PAPER Special Issue on JTC-CSCC '90

Channel Assignment Problem in a Cellular Mobile System and a New Coloring Problem of Networks

Hiroshi TAMURA†, Masakazu SENGOKU††, Shoji SHINODA††† and Takeo ABE††, Members

SUMMARY In a cellular mobile system, assigning a channel for a call in a cell so as to achieve high spectral efficiency is an important problem. In usual channel assignment for a cellular mobile system, a channel can be simultaneously assigned to some cells with a constant separation distance. This usual model of a cellular mobile system has been formulated using a graph and it is known that the channel assignment problem is equivalent to the coloring problem of graph theory. Recently, a new channel assignment scheme has been proposed. This scheme takes the degree of interference into consideration. In the scheme, a channel is simultaneously assigned if the CIR (carrier-to-interference ratio) is more than the desired value. In this paper, we formulate this new model using a network and a new coloring problem of networks. The new coloring problem of networks is a generalization of the usual coloring problem of graphs. One of the merits of this formulation is that the degree of cochannel interference between cells can be represented. In the usual formulation using a graph, the degree of cochannel interference between cells can not be represented. Therefore, spectral efficiency in the formulation using a network is higher than spectral efficiency in the formulation using a graph. In this paper, we show that the new coloring problem is an NP-hard problem. Subsequently, we rewrite the new coloring problem of networks to a coloring problem of graphs on some assumptions and consider the relation between the results on the new coloring and the results on the usual coloring.

1. Introduction

In a cellular mobile system, assigning a channel for a call in a cell so as to achieve high spectral efficiency is an important problem[(1),(2)]. A lot of studies on this assignment problem have been done[(3),(4)]. In usual channel assignment for a cellular mobile system, a channel can be simultaneously assigned to some cells with a constant separation distance. This usual model of a cellular mobile system has been formulated using a graph[(5)] and it is known that the channel assignment problem is equivalent to the coloring problem of graph theory.

There exist the merits of this formulation using a graph as follows. In usual, a service area in a cellular mobile system is divided into regular hexagon cells. A lot of studies on·channel assignment in a cellular mobile system depend on this regular location of cells. However, a cellular mobile system may heve different size cells and non-uniform interferences from cells to cells. If we use a graph, we can formulate all kinds of systems. Moreover, we can apply the results on graph theory to the channel assignment problem.

Recently, a new channel assignment scheme has been proposed[(6)-(8)]. This scheme takes the degree of interference into consideration. In the scheme, a channel is simultaneously assigned if the CIR (carrier-to-interference ratio) is more than the desired value. In this paper, we formulate this new model using a network and a new coloring problem of networks. We show that the new coloring problem is a generalization of the usual coloring problem of graphs. One of the merits of this formulation using a network is that the degree of cochannel interference between cells can be represented. In the formulation using a graph, the degree of cochannel interference between cells can not be represented. Subsequently, we rewrite the new coloring problem of networks to a coloring problem of graphs on some assumptions and consider the relation between the results on the new coloring and the results on the usual coloring. For general terminology in graph theory, we refer the reader to Refs. (9) and (10).

2. Formulation of a Cellular Mobile System Using a Network

When we assign a channel to the new call, we must consider cochannel interference. We assume that mobile unit transmission frequencies are different from cell site transmission frequencies. Therefore, we can neglect cochannel interference between mobile units and between cell sites. There are two cases to be considered for cochannel interference: 1) the signal and cochannel interference received by the cell site from mobile units and 2) the signal and cochannel interference received by the mobile unit from cell sites. We consider the first case hereafter. Since the same discussion applies to the last case, the last case is omitted in this paper.

We construct a directed network N from a cellular

Manuscript received February 15, 1991.
Manuscript revised June 3, 1991.
† The author is with Graduate School of Science and Technology, Niigata University, Niigata-shi, 950-21 Japan.
†† The authors are with the Faculty of Engineering, Niigata University, Niigata-shi, 950-21 Japan.
††† The author is with the Faculty of Science and Engineering, Chuo University Tokyo, 112 Japan.

mobile system. First, we define I_{ij} and C_i as follows.

