bequest levels of the altruistic and the non-altruistic agents when types are acquired, x^{Aa} and x^{Ae}, are shown in (4.22) and (4.23), respectively. Moreover, when types are acquired, the total bequest amount that the parent generation leaves to both agents, is calculated as follows:

$$px^{Aa}+(1-p)x^{Ae}=\frac{((1+n)(1+\beta)k-\beta w)}{\beta-\gamma p}. \tag{5.1}$$

We summarize the results of the bequest levels comparison in both cases in the following proposition.

Proposition 3.
(i) *Both x^{Aa} and x^{Ae} are essentially smaller than x^A. That is, $x^{Aa}<x^A$ and $x^{Ae}<x^A$.*
(ii) *The total bequest amount when types are acquired is less compared to when they are hereditary. That is, $px^{Aa}+(1-p)x^{Ae}<x^A$.*

***Proof*.** (i) $x^{Aa}\lesseqgtr x^A$ if and only if $p\lesseqgtr \beta/\gamma(\beta-\gamma)$. However, since $\beta/\gamma(\beta-\gamma)>1$ if and only if $\beta(1-\gamma)+\gamma^2>0$, $p<\beta/\gamma(\beta-\gamma)$ always holds and, as a result, $x^{Aa}<x^A$ is satisfied. $x^{Ae}<x^A$ is immediately proved because $x^{Ae}<x^A$ if and only if $\beta(1-p)+\gamma p^2(\beta-\gamma)>0$.
(ii) It is immediately obtained from Proposition 3(i), or easily proved by comparing (3.23) with (5.1). □

Proposition 3 claims that parent generations are able to save the amount of bequest they leave for children when agent types are acquired. At first glance, it seems that the required bequest amount in the acquired case is larger compared to the hereditary case, since parents are required to leave bequests to both types of agents. However, the result shows that when altruism is inherited, altruistic parents are required to pay a larger bequest to their altruistic children to achieve the equivalent of capital accumulation as when types are acquired. The reason that Proposition 3 holds, is attributed to the difference in the effect of children's utility on parents' utility when types are hereditary or acquired. When types are hereditary, the degree of influence that altruistic children's utility has on their altruistic parents' utility follows the degree of intergenerational altruism γ. Contrastingly, when types are acquired, the degree of influence that altruistic children's utility has on their altruistic parents' utility decreases at the ratio of γp. Even if parents obtain the additional increase in utility from non-altruistic children's utility, this portion of utility is not recursive and the effect disappears in only one period. The decrease in the importance of children utility decreases the marginal utility of bequest for both agents and leads to a smaller bequest than in the hereditary case.

Subsequently, we compare social welfare in the steady state using two cases. First, we compare the social welfare of the non-altruistic agents. The non-altruistic agent's utility is $U^E=\ln(c^E)+\beta\ln(d^E)$ in both cases and the consumption levels (c^E,d^E) are shown in (3.12) and (3.13) when types are hereditary and in (4.14) and (4.15) when types are acquired, respectively. Clearly, because the bequest exists only when types are acquired, both consumption levels are higher when types are acquired than when they are hereditary, that is, $c^E_{ac}>c^E_{he}$ and $d^E_{ac}>d^E_{he}$.[5] Thus, social welfare is greater when types are acquired

[5] The subscripts *he* and *ac* indicate the hereditary and acquired cases, respectively.

compared to when they are hereditary, that is, $U_{ac}^{E} > U_{he}^{E}$.

Second, we compare the life-cycle utility of altruistic agents. The altruistic agent's life-cycle utility is $U^A = \ln(c^A) + \beta \ln(d^A)$, which depends on the consumption levels (c^A, d^A). By substituting the bequest amount x^A, which is shown in (3.23), with the young-period consumption level c^A, which is shown in (3.19), we obtain the steady-state young-period consumption level when types are hereditary. Likewise, by substituting the bequest amounts x^{Aa} and x^{Ae}, which are shown in (4.22) and (4.23), with the young-period consumption level c^A, which are shown in (4.17), we obtain the steady-state young-period consumption level when types are acquired.

$$c_{he}^{A} = \frac{\big(p(\beta-\gamma) - \beta(1-\gamma)\big)w + (1+n)(1+\beta)(1-\gamma)k}{p(1+\beta)(\beta-\gamma)}, \tag{5.2}$$

$$c_{ac}^{A} = \frac{-\gamma p(1-\beta)w + (1+n)(1+\beta)(1-\gamma p)k}{(1+\beta)(\beta-\gamma p)}. \tag{5.3}$$

Under the assumption of a positive bequest, it is shown that $c_{he}^{A} > c_{ac}^{A}$ if and only if $\Gamma(p) \equiv \gamma(\beta-\gamma)p^2 - (\beta-\gamma^2)p + \beta(1-\gamma) > 0$. Since $\Gamma(0) > 0$, $\Gamma(1) = 0$, and $\Gamma(p)$, which is a quadratic function of p, takes the minimum when $p > 1$, $\Gamma(p) > 0$ holds for all $p \in (0,1)$. Thus, $c_{he}^{A} > c_{ac}^{A}$ and $d_{he}^{A} > d_{ac}^{A}$ hold. As a result, the life-cycle utility is larger when types are hereditary than when they are acquired, that is, $U_{he}^{A} > U_{ac}^{A}$.

