PAPER

Real-Time Restoration of Nonstationary Biomedical Signals under Additive Noises

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SUMMARY In the present paper we shall examine the real-time restoration of biomedical signals under additive noises. Biomedical signals measured by instruments such as catheter manometers, ambulatory electrocardiographs and thermo-dilution sensors are susceptible to distortion and noise. Therefore, such signals must be restored to their original states. In the present study, nonstationary biomedical signals are observed and described using a mathematical model, and several restoration filters that are composed of a series of applications of this model are proposed. These filters restored band-limited approximations of the original signals in real-time. In addition, redundancy is introduced into these restoration filters in order to suppress additive noise. Finally, an optimum filter that accounts for restoration error and additive noise is proposed.
key words: *restoration, band-limitation, real-time, additive noise, redundancy*

1. Introduction

Biomedical signals such as blood pressure signals, electrocardiograms, and blood temperatures are considered to be nonstationary processes because the properties of generators and of the heart tend to vary over time. During clinical analysis or diagnosis, nonstationary biomedical signals, commonly referred to as the patient's vital signs, are measured. However, the instruments involved in measuring nonstationary biomedical signals, such as catheter manometers, ambulatory electrocardiographs and thermo-dilution sensors, are susceptible to distortion and noise. Therefore, restoring these signals to their original states is of primary importance. Traditional restoration methods expand signals on a fixed functions basis, making accurate restoration of nonstationary signals difficult. Moreover, most traditional methods, such as Fourier analysis, require signals to be approximated from an infinite past to an infinite future time, whereas the present time remains undescribed.

Recently, various time-frequency analyses and multi-resolution analyses, such as Wavelet analysis and Wigner-Ville distribution [1]–[3], have been proposed. In these methods, nonstationary signals are represented on a local basis, however, such methods are not suitable for real-time implementation because the signal in the present is represented by reference to future information. If the transfer function of an observation system is known, the original signals can be theoretically restored using an infinite Neumann series [4]. Infinite series, however, cannot be reconstructed technically.

Several restoration filters base on the observation model have been proposed [5]–[7]. These filters restored band-limited approximations to their original signals in real-time. We applied these filters to catheter manometers [9], [10], ambulatory electrocardiographs [11] and thermo-dilution sensors [12]. However, the noise included in the signals observed using this equipment has not yet been critically discussed. We therefore attempted to suppress the additive noise by introducing redundancy into the proposed filter [13]. Since increasing the order of the band-limitation filter sharpens the attenuating characteristics, the order of the restoration filter was set higher than the order of the observation system. Unnecessary high-frequency noises of restored signals were found to be reduced effectively. In the present study, the theory underlying real-time restoration and its possible application are discussed.

2. Definition of Problem

We treat signals in $L^2(-\infty,\infty)$, i.e., signals for which the energy in all time-domains has a finite value. The inner product of two signals f and g in L^2 is defined as

$$(f,g)=\int_{-\infty}^{\infty} f(t)\overline{g(t)}dt \qquad (1)$$

and the norm of f is defined as

$$\|f\|=\sqrt{(f,f)}. \qquad (2)$$

Let f be an original signal in L^2, n be the additive noise, and A be the bounded-linear operator from L^2 to L^2 that represents the observation. The observed signal is defined by

$$a_0=Af+n. \qquad (3)$$

We call A the observation operator. The observed signal a_0 in Eq. (3) is distorted by the observation operator A and the noise n. Thus, we need to restore the original signal f from the observed signal a_0.

Let B be the restoration filter. Then a restored signal is defined as

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$$f_0 = Ba_0. \tag{4}$$

We consider the following limitations, in the restoration process described in Eq. (4):

(i) f_0 is approximately equivalent to f.

(ii) The restoration must be accomplished in real-time.

(iii) The additive noise, n, must be minimized.

To satisfy (ii), we restore the signal using a signal observed in the past.

Assume t to be the present time. If f is a nonstationary signal, that is, the characteristics of f varies over time, then a_0 must be nonstationary. The observation signal a_0 is obtained over the passage of time t in real-time. In real-time processing, the signal at the present time t must be represented using signals from the past to the present, because future signals are unavailable. Let A in Eq. (3) be

$$\Gamma(s)f(t) = \int_0^\infty \frac{1}{s}e^{-\tau/s}f(t-\tau)d\tau \quad (s>0). \tag{5}$$

$\Gamma(s)$ is the bounded linear operator from $L^2(-\infty,\infty)$ to $L^2(-\infty,\infty)$ (see Appendix in [5]). Equation (5) requires only the signal observed from the past to the present, and does not require knowledge of the future signal. Moreover, since the past signal decays exponentially, the present signal is emphasized in Eq. (5). Thus, $\Gamma(s)$ expresses the present-time signal in real-time. $\Gamma(s)$ corresponds to a first-order low-pass filter with a time constant of s. The transfer functions of many biomedical instruments are expressed using $\Gamma(s)$ [9]–[12]. In the following sections, the proposed restoration method is applied to the signals obtained by the observation system represented by the product of $\Gamma(s)$ in Eq. (5).