I_{ij} : the cochannel interference from the mobile unit in the cell z_j received by the cell site in cell z_i in a worst case.

C_i: the signal from the mobile unit in the cell z_i received by the cell site in z_i in a worst case.

A vertex v_i of N corresponds to a cell z_i in the cellular mobile system. The edge weight $w(v_i, v_j)$ is I_{ji}/C_j that represents the degree of cochannel interference from z_i to z_j. It the cochannel interference is negligible, then (v_i, v_j) dose not belong to the edge set of N. The network N is called the *interference network* of the cellular mobile system. A C/I ratio (carrier-to-interference ratio) that is not less than a desired value α is necessary to good voice quality in any cell z_i (for example, in the case of FM, 30 kHz channel bandwidth and analog system, it is reported that $\alpha = 18$ dB[(11),(12)]. We assume that the total cochannel interference received by a cell site is determined by the sum of each cochannel interference. Then,

$$\frac{C_i}{\sum_j I_{ij}} \geqq \alpha \quad (z_j \text{ uses the same channel as } z_i \text{ uses}),$$

namely,

$$\frac{1}{\alpha} \geqq \sum_j \frac{I_{ij}}{C_i}. \qquad (1)$$

We assign a color $f(v_i)$ to each vertex v_i of N. $f(v_i)$ corresponds to a channel assigned to the cell z_i. Let N_c be the induced subnetwork whose vertices are assigned a color c. If for each N_c and each vertex v of N_c, the sum of edge weights of edges which are incident to v is not greater than a desired value, the assignment $f(\cdot)$ is a channel assignment guaranteeing good voice quality from (1) ($e = (u, v)$ is an edge of N, then we say e is *incident from* u and *incident to* v).

We express the channel assignment problem in different words using the terminology of graph theory.

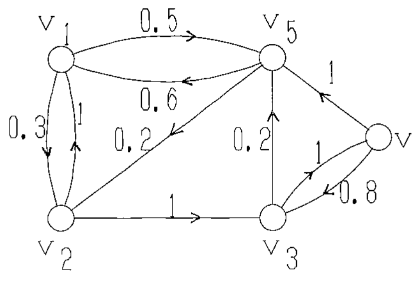
[Definitions]

Let N be a directed network. The *edge weight sum* $d_N(v)$ of a vertex v of N is the sum of edge weights of edges which are incident to v. The maximum value of edge weight sums of vertices of N is denoted by $d_{\max}(N)$.

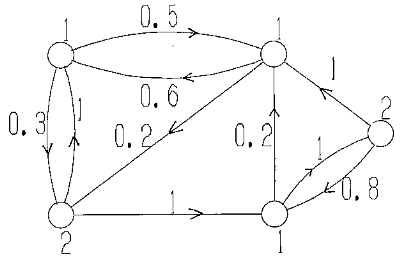
For each vertex v of N, we assign a color. The assignment is called the *coloring with defect h* if the following conditions are satisfied. Let N_c be the induced subnetwork whose vertices are assigned the color c. For each N_c, $d_{\max}(N_c)$ is not greater than h.

N is *k-colorable with defect h*, or simply *(k, h)-colorable* if there exists a coloring with defect h using $k'(\leqq k)$ colors. The *chromatic number with defect h* is the minimum k for which N is (k, h)-colorable, denoted by $x_h(N)$.

The similar coloring in the case of undirected graphs is defined in Ref. (13).



(a) A network N.



(b) In case that $h=1$, a coloring with defect h.

Fig. 1 A network N and a coloring of N.

In the formulation of a cellular mobile system using a network, for any channel assignment for a cellular mobile system, the number of channels assigned in the cellular mobile system is not less than the chromatic number with defect h of the interference network N.

For example, let us consider the network N shown in Fig. 1(a). In Fig. 1(a), the number on each edge represents the edge weight of the edge. From the above definition.

$$d_N(v_1) = 0.6 + 1 = 1.6 \quad \text{and}$$

$$d_{\max}(N) = d_N(v_3) = 1.8.$$

An example of a coloring with defect h of N is shown in Fig. 1(b). In Fig. 1(b), the number on each vertex represents the color assigned to the vertex. Since the network N is (2,1)-colorable and not (1, 1)-colorable, we obtain $x_1(N) = 2$.

