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A Theory and an Algorithm for Fault Diagnosis by Measuring Transmission Numbers in a Directed Network

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SUMMARY In location problems, the outtransmission and intransmission numbers are important indices to evaluate a directed network. We formulate and consider a new problem of fault diagnosis in a system modeled by a directed network in which to each edge a positive real number called the length of edge is assigned and to each vertex a positive real number called the weight of vertex is assigned. By a fault in a directed network, we mean any increase in the length of an edge with respect to its nominal length. A theory and an algorithm for detecting a fault edge in a directed network in which the above indices, i. e. outtransmission and/or intransmission numbers, are measurable, are presented.

1. Introduction

The outtransmission and intransmission numbers are used in location problems as important indices of evaluating the centrality of vertices in a directed network[(1),(2)]. In location theory on networks, there are two problems which have been studied. One is to obtain the indices such as transmission number or eccentricities for a given network. This is called an analysis problem. The other is to obtain a network from a given set of transmission number or a given set of eccentricities. This is called a synthesis problem. In this paper, we propose a new problem called a diagnosis problem. In the problem, a network is given and the transmission numbers of all vertices of the graph are measurable. The problem is to detect a fault edge in a network from the measurement of the transmission numbers.

We consider a directed network in which to each edge a positive real number called the length of the edge is assigned and to each vertex a positive real number called the weight of the vertex is assigned. By a fault in a directed network, we mean any increase in the length of an edge with respect to its nominal length which can cause any increase of the outtransmission or intransmission number of at least one vertex in the network under the assumption that the weight of any vertex in the network does not change in any situations. We give a theory and an algorithm for detecting a fault edge in a directed network from measurements of outtransmission and/or intransmission numbers of all vertices of the network.

The objective of this paper is to propose a diagnosis problem in the location theory. At present we do not have any concrete idea of applying this problem to practical problems but we hope that some practical applications will be motivated by this paper.

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2. Outtransmission and Intransmission Numbers

Let $N=(G, l, w)$ be a connected and directed network whose underlying graph is denoted by $G=(V, E)$ where V and E represent the vertex set and the edge set, respectively, of G. To each edge, a positive real number called the length of the edge is assigned and to each vertex, a positive real number called the weight of the vertex is assigned. The length of $e_k \in E$ is denoted by $l(e_k)$ and the weight of $v_i \in V$ is denoted by $w(v_i)$, where $l: E \rightarrow R_+$, $w: V \rightarrow R_+$, $(R_+=(0, \infty))$.

Now, for $v_j \in V$, let us consider the outtransmission number $t_o(v_j)$ and the intransmission number $t_i(v_j)$ which are defined by

$$t_o(v_j)=\sum_{v_k \in V} d(v_j, v_k) w(v_k) \qquad (1)$$

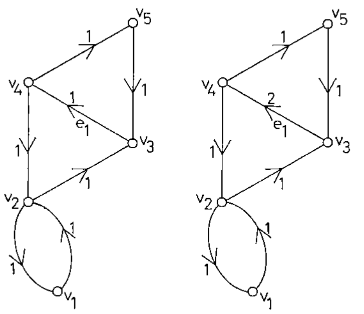
$$t_i(v_j)=\sum_{v_k \in V} d(v_k, v_j) w(v_k) \qquad (2)$$

where $d(v_i, v_j)$ is the length of the shortest path from vertex v_i to v_j which is called the distance from v_i to v_j. Let $|V|=n$ and let $D=[d_{ij}]$ be an $n \times n$ matrix whose i-th row and j-th column element is d_{ij} where $d_{ij}=d(v_i, v_j)$ and $|A|$ denotes the cardinal number of the set A. D is called the distance matrix of N.
Let

$$T_o=\begin{bmatrix} t_o(v_1) \\ t_o(v_2) \\ \vdots \\ t_o(v_n) \end{bmatrix}, \quad T_i=\begin{bmatrix} t_i(v_1) \\ t_i(v_2) \\ \vdots \\ t_i(v_n) \end{bmatrix} \quad \text{and} \quad W=\begin{bmatrix} w(v_1) \\ w(v_2) \\ \vdots \\ w(v_n) \end{bmatrix}$$

Then, Eqs. (1) and (2) are rewrited as follows.

$$T_o=DW \qquad (3)$$



(a) N (b) N'
Fig. 1 Networks N and N'.

$$T_i=D^tW \tag{4}$$

where D^t is the transpose of D.