3. Restoration Method

3.1 Band-Limited Restoration

Rather than restoring the complete original signal, the band-limited signal is restored. Theoretically, band-limitation means that the frequency characteristics of the eliminated domain are all zero. In the present study, band-limitation refers to the gradual decay in frequency that is realized by the linear filter.

Let P be the band-limited operator. We call Pf the band-limited signal for f. In this section, we consider the problem of restoring the band-limited signal Pf from the observed signal a_0 (Fig. 1). That is, we attempt to obtain the restoration filter, B, that satisfies the equation

$$Ba_0 = Pf. \tag{6}$$

The observation systems can be represented not only by the first-order system in Eq. (5), but also by higher-order systems. We define the Lth-order observation operator as

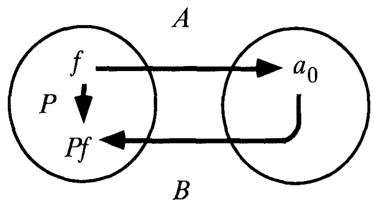


Fig. 1 Band-limited restoration. f: Original signal; a_0: Observed signal; Pf: Band-limited signal (= restored signal); A: Observation operator; B: Restoration filter; P: Band-limited operator.

$$A = \prod_{l=1}^{L}\Gamma(s_l) \quad (Re\{s_l\}>0, l=1,2,\cdots,L) \tag{7}$$

and the Mth-order band-limited operator as

$$P = \Gamma(s_0)^M \quad (s_0 < Re\{s_l\}, l=1,2,\cdots,L) \tag{8}$$

where $\{s_l\}$ are any complex values and s_0 is a real value. $Re\{\cdot\}$ is the real part of $\cdot$. A detailed description of the method for determining s_0 is presented in Sect. 4.1.

$$\|f - Pf\| < \|f - Af\| \tag{9}$$

is realized by $s_0 < Re\{s_l\} \quad l=1,2,\cdots,L$. Thus, we can always obtain a signal that is nearer to the original than the observed signal. To satisfy limitation (i) mentioned previously in Sect. 2, s_0 must be set to a small value.

We set

$$\Lambda(s_0) = I - \Gamma(s_0). \tag{10}$$

$\Lambda(s_0)$ is a first-order high-pass filter with a time constant of s_0. We propose a restoration filter, which is composed of the high-pass filter, $\Lambda(s_0)$. The fundamental observation signals, $\{a_m\}$, are recursively defined using $\Lambda(s_0)$ as

$$a_m = \Lambda(s_0)a_{m-1} \quad (m=1,2,\cdots,M). \tag{11}$$

By using $\{a_m\}$, the band-limited signal may be expressed as

$$Pf = \sum_{m=0}^{M} b_m a_m \tag{12}$$

where $\{b_m\}$ are the linear combination coefficients and M is the order. We derive M and $\{b_m\}$, which satisfy Eq. (12). From Eqs. (6), (11) and (12), the restoration filter for Eq. (7) is derived using

$$B = \sum_{m=0}^{M} b_m \Lambda(s_0)^m. \tag{13}$$

Equation (13) includes $\Lambda(s_0)$, which requires no future-time signal. Thus, the restoration filter, B, satisfies limitation (ii) from Sect. 2 above.

3.2 Restoration Filter ($M=L$) [5]

To satisfy Eq. (12), the order of the restoration filter M must be greater than or equal to the order of the observation system, L. We have previously reported on a restoration filter foe which $M=L$ [5], i.e., for the Lth-order observation operator,

$$A=\prod_{l=1}^{L}\Gamma(s_l), \tag{14}$$

and the Mth-order band-limited operator is defined by

$$P=\Gamma(s_0)^L. \tag{15}$$

Before deriving the restoration filter, we introduce the following lemmas.

Lemma 1: For the operator $\Gamma(s)$, we have derived the following equations:

$$\Gamma(s_0)\Gamma(s_l)^{-1}=I+\frac{s_l-s_0}{s_0}\Lambda(s_0) \quad (l=1,\cdots,L). \tag{16}$$

A proof of Eq. (16) appears below in the Appendix.

The following theorem can also be derived:

Theorem 1: The restoration filter for the Lth-order observation operator in Eq. (14) is represented by

$$B=\sum_{m=0}^{L}b_m\Lambda(s_0)^m \tag{17}$$

where

$$b_m=\sum_{i=1}^{\binom{L}{m}}\prod_{j=1}^{m}\frac{s_{d(i,j)}-s_0}{s_0} \quad (m=0,1,\cdots,L). \tag{18}$$

$\binom{L}{m}=0$ if $m<0$ or $L<m$. Otherwise, $\binom{L}{m}$ is a bilinear coefficient. The quantity $d(i,j)$ is the natural number that satisfies $1\le d(i,1)<\cdots<d(i,m)\le L$ and $\{d(i',1),\cdots,d(i',m)\}\ne\{d(i'',1),\cdots,d(i'',m)\}$ while $i'\ne i''$.