3. Usual Formulation of a Cellular Mobile System Using a Graph

The usual model of a cellular mobile system has been formulated using a graph. We construct an undirected graph G for a cellular mobile system. G is a graph in which a vertex v_i corresponds to a cell z_i in the cellular mobile system and an edge $e = (v_i, v_j)$ represents that the cochannel interference between cells z_i and z_j can not be neglected. The graph G is called the *interference graph* of the cellular mobile system. In the interference graph, assigning channels to cells is equal to assigning colors to vertices. So, in the formulation of a cellular mobile system using a graph, for any channel assignment for a cellular mobile system, the number of channels assigned in the cellular mobile system is not less than the chromatic number $x(G)$ of the interference graph G.

4. Comparison between the Usual Coloring and the New Coloring

We compare the new coloring with the usual coloring using an example. A service area is shown in Fig. 2.

First, We construct the interference network from the service area. We assume the 40 dB/dec mobile radio propagation path loss rule[(11),(12)] (in propagation path loss, radio signal strength drops proportionate to distance). Then, $C_i/I_{ji}=R_i^{-4}/D_{ji}^{-4}$, where R_i is the distance between the cell site in z_i and the farthest point in z_i from the cell site, and D_{ji} is the distance between the cell site in z_i and the nearest point in z_j from the cell site (see Fig. 3). The interference network is shown in Fig. 4. In Fig. 4, in the case that C_i/I_{ji} is very small, We omit the edge (v_j, v_i) of the interference network. If we assume $\alpha=18$ dB, then the left side in Eq. (1) is $1/\alpha \fallingdotseq 0.016$. We assign a channel to each cell. For example, since $w(v_2, v_3)=0.037>1/\alpha$, namely, inequality Eq. (1) dose not hold, a channel can not be assigned simultaneously to z_2 and z_3. Since $w(v_1, v_2)=0.008$ and $w(v_2, v_1)=0.003$, a channel can be assigned simultaneously to cells z_1 and z_2. In the network, we can assign Channel 1 to cell z_2 and Channel 2 to cells z_1, z_3 and z_4.

On the other hand, in the formulation using a graph, the interference graph is shown in Fig. 5. In this case, we need three channels. Therefore, spectral efficiency in the formulation using a network is higher than spectral efficiency in the formulation using a graph.

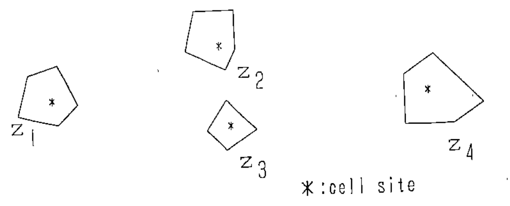


Fig. 2 A service area.

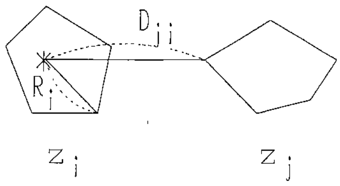


Fig. 3 Explanation for R_i and D_{ji}.

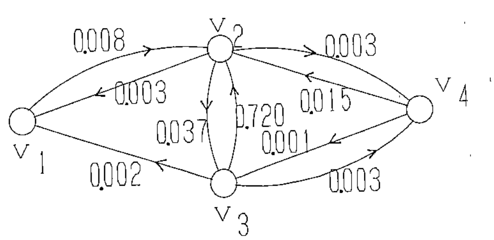


Fig. 4 The interference network of the service area.

5. Results on a New Coloring of Networks

We have considered assigning a channel to each cell. This means that only one user is in a cell. If some users are in a cell, we have to assign some channels to each cell. In a system where a cell have more than one user, we reconstruct the interference network as follows. For example, we consider assigning t channels to a cell z_i. We replace the vertex v_i with the directed network where the vertex set is $\{v_{i1}, \cdots, v_{it}\}$, any ordered vertex pair (u, v) in $\{v_{i1}, \cdots, v_{it}\}$ belongs to the edge set and all edge weights are $s(>1/\alpha)$. An example of the replacement in the case that $t=3$ is shown in Fig. 6. Therefore, we consider only assigning a channel to each vertex of the interference network hereafter.