For example, matrices D, T_o and T_i of a network N of Fig. 1(a), in which the weight given to each edge indicates the edge length of the edge, and $w(v_1)=1$, $w(v_2)=2$, $w(v_3)=3$, $w(v_4)=4$ and $w(v_5)=5$, are given below:

$$D=\begin{bmatrix}0&1&2&3&4\\1&0&1&2&3\\3&2&0&1&2\\2&1&2&0&1\\4&3&1&2&0\end{bmatrix} \tag{5}$$

$$T_o=DW=D\begin{bmatrix}1\\2\\3\\4\\5\end{bmatrix}=\begin{bmatrix}40\\27\\21\\15\\21\end{bmatrix} \tag{6}$$

$$T_i=D^tW=D^t\begin{bmatrix}1\\2\\3\\4\\5\end{bmatrix}=\begin{bmatrix}39\\26\\17\\20\\20\end{bmatrix} \tag{7}$$

3. Assumptions and a Fault of an Edge

The outtransmission and intransmission numbers are used in the so-called minisum location problem. An edge in a network corresponds to a communication link in a communication system or a road in a road distribution network. The length of an edge in the network represents the transmission or transportation time between the two vertices. A fault of a link or a road in these systems generally causes the increase of the transmission or transportation time.

In this paper, therefore, by a fault in a directed network, we mean any increase in the length of an edge with respect to its nominal length which can cause any increase of the outtransmission or intransmission number of at least one vertex in the network under the assumption that the weight of any vertex in the network does not change in any situations.

It should be noted that an increase in the length of an edge in a directed network can not be considered to be a fault in the network if it does not cause any increase of any outtransmission or intransmission number in the network.

Now, we say that a directed network is k-edge fault detectable if one will be able to determine a set of k edges containing at least one fault edge from measurements of outtransmission and/or intransmission numbers of the network. We say vertices u and v of a fault edge fault vertices. And we say that a directed network is k-vertex fault detectable if one will be able to determine a set of k vertices containing at least one fault vertex from measurements of outtransmission and/or intransmission numbers of the network.

In this paper, we assume that a fault occurs at an edge only. And our purpose is to detect a fault edge in a network from measurements of outtransmission and/or intransmission numbers.

4. Increment of Distance by Increase of the Length of an Edge

Let $N'=(G, l', w)$ be a network obtaining from $N=(G, l, w)$ after increase of the length of an edge $e_m=(v_p, v_q)$ from $l(e_m)$ to $l'(e_m)$ by δ. That is,

$$\left.\begin{aligned}&l'(e_m)=l(e_m)+\delta,\ (\delta>0,\quad l(e_m)>0)\\&l'(e_j)=l(e_j),\ (j\neq m,\quad e_j\in E)\end{aligned}\right\} \tag{8}$$

Let $d'(v_i, v_j)$ be the distance from vertex v_i to vertex v_j in N'.

Then, we get the following lemmas.

[Lemma 1] For any vertices $v_i, v_j, v_k \in V$

$$d(v_i, v_k) \leqq d(v_i, v_j)+d(v_j, v_k) \tag{9}$$

$$d'(v_i, v_k) \leqq d'(v_i, v_j)+d'(v_j, v_k) \tag{10}$$ □

[Lemma 2] For any vertices $v_i, v_j, v_k \in V$,

$$\begin{aligned}d(v_j, v_i)&=\min\{d'(v_j, v_i),\ d'(v_j, v_p)+d'(v_q, v_i)+l'(e_m)-\delta\}\\&=\min\{d'(v_j, v_i),\ d'(v_j, v_p)+d'(v_q, v_i)+d'(v_p, v_q)-\delta'\}\end{aligned} \tag{11}$$

where $\delta \geqq \delta' \geqq 0$. □

From Lemma 1 and Lemma 2, we get the following theorem.

[Theorem 1] If for any two distinct vertices v_i and v_j of V

$$d'(v_j, v_i)>d(v_j, v_i) \tag{12}$$

then

$$d(v_j, v_i) > d(v_p, v_i), \ (p \neq j) \quad (13)$$

and

$$d'(v_p, v_i) - d(v_p, v_i) \geqq d'(v_j, v_i) - d(v_j, v_i) \quad (14)$$

(proof) Since the distance between any two vertices does not decrease by the modification (8) of N, for any $v_i \in V$

$$d'(v_p, v_i) \geqq d(v_p, v_i) \quad (15)$$

Assume that

$$d'(v_p, v_i) = d(v_p, v_i) \quad (16)$$

Then, from Eq. (11)

$$d(v_p, v_i) = d'(v_p, v_i) \leqq d'(v_q, v_i) + l'(e_m) - \delta \quad (17)$$

Using Eqs. (11) and (12) for any vertex v_j $(j \neq i, \ j \neq p)$,

$$d'(v_j, v_i) > d(v_j, v_i) = d'(v_j, v_p) + d'(v_q, v_i) + l'(e_m) - \delta \quad (18)$$