Proof: We first prove that a_L can be represented by Pf, a_0, a_1, $\cdots$, a_{L-1}. That is, when we set $M=L$ in Eq. (12), by Eqs. (11), (7), and (8), we obtain $\{b_m\}$ that satisfies

$$\sum_{m=0}^{L}b_m\Lambda(s_0)^m\prod_{l=1}^{L}\Gamma(s_l)=\Gamma(s_0)^L. \tag{19}$$

By Eq. (19),

$$\sum_{m=0}^{L}b_m\Lambda(s_0)^m=\prod_{l=1}^{L}\Gamma(s_0)\Gamma(s_l)^{-1}. \tag{20}$$

By Eq. (16),

$$=\prod_{l=1}^{L}\left\{I+\frac{s_l-s_0}{s_0}\Lambda(s_0)\right\}$$

$$=I+\left(\frac{s_1-s_0}{s_0}+\frac{s_2-s_0}{s_0}+\cdots+\frac{s_L-s_0}{s_0}\right)\Lambda(s_0)$$

$$+\left(\frac{s_1-s_0}{s_0}\frac{s_2-s_0}{s_0}+\cdots+\frac{s_{L-1}-s_0}{s_0}\frac{s_L-s_0}{s_0}\right)$$

$$\cdot\Lambda(s_0)^2$$

$$+\cdots+\left(\frac{s_1-s_0}{s_0}\frac{s_2-s_0}{s_0}\cdots\frac{s_L-s_0}{s_0}\right)\Lambda(s_0)^L$$

Thus, Eq. (18) holds. □

We can restore the Lth-order band-limited signal by substituting $\{b_m\}$ from Eq. (18) into Eq. (12). That is, in the case of an Lth-order observation system, a band-limited signal can be restored using the same Lth-order restoration filter. The influence of noise was not considered in this system.

3.3 Redundant Restoration Filter ($M>L$)

In this section, we try to suppress the noise by introducing redundancy into the order of the restoration filter. We propose an $M(>L)$th-order restoration filter that restores the signals observed by the Lth-order system. The Mth-order restoration filter is derived for a first-order observation operator and for an Lth-order observation operator.

3.3.1 Derivation of the Mth-Order Restoration Filter for a First-Order Observation System

For a first-order observation operator,

$$A=\Gamma(s_1), \tag{21}$$

we define the Mth-order band-limited operator follows:

$$P=\Gamma(s_0)^M \quad (M>1). \tag{22}$$

Theorem 2: The Mth-order restoration filter for a first-order observation system can be derived by

$$B=\sum_{m=0}^{M}b_m\Lambda(s_0)^m \tag{23}$$

where

$$b_m=(-1)^m\left\{\binom{M-1}{m}-\binom{M-1}{m-1}\frac{s_1-s_0}{s_0}\right\}$$

$$(m=0,1,\cdots,M). \tag{24}$$

Proof: We can show that

$$\sum_{m=0}^{M}b_m\Lambda(s_0)^m\Gamma(s_1)=\Gamma(s_0)^M. \tag{25}$$

By Eq. (25),

$$\sum_{m=0}^{L} b_m \Lambda(s_0)^m = \Gamma(s_0)^{M-1}\Gamma(s_0)\Gamma(s_1)^{-1}. \tag{26}$$

By Eqs. (16) and (11),

$$= \{I - \Lambda(s_0)\}^{M-1}\left\{I + \frac{s_1 - s_0}{s_0}\Lambda(s_0)\right\}$$

$$= \sum_{m=0}^{M-1}(-1)^m\binom{M-1}{m}\Lambda(s_0)^m\left\{I + \frac{s_1 - s_0}{s_0}\Lambda(s_0)\right\}$$

$$= \sum_{m=0}^{M}(-1)^m\left\{\binom{M-1}{m} - \binom{M-1}{m-1}\frac{s_1 - s_0}{s_0}\right\}\Lambda(s_0)$$

Thus, Eq. (24) is valid. □

3.3.2 Derivation of the Mth-Order Restoration Filter for an Lth-Order Observation System

For an Lth-order observation operator,

$$A = \prod_{l=1}^{L}\Gamma(s_l), \tag{27}$$

we define the Mth-order band-limited operator as follows:

$$P = \Gamma(s_0)^M \quad (M > L) \tag{28}$$

Theorem 3: The restoration filter for an Lth-order observation system can be derived as follows:

$$B = \sum_{m=0}^{M} b_m \Lambda(s_0)^m \tag{29}$$

where

$$b_m = \sum_{l=0}^{m}(-1)^l\binom{M-L}{m-l}\sum_{i=1}^{\binom{L}{l}}\prod_{j=1}^{l}\frac{s_{d(i,j)} - s_0}{s_0}$$

$$(m = 0, 1, \cdots, M) \tag{30}$$

Proof: First we show

$$\sum_{m=0}^{M} b_m \Lambda(s_0)^m \prod_{l=1}^{L}\Gamma(s_l) = \Gamma(s_0)^M. \tag{31}$$