The new coloring problem is a generalization of the usual coloring problem of graphs. The reason is following. Let G be an undirected graph and N be a network whose underlying graph is G and each edge weight is 1. In this case, G is k-colorable for the usual coloring if and only if N is $(k, 0)$-colorable. So, we obtain the following theorem.

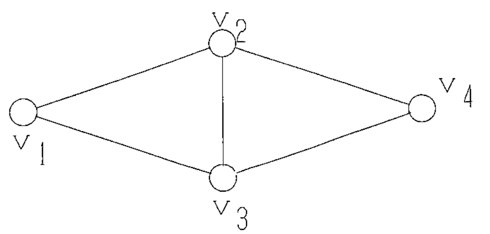


Fig. 5 The interference graph of the service area.

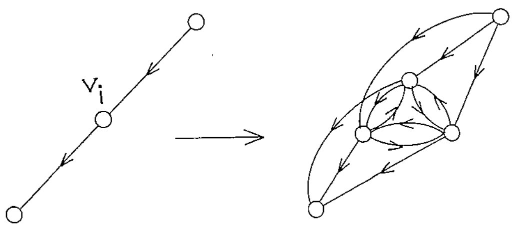


Fig. 6 The replacement in the case that $t=3$.

[Theorem 1]

Given a network N and nonnegative real number h, the problem of determining whether a network N is (k, h)-colorable or not is an NP-complete problem(14),(15) and the problem of determining the chromatic number with defect h is an NP-hard problems(14),(15). These problem are intractable problems if N has a great number of vertices and edges. □

Since it is not easy to obtain the chromatic number with defect h of a network N from Theorem 1, it is important to examine upper bounds for the chromatic number with defect h. These bounds are rough standard of channel assignments in cellular mobile systems.

A nontrivial connected graph with no cut vertices is called a *block*. A *block of a graph G* is a subgraph of G, which is itself a block and which is maximal with respect to that property. If the underlying graph of a directed network N is a block, then N is called a *block*. A *block of a network N* is a subnetwork of N, which is itself a block and which is maximal with respect to that property.

[Theorem 2]

Let N_1 and N_2 be all blocks of a network N, then

$$x_h(N) \leqq \max\{x_h(N_1), x_h(N_2)\}+1. \quad (2)$$

(proof)

Let v be a cut vertex of N (see Fig. 7). Let $f(\cdot)$ be an optimal coloring with defect h in N_1 and $f(v)=c(1 \leqq c \leqq x_h(N_1))$. Then there exists a coloring with defect h $f'(\cdot)$ using colors $1, 2, \cdots, c-1, c+1, \cdots, x_h(N_2)+1$ in N_2-v. Putting together $f(\cdot)$ and $f'(\cdot)$, we obtain the coloring with defect h of N. So, N is $(\max\{x_h(N_1), x_h(N_2)+1\}, h)$-colorable. Therefore,

$$x_h(N) \leqq \max\{x_h(N_1), x_h(N_2)+1\}$$
$$\leqq \max\{x_h(N_1), x_h(N_2)\}+1. \quad □$$

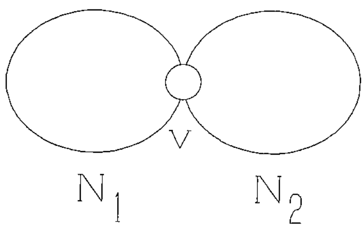


Fig. 7 A network N and the cut vertex v of N.

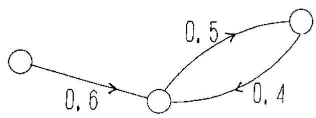


Fig. 8 A network N.

An example where the equality of Eq. (2) holds is shown in Fig. 8. There exist just two blocks in the network N in Fig. 8. Clearly, $x_{0.7}(N)=2$ and $\max\{x_{0.7}(N_1), x_{0.7}(N_2)\}+1=1+1=2$.

Using the coloring in the proof of Theorem 2 repeatedly, we obtain the following corollary.