From Eqs. (17) and (18),

$$d'(v_j, v_i) > d(v_j, v_i) \geqq d'(v_j, v_p) + d'(v_p, v_i) \quad (19)$$

This is cotradictory to (10). Hence, $d'(v_p, v_i) \neq d(v_p, v_i)$ and in Eq. (15)

$$d'(v_p, v_i) > d(v_p, v_i) \quad (20)$$

Then, from Eqs. (11) and (20),

$$d(v_p, v_i) = d'(v_q, v_i) + l'(e_m) - \delta \quad (21)$$

Using Eqs. (11), (12), and (21),

$$d(v_j, v_i) = d'(v_j, v_p) + d(v_p, v_i) \quad (22)$$

Since $d'(v_j, v_p) > 0$, $(p \neq j)$,

$$d(v_j, v_i) > d(v_p, v_i) \quad (23)$$

Hence we get Eq. (13).

Using Eq. (10),

$$d'(v_j, v_i) - d(v_j, v_i) \leqq \{d'(v_j, v_p) + d'(v_p, v_i)\} - d(v_j, v_i) \quad (24)$$

From Eqs. (22) and (24),

$$d'(v_j, v_i) - d(v_j, v_i) \leqq d'(v_p, v_i) - d(v_p, v_i) \quad (25)$$

Hence we get Eq. (14). □

Similarly, we get the following theorem.

[Theorem 2] If for any two distinct vertices v_i and v_j of V

$$d'(v_j, v_i) > d(v_j, v_i) \quad (26)$$

then

$$d(v_j, v_i) > d(v_j, v_q), \quad (q \neq i) \quad (27)$$

and

$$d'(v_j, v_q) - d(v_j, v_q) \geqq d'(v_j, v_i) - d(v_j, v_i) \quad (28)$$ □

we also get the following theorem using Lemma 1 and Lemma 2.

[Theorem 3] Let v_i be a vertex such that $d'(v_i, v_p) > d'(v_i, v_q)$. Then, for any vertex $v_j \in V$,

$$d'(v_i, v_j) = d(v_i, v_j) \quad (29)$$

Let v_i be a vertex such that $d'(v_p, v_i) < d'(v_q, v_i)$. Then, for any vertex $v_j \in V$,

$$d'(v_j, v_i) = d(v_j, v_i) \quad (30)$$

(Proof) We show Eq. (29). In general, since the distance between any two vertices does not decrease by the modification (8) of N, $d'(v_i, v_j) \geqq d(v_i, v_j)$. Assume that $d'(v_i, v_j) > d(v_i, v_j)$. From Eqs. (11) and (8),

$$d'(v_i, v_j) > d(v_i, v_j) = d'(v_i, v_p) + d'(v_q, v_j) + l'(e_m) - \delta > d'(v_i, v_p) + d'(v_q, v_j). \quad (31)$$

Since $d'(v_i, v_p) > d'(v_i, v_q)$ from the hypothesis,

$$d'(v_i, v_j) > d'(v_i, v_q) + d'(v_q, v_j) \quad (32)$$

This is contradictory to Eq. (10). Hence we get Eq. (29). Similarly we get Eq. (30). □

5. Detection of a Fault Edge

Let $t_o'(v_j)$ and $t_i'(v_j)$ be outtransmission number and intransmission number of a vertex in N', respectively.

$$\Delta t_o(v_j) \triangleq t_o'(v_j) - t_o(v_j) \quad (33)$$

$$\Delta t_i(v_j) \triangleq t_i'(v_j) - t_i(v_j) \quad (34)$$

We call $\Delta t_o(v_j)$ and $\Delta t_i(v_j)$ the increment of outtransmission number and intransmission number of v_j, respectively. From the definition of outtransmission and intransmission numbers,

$$\Delta t_o(v_j) = \sum_{v_k \in V} (d'(v_j, v_k) - d(v_j, v_k)) w(v_k) \quad (35)$$

$$\Delta t_i(v_j) = \sum_{v_k \in V} (d'(v_k, v_j) - d(v_k, v_j)) w(v_k) \quad (36)$$

For any vertex $v_i, v_j \in V$, $d'(v_i, v_j) \geqq d(v_i, v_j)$ since the distance between any two vertices does not decrease by the modification (8) of N. Then, we get the following lemma.