By Eq. (31),

$$\sum_{m=0}^{L} b_m \Lambda(s_0)^m = \Gamma(s_0)^{M-L}\prod_{l=1}^{L}\Gamma(s_0)\Gamma(s_l)^{-1}. \tag{32}$$

By Eqs. (16) and (11),

$$= \{I - \Lambda(s_0)\}^{M-L}\prod_{l=1}^{L}\left\{I + \frac{s_l - s_0}{s_0}\Lambda(s_0)\right\}$$

$$= \sum_{m=0}^{M-L}(-1)^m\binom{M-L}{m}\Lambda(s_0)^m\prod_{l=1}^{L}\left\{I + \frac{s_l - s_0}{s_0}\Lambda(s_0)\right\}$$

$$= \sum_{m=0}^{M}\sum_{l=0}^{m}(-1)^l\binom{M-L}{m-l}\left\{\sum_{i=1}^{\binom{L}{l}}\prod_{j=1}^{l}\frac{s_{d(i,j)} - s_0}{s_0}\right\}$$

$$\cdot\,\Lambda(s_0)^m$$

Thus, Eq. (30) is valid. □

Figure 2 shows a block diagram of the Mth-order restoration filter for an Lth-order observation system.

Most biomedical signals are band-limited in frequency. Considering the frequency band of the original signals, the proposed filters in Eq. (29) can restore signals while suppressing unnecessary high-frequency noise. The restoration filters are a simple composition of successive fundamental first-order high-pass filters, $\Lambda(s_0)$, where the number of filters adjusted is set equal to or higher than the order of the observation system. The higher the order of the restoration filter, i.e., the higher the redundancy, the sharper the characteristics of high-frequency attenuation. Using this property, the high-frequency noises that are produced during restoration can be suppressed effectively.

4. Parameter Estimation under Additive Noise

To realize the restoration filter, B, parameters such as s_0, $\{b_m\}$ and M, are determined in advance. In this section, methods for determining these parameters under additive noise are discussed.

4.1 Time Constant of Band-Limited Operator, s_0

The parameter of the band-limited operator, s_0, affects the precision of the restoration. The band-limited signal, Pf, approaches f by setting s_0 as small as possible. If we set to $s_0 = 0$, that is, if the restoration filter restores the original signal, the parameters of the

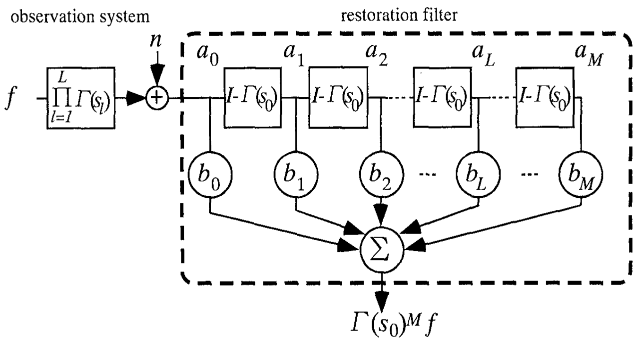


Fig. 2 Mth-order restoration filter for an Lth-order observation system.

restoration filter $\{b_m\}$ attain infinite values, and the restoration filter cannot be constructed. Thus, we have to decide s_0 in an Mth-order restoration filter. Let ω_c be the maximum angular frequency of the original signal and $P(\omega)$ be the frequency characteristic of the operator P in Eq. (28), where ω is the angular frequency. In order to determine s_0, $|P(\omega_c)|$ must satisfy

$$20\log|P(\omega_c)| \geq -3[dB]. \tag{33}$$

The amplitude characteristic of $P(\omega)$ is given by

$$|P(\omega)| = \left(\frac{1}{\sqrt{1+\omega^2{s_0}^2}}\right)^M. \tag{34}$$

By Eqs. (33) and (34),

$$s_0 \leq \frac{1}{\omega_c}\sqrt{2^{1/M}-1}. \tag{35}$$

If the order of the restoration filter, M, and the maximum angular frequency, ω_c, are predetermined, the parameter, s_0, in the restoration filter is determined by Eq. (35).

4.2 Determination of $\{b_m\}$ when $\{s_l\}$ Parameters are Unknown

If the parameters of the observation operator, $\{s_l\}$, are known, the parameters of the restoration filter, $\{b_m\}$, can be obtained using Eq. (30). However, $\{s_l\}$ are unknown in most observation systems. When $\{s_l\}$ are unknown, $\{b_m\}$ are calculated by taking a known step signal to be f, and taking its response as a_0 and Pf.