Finally, we compare the type A's social welfare including altruism toward their children. Type A's social welfare is shown in (3.24) when types are hereditary, and shown in (4.29) when types are acquired. When comparing both social welfare levels, we can derive that $V_{he}^{A} > V_{ac}^{A}$. We summarize the results of social welfare comparison in both cases in the following proposition.

Proposition 4.
(i) The social welfare of non-altruistic agents is greater when types are acquired than when they are hereditary. That is, $U_{ac}^{E} > U_{he}^{E}$.
(ii) Both the life-cycle utility and social welfare, including altruism for the altruistic agents, are greater when types are hereditary than when they are acquired. That is, $U_{he}^{A} > U_{ac}^{A}$ *and* $V_{he}^{A} > V_{ac}^{A}$.

***Proof*.** The proofs of $U_{ac}^{E} > U_{he}^{E}$ and $U_{he}^{A} > U_{ac}^{A}$ are already in the abovementioned. The proof of $V_{he}^{A} > V_{ac}^{A}$ is as follows: when we suppose that $U_{he}^{A} = U_{ac}^{A}$ were satisfied, it is shown that $V_{he}^{A} > V_{ac}^{A}$ if and only if $1 + p(1-\gamma) > 0$ by substituting (3.24) and (4.29). Since $U_{he}^{A} > U_{ac}^{A}$ in reality, $V_{he}^{A} > V_{ac}^{A}$ continues to hold. □

Proposition 4 implies that altruistic agents prefer a situation in which types are hereditary and when their dynasty can continue indefinitely by altruistic agents, whereas non-altruistic agents prefer a situation in which types are acquired because they are able to obtain the bequest from their parents. When agent types are acquired, the altruistic parents are required to leave the bequest to both types of children. As shown in Proposition 3(ii), the bequest amount that the altruistic parents pay is smaller when types are acquired than when they are hereditary. However, when types are acquired, the marginal utility of the bequest decreases because the ratio to which the bequest that a parent generation leaves to their children is inherited by future offspring generations declines. Therefore, the altruistic agents' social welfare decreases when types are acquired. In contrast, non-altruistic agents prefer the situation in which types are acquired because they can obtain the positive bequest. Thus, the most preferable environment for agent heterogeneity depends on the agent types of altruism. For altruistic agents, type inheritance guarantees greater social welfare from the permanent family dynasty. Alternatively, non-altruistic agents prefer types that are not genetically determined.

6 Concluding remarks

This study examined the steady-state bequest behavior when altruism is hereditary or acquired, using an OLG model with both altruistic and non-altruistic agents. Unlike the assumption of existing studies that agent types are hereditary, we considered a situation in which altruism is acquired and in which parents are altruistic to both heterogeneous children. We presented the following main results. First, in Propositions 1(iii) and 2(iii), when agent types are hereditary, the optimal bequest decreases according to the ratio of altruistic agents, whereas when types are acquired, the optimal bequests for both agent types increase according to the ratio of altruistic agents. Second, in Propositions 1(iv) and 2(iv), the altruistic agent's social welfare decreases (increases) with the ratio of altruistic agents in the hereditary (acquired) case. Third, as per Proposition 3, the bequest amount is smaller when types are acquired than when they are hereditary. Finally, in Proposition 4, the non-altruistic agent's social welfare is greater when types are acquired than when hereditary, whereas the altruistic agent's social welfare is smaller. The most preferable environment for agent heterogeneity depends on the agent types of altruism. In addition, these results suggest that the difference in the environment in which agent types are determined affects the bequest behavior and social welfare of both agent types.

Finally, we conclude by suggesting scope for future research based on this study. First, we limited the argument to only two extreme cases, in which types are hereditary or acquired. A natural way to expand on our study is to investigate a more general situation in which some agent types are hereditary and others are acquired. In addition, whether children acquire altruism is greatly affected by emotional learning from parents. As a future challenge, we should consider the endogeneity of altruism and incorporate the investment of emotional learning into the model to investigate the bequest behavior in an OLG model. Second, one of the interesting issues among existing studies that examined altruism in an OLG model, is the effectiveness of fiscal policy. In this study, we only considered the steady-state bequest behavior in which agent types are acquired and compared these results with the hereditary case. However, as the circumstances of agent heterogeneity on altruism changes, the effectiveness of fiscal policy could also develop. Another extension of our study could include an investigation regarding whether or how the effectiveness of fiscal policy, such as public debt, pay-as-you-go pension plans, and inheritance taxation, transform when agent types are acquired.

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