[Lemma 3] The increments of outtransmission and intransmission numbers of any vertex of N by the modification (8) of N are nonnegative. That is, for any vertex $v_j \in V$,

$$\Delta t_o(v_j) \geqq 0 \quad (37)$$

$$\Delta t_i(v_j) \geqq 0 \quad (38)$$ □

Then, we get the following theorem using the theorems of previous section.

[Theorem 4] For any vertex $v_j \in V$,

$$\Delta t_o(v_p) \geqq \Delta t_o(v_j) \geqq 0 \quad (39)$$

and

$$\Delta t_o(v_q) = 0. \quad (40)$$

(Proof) From Eqs. (35) and (36),

$$\Delta t_o(v_p) - \Delta t_o(v_j) = \sum_{v_k \in V} \{(d'(v_p, v_k) - d(v_p, v_k)) - (d'(v_j, v_k) - d(v_j, v_k))\} w(v_k) \quad (41)$$

Let

$$Y = (d'(v_p, v_k) - d(v_p, v_k)) - (d'(v_j, v_k) - d(v_j, v_k)) \quad (42)$$

Since the distance between any two vertices v_i and v_j does not decrease by the modification (8) of N,

$$d'(v_i, v_j) \geqq d(v_i, v_j) \quad (43)$$

In Eq. (42), $d'(v_j, v_k) \geqq d(v_j, v_k)$. If $d'(v_j, v_k) = d(v_j, v_k)$, Y is nonnegative because $d'(v_p, v_k) \geqq d(v_p, v_k)$ in general. If $d'(v_j, v_k) > d(v_j, v_k)$, Y is also nonnegative from Eq. (14) of Theorem 1. Hence, $\Delta t_o(v_p) \geqq \Delta t_o(v_j) \geqq 0$ using Lemma 3.

Next, in general $d'(v_q, v_p) > d'(v_q, v_q) = 0$. From Eq. (29) of Theorem 3, $d'(v_q, v_k) = d(v_q, v_k)$ for any vertex $v_k \in V$. Hence $\Delta t_o(v_q) = 0$. □

Similary we get the following theorem.

[Theorem 5] For any vertex $v_j \in V$,

$$\Delta t_i(v_q) \geqq \Delta t_i(v_j) \geqq 0 \quad (44)$$

and

$$\Delta t_i(v_p) = 0. \quad (45)$$ □

Next let us define the following subsets of vertices.

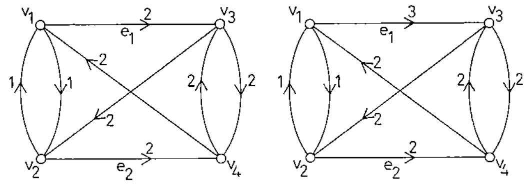
$$V_{mo} = \{v_k | \Delta t_o(v_k) \geqq \Delta t_o(v_j), \quad v_k, v_j \in V\} \quad (46)$$

$$V_{mi} = \{v_k | \Delta t_i(v_k) \geqq \Delta t_i(v_j), \quad v_k, v_j \in V\} \quad (47)$$

$$V_{zo} = \{v_k | \Delta t_o(v_k) = 0, \quad v_k \in V\} \quad (48)$$

$$V_{zi} = \{v_k | \Delta t_i(v_k) = 0, \quad v_k \in V\} \quad (49)$$

First, let us consider a network in which only outtransmission number is measurable. We show an example. Fig. 1(b) is a network N' obtaining from N of Fig. 1(a) by increasing the length of an edge $e_1 = (v_3, v_4)$ by 1. The weight of every vertex of N' is the same as that of N. The distance matrix D' and the outtransmission number matrix T_o' of N' are given bellow :



(a) N (b) N'

Fig. 2 Networks N and N'.

$$D' = \begin{bmatrix} 0 & 1 & 2 & 4 & 5 \\ 1 & 0 & 1 & 3 & 4 \\ 4 & 3 & 0 & 2 & 3 \\ 2 & 1 & 2 & 0 & 1 \\ 5 & 4 & 1 & 3 & 0 \end{bmatrix} \quad (50)$$

$$T_o' = D'W = \begin{bmatrix} 49 \\ 36 \\ 33 \\ 15 \\ 28 \end{bmatrix} \quad (51)$$