$\{b_m\}$ are obtained by satisfying

$$Pf = \sum_{m=0}^{M} b_m a_m. \tag{36}$$

In this method, the restoration filter is realized by determining $\{b_m\}$ without $\{s_l\}$.

Digital signals were acquired via A/D conversion and used to estimate $\{b_m\}$ in Eq. (36). Replacing the function space $L^2(-\infty,\infty)$ with Kth-order vector space R^K, the Eq. (36) is represented by

$$Pf(k) = \sum_{m=0}^{M} b_m a_m(k) + e(k) \quad (k = 1, 2, \cdots, K) \tag{37}$$

where k is the sampled discrete time and $e(k)$ is the residual. The time constant s_0 and the order M were obtained previously. The observation operator, $\Gamma(s)$, in Eq. (5) is designed by the bilinear z-transformation. Defining

$$\boldsymbol{Pf} = (Pf(1), \cdots, Pf(K))^T, \tag{38}$$

$$\boldsymbol{a}_m = (a_m(1), \cdots, a_m(K))^T, \tag{39}$$

$$\boldsymbol{A} = (\boldsymbol{a}_0, \cdots, \boldsymbol{a}_M), \tag{40}$$

$$\boldsymbol{b} = (b_0, \cdots, b_M)^T \tag{41}$$

and

$$\boldsymbol{e} = (e(1), \cdots, e(K))^T, \tag{42}$$

then, Eq. (37) is rewritten as

$$\boldsymbol{Pf} = \boldsymbol{Ab} + \boldsymbol{e} \tag{43}$$

The value $\boldsymbol{b}_0$ obtained by

$$\boldsymbol{b}_0 = \boldsymbol{A}^+\boldsymbol{Pf} \tag{44}$$

is one of the solutions that minimizes the evaluation function

$$J[\boldsymbol{b}] = \|\boldsymbol{Pf} - \boldsymbol{Ab}\|^2. \tag{45}$$

We adopt $\boldsymbol{b}_0$ in Eq. (44) as the approximation of $\{b_m\}$ in Eq. (36).

4.3 Order of Restoration Filter, M

To determine the order of the restoration filter, M, we employ an evaluation function that indicates both the precision of restoration and the amplitude of noise. At first, we consider two types of relative errors. One is the error between a band-limited signal and a restored signal,

$$J_1(M) = \frac{\|Pf - Ba_0\|}{\|Pf\|} \tag{46}$$

where J_1 expresses the amplitude of noise. The other is the error between an original signal and a restored signal,

$$J_2(M) = \frac{\|f - Ba_0\|}{\|f\|} \tag{47}$$

where J_2 expresses the distortion of the signal. We confirmed the restorative capabilities of the proposed filter using the step signal. Figure 3 shows the relationship between the order of the restoration filter and

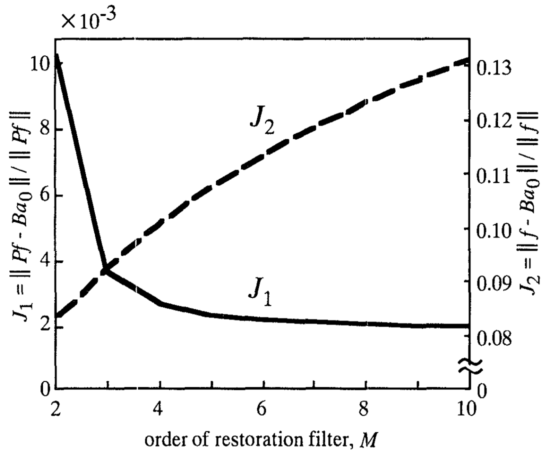


Fig. 3 Relative errors. J_1: Error between Pf and Ba_0. J_2: Error between f and Ba_0.

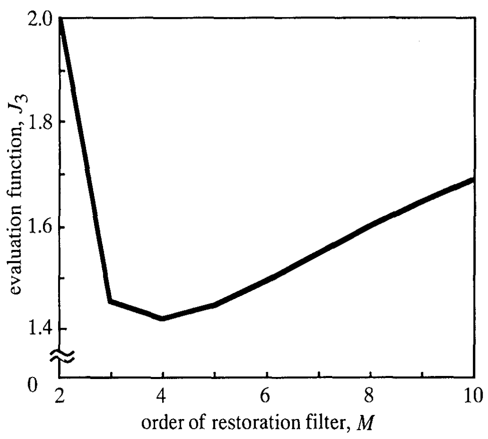


Fig. 4 Evaluation function J_3 for determining the optimum order of the restoration filter.

the evaluation functions, J_1 and J_2. The observed signal was obtained by the second-order system in which s_1 and $s_2 = 0.03 \pm j0.04$. Random noise having variance $\sigma = 10^{-8}$ was added to the observed signal. The time constant s_0 was set using Eq. (35) with equality holding in order to fix the frequency bandwidth, ω_c, while changing the order M. J_1 and J_2 inversely proportional and directly proportional, respectively, to the order of the restoration filter.