[Corollary 2]

Let $N_1, N_2, \cdots, N_t$ be all blocks of a network N. Then

$$x_h(N) \leqq \max\{x_h(N_i) | i=1, \cdots, t\}+1. \quad □$$

6. Definitions and Results on a New Coloring of Graphs

In the previous chapter, we use a directed network whose edge weights generally have different values. In this section, we use assumptions for simplicity and consider the relation between the new coloring problem and the usual coloring problem. The assumptions are following.

1) The network is symmetric, namely whenever (u, v) is an edge of the network, then so too is (v, u). Moreover, $w(u, v)$ and $w(v, u)$ are almost same value for each edge pair (u, v) and (v, u).

In this case, we can use an undirected network instead of a directed network as the interference network.

2) Each edge weight is a rational number.

In this case, since the ratio of all edge weights can be represented using natural numbers, we can reconstruct the interference network, that admits multiple edges, such that all edges have same weight.

Under the above assumptions, the new coloring problem is rewritten into the following coloring problem of graphs.

[Definitions]

Let G be an undirected graph that admits multiple edges and no loops. The degree of vertex v of G is denoted by $d_G(v)$ and the maximum degree of G is denoted by $d_{\max}(G)$. For each vertex v of G, we assign a color. The assignment is called the *coloring with defect h* if the following conditions are satisfied. Let G_c be the induced subgraph whose vertices are assigned the color c. For each G_c, $d_{\max}(G_c)$ is not greater than h.

G is *k-colorable with defect h* or simply *(k, h)-colorable* if there exists a coloring with defect h using $k'(\leqq k)$ colors. The *chromatic number with defect h* is the minimum k for which G is (k, h)-colorable, denoted by $x_h(G)$. □

An example of a coloring with defect h is shown in Fig. 9.

From the above definitions, in the case that $h=0$, the coloring problem coincides with the usual coloring problem. So, $x_0(G)=x(G)$. Therefore, given a graph G, the problem of determining whether G is $(k, 0)$-

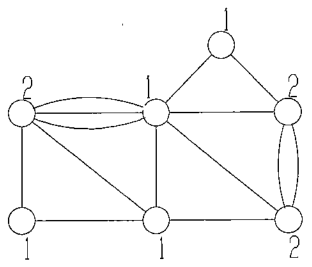


Fig. 9 In the case that $h=2$, a coloring with defect h.

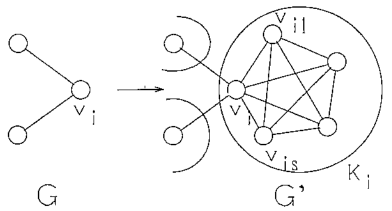


Fig. 10 Explanation for the proof of Theorem 3.

colorable or not is an NP-complete problem. Moreover, for any h, we obtain the similar result.

[Theorem 3]

Given a graph G, for any nonnegative integer h_0, the problem of determining whether a graph G is (k, h_0)-colorable or not is an NP-complete problem.

(proof)

Let G be a graph. We construct a graph G' from G and we show that the problem of determining whether a graph G is k-colorable for the usual coloring or not is rewritten into the problem of determining whether a graph G' is (k, h_0)-colorable or not.

For each vertex v_i of G, we add the vertices $v_{i1}, \cdots, v_{is}$ where $s=k(h_0+1)-1$ and add edges (x, y) for any vertex pair x, y in the vertex set $\{v_i, v_{i1}, \cdots, v_{is}\}$. Therefore, the induced subgraph of $\{v_i, v_{i1}, \cdots, v_{is}\}$ becomes a clique that is denoted by K_i. Let G' be a graph such that vertices and edges are added for all vertices of G (see Fig. 10). We claim that G' is (k, h_0)-colorable if and only if G is k-colorable for the usual coloring.

Assume that G' is (k, h_0)-colorable and $f'(\cdot)$ is the coloring with defect h_0. Since the order of each clique K_i is $k(h_0+1)$, the number of vertices assigned a color c is correctly h_0+1 for each color c. So if $(u, v)\in E(G)$, then $f'(u)\neq f'(v)$. $f'(\cdot)$ is a usual coloring of G. Therefore, G is k-colorable for the usal coloring.