Thus, we propose a new evaluation function,

$$J_3(M) = \frac{J_1(M)}{J_1(L)} + \frac{J_2(M)}{J_2(L)}, \tag{48}$$

in order to determine the order of the restoration filter. L is the order of the observation operator. J_3 represents an evaluation of both shape and noise, i.e., the smaller J_3, the better the restoration. The order, M, is optimized for restoration by minimizing J_3. Figure 4 shows the relationship between the order of the restoration filter and the evaluation function, J_3. Here, the optimum order was the fourth-order.

5. Simulation

If the order of the observation system, L, is known, the Mth-order redundant restoration filter in Eq. (29) is also represented using the Lth-order restoration filter in Eq. (17), as follows:

$$B = \Gamma(s_0)^{M-L} \sum_{m=0}^{L} b_m \Lambda(s_0)^m. \tag{49}$$

where $\{b_m\}$ is given by Eq. (18). We consider the restorative ability by comparing filter proposed in Eq. (29) with filter in Eq. (49) using a step signal as the original signal f (Fig. 5 (a)). Figure 5 (b) shows the observed signal a_0 obtained by the second-order observation system with s_1 and $s_2 = 0.03 \pm j0.04$, including additive noises ($\sigma = 10^{-8}$), and Fig. 5 (c) shows

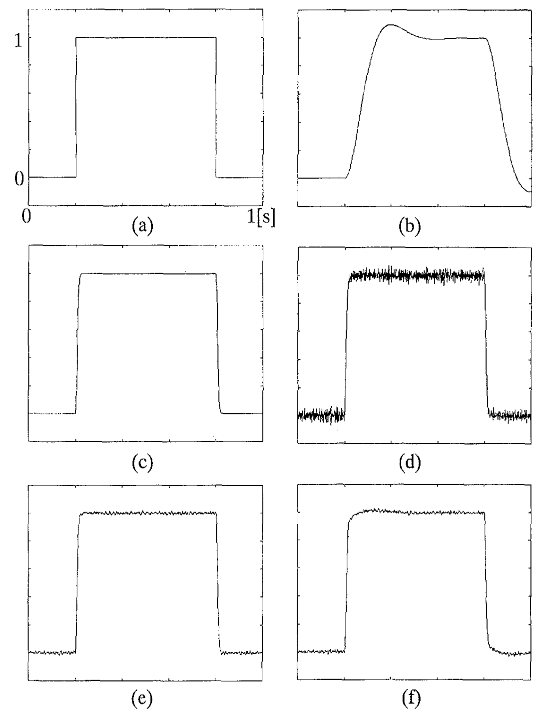


Fig. 5 Restoration of a step signal. (a) Original signal f. (b) Observed signal a_0. (c) Band-limited signal Pf. (d) Restored signal Ba_0 (Eq. (17), $M = 2$). (e) Restored signal Ba_0 (Eq. (29), $M = 4$). (f) Restored signal Ba_0 (Eq. (49), $L = 2$, $M = 4$).

Table 1 Evaluation results of restoration filters.

restoration filter	J_1	J_2	J_3
Eq. (17), $M = 2$	0.1202	0.0295	2.0000
Eq. (29), $M = 4$	0.1439	0.0077	1.4583
Eq. (49), $L = 2$, $M = 4$	0.1478	0.0170	1.8052

the forth-order band-limited signal, Pf. The time constant, s_0, is set using Eq. (35) with equality holding. The signals restored by the second-order restoration filter in Eq. (17) and by the fourth-order restoration filter in Eq. (29) are shown in Figs. 5 (d) and (e), respectively. The signals restored by the filter in Eq. (49) is shown in Fig. 5 (f). The coefficients $\{b_m\}$ in all restoration filters were calculated using the method described in Sect. 4.2. The relative errors, J_1 and J_2, and the values of evaluation functions, J_3, obtained by the three restoration filters are shown in Table 1. The noise of the signal in Fig. 5 (e) is smaller than that in Fig. 5 (d). Figure 5 (e) is in good agreement with Fig. 5 (d), whereas Fig. 5 (f) has slightly damped oscillations due to the estimation error of the coefficients, i.e., the sensitivity for noises is decreased by increasing the number of coefficients. Moreover, the restoration filter in Eq. (49) can not be applied to most observation systems in which the order, L, is unknown. Thus, the redundant restoration filter in Eq. (29) provides more precise signals than other methods.