Conversely, assume that G is k-colorable for the usual coloring and $f(\cdot)$ is the coloring. if $f(v_i)=c$, we assign colors to vertices $v_{i1}, \cdots, v_{is}$ of K_i as follows. We assign a color c' to h_0+1 vertices for each color c' $(\neq c)$ and assign a color c to h_0 vertices. Then $f(\cdot)$ is a coloring with defect h_0 of G'. Therefore, G' is (k, h_0)-colorable. □

Given a graph G, for any nonnegative integer h_0, the problem of determining the chromatic number with defect h_0 is obviously an NP-hard problem from Theorem 3.

Next, we show some upper bounds of the chromatic number with defect h.

[Theorem 4]

If G is a network such that $d_G(v)=r$ for all $v\in V(G)$ and h is a nonnegatie integer, then

$$x_h(G)\leqq\{(r+1)/(h+1)\},$$

where $\{x\}$ is the minimum integer which is not less than x.

(proof)

In the case that $\{(r+1)/(h+1)\}=1$, r is not greater than h. Since $x_h(G)=1$, the above inequality holds.

We assume that $\{(r+1)/(h+1)\}\geqq 2$. We assign a color to each vertex of G using $b=\{(r+1)/(h+1)\}$ colors. For any assignment, let m_c be the number of edges of the induced subgraph G_c whose vertices are assigned a color c and define $m=m_1+m_2+\cdots m_b$. Choose an assignment $f(\cdot)$ using b colors such that m is a minimum. We claim that for each $1\leqq c\leqq b$, the maximum degree of G_c is at most h, namely, $f(\cdot)$ is a coloring with defect h of G. Suppose that there exists a vertex v of G_c, where $d_{Gc}(v)=t(>h)$. Then, the number of edges whose one end vertex is v and the other is a vertex colored $c'(\neq c)$ in G is at most $r-t$. So,

$$\begin{aligned}\frac{r-t}{b-1}&=\frac{r-t}{\{(r+1)/(h+1)\}-1}\\&\leqq\frac{r-t}{(r+1)/(h+1)-1}=(h+1)\left(\frac{r-t}{r-h}\right)\\&<(h+1)\left(\frac{r-h}{r-h}\right)\leqq t.\end{aligned}$$

Therefore, there exists a color c_0 such that the number of edges whose one end vertex is v and the other is a vertex colored c_0 in G is less than t. If v is recolored by c_0, the value of m will decrease and this is a contradiction. □

For any graph G, there is a graph G' such that $d_{G'}(v)=d_{\max}(G)$ for all $v\in V(G')$ and G is a subgraph of G'. Because, if G does not satisfy the condition, then we construct G' as follows. G' consists of two disjoint copies of G and edges joining identically labeled vertices for each vertex v of G such that $d_G(v)\neq d_{\max}(G)$. And the number of edges joining identical labeled vertices for each vertex v is $d_{\max}(G)-d_G(v)$. Since G' satisfies the condition, we obtain the following corollary.

[Corollary 4]

Let G be a graph whose maximum degree is r and h be a nonnegative integer, then

$x_h(G) \leqq \{(r+1)/(h+1)\}$. □

(Remark 1)

From the result of graph theory, it is wellknown that

$x_0(G) \leqq r+1$. □

Another coloring problem is defined in Ref. (16). In the case that $h=1$, Corollary 4 is equivalent to the result in Ref. (16).

Using the method of the proofs of Theorem 2 and Corollary 2, we obtain the following theorem.

[Theorem 5]

Let $G_1, G_2, \cdots, G_t$ be all blocks of a graph G. Then

$x_h(G) \leqq \max\{x_h(G_i) | i=1, \cdots, t\}+1$. □

(Remark 2)

From the result of graph theory, it is wellknown that

$x_0(G) = \max\{x_0(G_i) | i=1, \cdots, t\}$. □

The following inequalities are concerned with the chromatic number with defect h and the usual chromatic number.