The proposed theory was applied to the ambulatory electrocardiogram system (Fig. 6). The ECG sig-

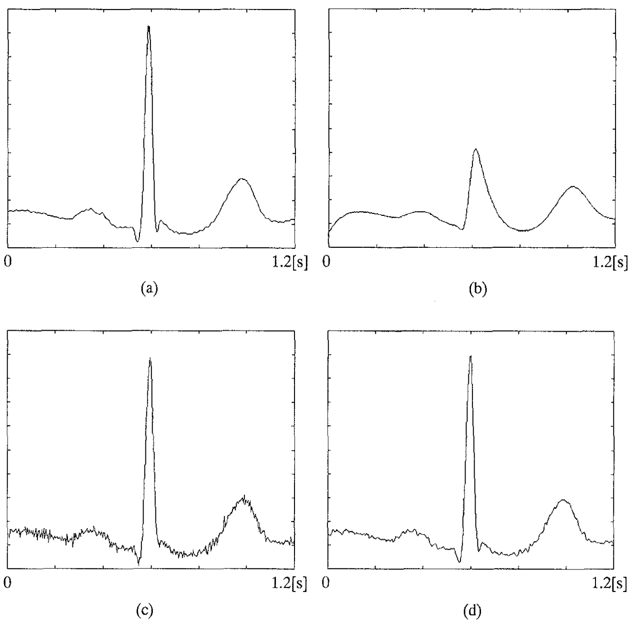


Fig. 6 Restoration of an electrocardiogram measured by an ambulatory system. (a) Original signal f. (b) Observed signal a_0. (c) Restored signal Ba_0 ($M = 2$). (d) Restored signal Ba_0 ($M = 4$).

nals obtained by the ambulatory system tend to be distorted because the frequency responses of these signals do not satisfy the AHA recommendations, which are generally accepted as the standard for conventional ECG systems. We assumed the observation system to be of second-order. When the order of the restoration filter was equal to that of the observation system, the coefficients $\{b_m\}$ were calculated as follows:

$$\begin{cases} b_0 = 1 \\ b_1 = \dfrac{s_1 - s_0}{s_0} + \dfrac{s_2 - s_0}{s_0} \\ b_2 = \dfrac{s_1 - s_0}{s_0} \dfrac{s_2 - s_0}{s_0} \end{cases} \tag{50}$$

The high noise level of the restored signal occurred due to the higher frequency compensation (Fig. 6 (c)). For the fourth-order restoration filters, the coefficients $\{b_m\}$ were calculated as follows:

$$\begin{cases} b_0 = 1 \\ b_1 = \dfrac{s_1 - s_0}{s_0} + \dfrac{s_2 - s_0}{s_0} - 2 \\ b_2 = \dfrac{s_1 - s_0}{s_0} \dfrac{s_2 - s_0}{s_0} \\ \qquad - 2\left(\dfrac{s_1 - s_0}{s_0} + \dfrac{s_2 - s_0}{s_0}\right) + 1 \\ b_3 = -2\dfrac{s_1 - s_0}{s_0} \dfrac{s_2 - s_0}{s_0} + \dfrac{s_1 - s_0}{s_0} + \dfrac{s_2 - s_0}{s_0} \\ b_4 = \dfrac{s_1 - s_0}{s_0} \dfrac{s_2 - s_0}{s_0} \end{cases} \tag{51}$$

The noise of the restored signal was suppressed (Fig. 6 (d)), and the restored signal was approximately equivalent to the original signal.

6. Conclusions

Optimum filters for the restoration of signals under additive noise were proposed. These filters, composed of redundant linear combinations of fundamental filters, restored band-limited approximations of signals to the original signals in real-time. These filters were applicable, not only to first-order measurement systems, but also to systems of higher order. Furthermore, these filters were applicable to both high-pass and low-pass measurement systems. The proposed method was determined to be useful for a broad range of practical situations.