[Theorem 6]

Let G be a graph and h be a nonnegative integer, then

$$\frac{x(G)}{h+1} \leqq x_h(G) \leqq x(G).$$

(proof)

The usual coloring of G is also the coloring with defect h of G. Therefore,

$x_h(G) \leqq x(G)$.

Next, we consider the coloring with defect h of G. Let G_c be the induced subgraph whose vertices are assigned a color $c\,(1 \leqq c \leqq x_h(G))$. Since the maximum degree of G_c is not greater than h, $x(G_c) \leqq h+1$ from Remark 1. Therefore, there exists a usual coloring of G using at most $x_h(G) \cdot (h+1)$ colors. From the definition of the chromatic number, $x(G) \leqq x_h(G) \cdot (h+1)$, that is to say,

$$\frac{x(G)}{h+1} \leqq x_h(G).$$ □

7. Conclusion

In a cellular mobile system, assigning a channel for a call in a cell so as to achieve high spectral efficiency is an important problem. Recently, a new channel assignment scheme has been proposed. The scheme takes the degree of interference into consideration. In this paper, we formulated this new model using a directed network and a new coloring problem of networks. Spectral efficiency in the formulation using a network is higher than spectral efficiency in the formulation using a graph. We showed that the new coloring problem was an NP-hard problem. Next, we rewrote the new coloring problem of networks to a coloring problem of graphs on some assumptions and considered the relation between the results on the new coloring and the results on the usual coloring.

References

(1) Schiff L.: "Traffic Capacity of Three Types of Common-User Mobile Radio Communication Systems", IEEE Trans. On Communication Technology, **COM-18**, 1, pp. 12-21 (Feb. 1970).

(2) MacDonald V. H.: "The Cellular Concept", The Bell System Technical Journal, **58**, 1, pp. 15-41 (Jan. 1979).

(3) Sengoku M.: "Efficient Utilization of Frequency Spectrum for Mobile Radio Communication Systems—Algorithms for Channel Assignments", J. IECE Japan, **69**, 4, pp. 350-356 (April 1986).

(4) Yokoyama M.: "Decentralization and Distribution in Network Control of Mobile Communications", Trans. ICICE, **E73**, 10, pp. 1579-1586 (Oct. 1990).

(5) Sengoku M., Itoh K. and Matsumoto T.: "A Dynamic Frequency Assignment Algorithm in Mobile Radio Communication Systems", Trans. IECE Japan, **E61**, 7, pp. 527-533 (July 1978).

(6) Nakano K.: "Dynamic Channel Assignment in Channel Offset Systems of a Mobile Communication System", Graduate Thesis of Faculty of Engineering, Niigata University (Feb. 1989).

(7) Yasuda S.: "Flexible Re-use for Dynamic Channel Assignments in Mobile Radio Systems", 1989 National Convention Record of IEICE of Japan, B-860 (March 1989).

(8) Nettleton R. W. and Schloemer G. R.: "A high capacity assignment method for cellular mobile telephone systems", Proc. 39th. IEEE Vehicular Technology Conference, pp. 359-367 (1989).

(9) Behzad M., Chartrand G. and Foster L. L.: "Graphs & Digraphs", Prindle, Weber & Schmidt (1979).

(10) Berge C.: "Graphs", North-Holland (1985).

(11) Lee W. C. Y.: "New Cellular Schemes for Spectral Efficiency", IEEE Trans. On Vehicular Technology, **VT-36**, 4, pp. 188-192 (1987).

(12) Lee W. C. Y.: "Narrowbanding in cellular mobile systems", Telephony, pp. 44-46 (1986).

(13) Cowen L. J., Cowen R. H. and Wooddall D. R.: "Defective colorings of graphs in surfaces: partition into subgraphs of bounded valency", J. Graph Theory, 10, pp. 187-195 (1986).

(14) Aho A. W., Hopcroft J. E. and Ullman J. D.: "The Design and Analysis of Computer Algorithm", Addison-Wesley (1974).

(15) Hopcroft J. E. and Ullman J. D.: "Introduction to Automata Theory, Languages and Computation", Addison-Wesley (1979).

(16) Akiyama J., Era H., Gervacio E. V. and Watanabe M.: "Path chromatic Number of Graphs", J. of Graph Theory, **13**, 5, pp. 569-575 (1989).