References

[1] M.J.T. Smith and T.P. Barnwell, "A new filter bank theory for time-frequency representation," IEEE Trans., vol.ASSP-35, no.3, pp.314–327, March 1987.
[2] M. Vetterli, "A theory of multirate filter banks," IEEE Trans., vol.ASSP-35, no.3, pp.356–372, March 1987.
[3] I. Daubechies, "The wavelet transform, time-frequency localization and signal analysis," IEEE Trans. Inf. Theory, vol.36, no.5, pp.961–1005, May 1992.
[4] C.W. Groetsch, Generalized Inverses of Linear Operator, Marcel Dekker Inc., 1977.
[5] J. Hori and H. Ogawa, "Real-time restoration of nonstationary signals," IEICE Trans., vol.J79-D-II, no.3, pp.430–438, March 1996.
[6] J. Hori, Y. Saitoh, and T. Kiryu, "Real-time restoration of signals by the natural observation system," Proc. 16th Ann. Int. Conf. IEEE EMBS, vol.16, pp.1326–1327, Oct. 1994.
[7] J. Hori, Y. Saitoh, and T. Kiryu, "Real-time restoration of nonstationary biomedical signals," Proc. 2nd IMIA-IFMBE Int. Workshop Biosignal Interpretation, vol.2, pp.119–120, July 1996.
[8] J. Hori, Y. Saitoh, and T. Kiryu, "Real-time restoration of biomedical signals under additive noise," Proc. 19th Ann. Int. Conf. IEEE EMBS, vol.19, pp.1406–1408, Oct. 1997.
[9] J. Hori, Y. Saitoh, T. Kiryu, H. Makino, K. Tamura, T. Yoshizaki, and T. Iijima, "Correction of pressure waveforms in a catheter- manometer system by natural observation method," JJME, vol.26, no.3, pp.133–138, Sept. 1988.
[10] J. Hori, Y. Saitoh, T. Kiryu, and T. Iijima, "Automatic correction of left-ventricular pressure waveform using the natural observation method," IEICE Trans. Inf. & Syst., vol.E75-D, no.6, pp.909–915, Nov. 1992.
[11] J. Hori, Y. Saitoh, T. Kiryu, T. Asakawa, T. Tamura, and T. Iijima, "Improvement of frequency responses in an ambulatory ECG system by natural observation method," Frontiers Med. Biol. Eng., vol.2, no.2, pp.134–145, April 1990.
[12] J. Hori, Y. Saitoh, and T. Kiryu, "Improvement of the time-domain response of a thermodilution sensor by the natural observation method," IEICE Trans. Fundamentals, vol.E77-A, no.5, pp.784–791, May 1994.
[13] N. Nakamura and H. Ogawa, "Optimum digital image restoration under additive noises," IEICE Trans., vol.J67-D, no.5, pp.563–570, May 1984.

Appendix: Proof of Lemma 1

By Eqs. (16) and (10), the following equations are derived.

$$\Gamma(s_0)\Gamma(s_l) = \frac{s_l}{s_l - s_0}\Gamma(s_l) + \frac{s_0}{s_0 - s_l}\Gamma(s_0) \quad (l = 1, 2, \cdots, L). \qquad \text{(A· 1)}$$

For f in L^2, we prove the following equation:

$$\Gamma(s_0)\Gamma(s_1)f = \left\{ \frac{s_1}{s_1 - s_0}\Gamma(s_1) + \frac{s_0}{s_0 - s_1}\Gamma(s_0) \right\} f. \qquad \text{(A· 2)}$$

By Eq. (5),

$$\Gamma(s_0)\Gamma(s_1)f = \int_0^\infty \frac{1}{s_0} e^{-\frac{\tau}{s_0}} \left\{ \int_0^\infty \frac{1}{s_1} e^{-\frac{\tau_1}{s_1}} f(t-\tau-\tau_1)d\tau_1 \right\} \cdot d\tau. \qquad \text{(A· 3)}$$

When $\tau' = \tau + \tau_1$,

$$= \int_0^\infty \frac{1}{s_0} e^{-\frac{\tau}{s_0}} \left\{ \int_\tau^\infty \frac{1}{s_1} e^{-\frac{\tau'-\tau}{s_1}} f(t-\tau')d\tau' \right\} d\tau$$
$$= \int_0^\infty \frac{1}{s_0} e^{-(\frac{1}{s_0}-\frac{1}{s_1})\tau} \left\{ \int_\tau^\infty \frac{1}{s_1} e^{-\frac{\tau'}{s_1}} f(t-\tau')d\tau' \right\} \cdot d\tau. \qquad \text{(A· 4)}$$

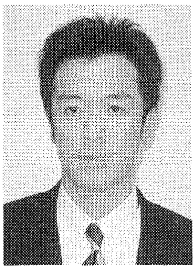
We apply the partial integral to Eq. (A· 4),

$$= \int_0^\infty \frac{1}{s_0} e^{-(\frac{1}{s_0}-\frac{1}{s_1})\tau} d\tau \int_0^\infty \frac{1}{s_1} e^{-\frac{\tau}{s_1}} f(t-\tau)d\tau - \int_0^\infty \frac{1}{s_0} e^{-\frac{\tau}{s_1}} f(t-\tau) \left\{ \int_\tau^\infty \frac{1}{s_1} e^{-(\frac{1}{s_0}-\frac{1}{s_1})\tau'} d\tau' \right\} \cdot d\tau. \qquad \text{(A· 5)}$$

By $0 < s_0 < s_1$,

$$= \left\{ \frac{s_1}{s_1 - s_0}\Gamma(s_1) + \frac{s_0}{s_0 - s_1}\Gamma(s_0) \right\} f. \qquad \text{(A· 6)}$$

Eqs. (A· 2) and (A· 1) exist. Therefore, Eq. (16) is valid. □



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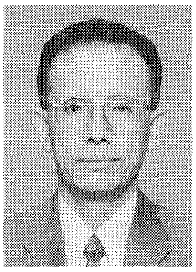

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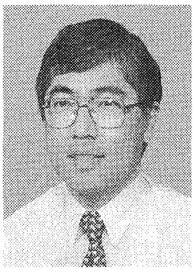

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