(17) Tamura H., Sengoku M., Shinoda S. and Abe T.: "Channel Assignment Taking into Account Interference in a Cellular Mobile System and Color Problem of a Graph", IEICE Technical Report, **CAS89**-43 (July 1989).

(18) Sengoku M., Nakano K., Shinoda S., Yamaguchi Y. and Abe T.: "Cellular Mobile Communication Systems and Channel Assignment Using Neural Networks", Proc. 33rd Midwest Symposium on Circuits and Systems, IEEE, pp. 1-4 (Aug. 1990).

Hiroshi Tamura was born in Saitama prefecture, Japan, November 16, 1959. He received the B. Educ., M. S. and Ph. D. degrees from Niigata University in 1982, 1986 and 1990, respectively. In 1990, he joined the staff at the Graduate School of Science and Technology, Niigata University as a Research Associate. His research interests are in computational geometry, network theory and graph theory. Dr. Tamura is a member of the Mathematical Society of Japan.

Masakazu Sengoku was born in Nagano prefecture, Japan, on October 18, 1944. He received the B. E. degree in electrical engineering from Niigata University, Niigata, Japan, in 1967 and the M. E. and Ph. D. degrees from Hokkaido University in 1969 and 1972, respectively. In 1972, he joined the staff at the Department of Electronic Engineering, Hokkaido University as a Research Associate. In 1978, he was an Associate Professor at the Department of Information Engineering, Niigata University, where he is presently a Professor. His research interests include network theory, graph theory, transmission of information and mobile communications. Dr. Sengoku is a member of IEEE and IPS of Japan.

Shoji Shinoda was born on December 15, 1941 in Hokkaido, Japan. He received the B. E., M. E., and Dr. E. degrees, all in electrical engineering, from Chuo University in 1964, 1966 and 1973, respectively. He joined the Department of Electrical Engineering at Chuo University in 1965 as a Research Assistant, where he was promoted to an Assistant Professor in 1970, to an Associate Professor in 1974 and then to a Professor in 1982. Currently, he is a Professor of the Department of Electrical and Electronic Engineering at the same university. Meanwhile, he was with the Coordinate Science Laboratory, University of Illinois at Urbana-Champaign, as a visiting scholar for a half year starting from March 30, 1981, and with the Electronics Laboratory, University of California at Berkeley, as a visiting scholar for three weeks of December 1988. His recent research interest has been in fault-diagnosis and analysis of analog integrated circuits, optimal location of guards in a closed region with holes, and optimal channel assignment in a cellular mobile system. He has co-authored eight books concerning basic circuit theory, applied graph theory and applied mathematics, one of which is the book "Foundations of Circuit Theory" (Tokyo: Corona Publishing. Co., 1990). He was a General Secretary of the Steering Committee of the IEICE Engineering Sciences Group for two years starting from May 1985, and a Vicechairperson of the same Committee for one year starting from May 1989. Also, he was the Editor of the Japanese edition of the Transactions of the IEICE, Part A (Engineering sciences), for two years starting from May 1987 and the Chairperson of the IEICE Technical Group on Circuits and System for one year starting from May 1989. He is now a Councilor of the IEICE.

Takeo Abe was born in Niigata, Japan, on March 8, 1926. He received the B. E. and Dr. Eng. degrees from Tokyo Institute of Technology, Tokyo, Japan, in 1949 and 1966, respectively. From 1950 to 1959, he was a Research Scientist at the Electrotechnical Laboratory, Tokyo, Japan. From 1962 to 1966, he was an Associate Professor at Tokyo Institute of Technology. From 1966 to 1978, he was a Professor at the Department of Electronic Engineering, Niigata University. From 1978 to 1991, he was a Professor at the Department of Information Engineering, Niigata University and from 1987 to 1991, he was Dean of the Faculty of Engineering, Niigata University. He is now a Professor at the Department of Electronic Engineering, Chiba Institute of Technology. He has been engaged in research and education in electromagnetic theory, microwave engineering, transmission of information and network theory. Dr.Abe is a member of IEEE, the Japanese Society of Snow and Ice and the Japan Society of Snow Engineering.