Then,

$$\Delta T_o \triangleq T_o' - T_o = \begin{bmatrix} \Delta t_o(v_1) \\ \cdot \\ \cdot \\ \cdot \\ \Delta t_o(v_5) \end{bmatrix} = \begin{bmatrix} 9 \\ 9 \\ 12 \\ 0 \\ 7 \end{bmatrix} \quad (52)$$

If only outtransmission number is measurable, that is, T_o and T_o' are given, then from ΔT_o we can get the subsets $V_{mo} = \{v_3\}$ and $V_{zo} = \{v_4\}$. Then, from Theorem 4, we can say that a vertex v_3 in V_{mo} is a fault vertex and a vertex v_4 in V_{zo} is also a fault vertex, because $|v_{mo}| = 1$ and $|V_{zo}| = 1$. Thus, this fault of N is 1-vertex fault detectable and is also 1-edge fault detectable because the only edge between v_3 and v_4 is e_1.

[Theorem 6] Let N be a network in which only the vertices with the maximum increments of outtransmission number are distinguishable. Then, N is k-vertex fault detectable if $\min\{|v_{mo}|, |v_{zo}|\} = k$. □

[Theorem 7] Let N be a network in which only the vertices with the maximum increments and the minimum increments of outtransmission number are distinguishable. Then, for any network N, N is 1-edge fault detectable if and only if $|v_{zo}| = 1$.

(proof) Sufficiency : From Eq. (40) of Theorem 4

$$V_{zo} = \{v_q\}, \text{ since } |V_{zo}| = 1.$$

Let

$$V_s=\{v_i|d(v_i, v_q)\leqq d(v_j, v_q), v_i, v_j\in V_{mo}\} \qquad (53)$$

Assume that $|V_s|>1$. Let v_i, $v_p\in V_s$. Since N' is a fault network of N, $\Delta t_o(v_p)>0$ and $d'(v_p, v_q)>d(v_p, v_q)$ from the definition of a fault network in this paper and Theorem 4. Then, since $v_i\in V_s$ and $\Delta t_o(v_p)=\Delta t_o(v_i)$, $d'(v_i, v_q)>d(v_i, v_q)$. Therefore, from Theorem 1, $d(v_i, v_q)>d(v_p, v_q)$. This is contradictory to Eq. (53). Hence, $|V_s|=1$. That is, we obtain a fault edge $e=(v_p, v_q)$. If N has more than one edge associated with a pair of vertices v_p and v_q, the edge with minimum length in the edges is the fault edge.

Necessity : We show a network N including two vertices with zero increment of outtransmission number, that is, a network with $|v_{zo}|=2$, which is not 1-edge fault detectable. Matrices D, D', T_o, T_o' and ΔT_o of networks N and N' of Fig. 2(a) and (b), in which the weight given to each edge indicates the edge length of the edge and the weight of every vertex is 1, are given below.

$$D=\begin{bmatrix}0&1&2&3\\1&0&3&2\\3&2&0&2\\2&3&2&0\end{bmatrix} \qquad (54)$$

$$D'=\begin{bmatrix}0&1&3&3\\1&0&4&2\\3&2&0&2\\2&3&2&0\end{bmatrix} \qquad (55)$$

$$T_o=\begin{bmatrix}6\\6\\7\\7\end{bmatrix},\quad T_o'=\begin{bmatrix}7\\7\\7\\7\end{bmatrix} \qquad (56)$$

$$\Delta T_o=T_o'-T_o=\begin{bmatrix}1\\1\\0\\0\end{bmatrix} \qquad (57)$$

From Eq. (57), we get V_{mo} and V_{zo}.

$$V_{mo}=\{v_1, v_2\},\quad V_{zo}=\{v_3, v_4\} \qquad (58)$$

The number of vertices with zero increment of outtransmission number is 2. N' of Fig. 2(b) is obtained from N by increasing the length of e_1 by 1. On the other hand, if N' is obtained from N by increasing the legth of e_2 by 1, then ΔT_o obtained from N and N' is clearly the same as Eq. (57). This means that the fault of e_1 and e_2 is not distinguishable by ΔT_o. It is simple to realize a network with $|V_{mo}|=1$ and $|V_{zo}|=2$ that is not 1-edge fault detectable. Therefore there are some networks which have $|V_{zo}|>1$ and are not 1-edge fault detectable. □

We get similar theorems of a network in which only intransmission number is measurable.

[Theorem 8] Let N be a network in which only intransmission number is measurable. Then, N is k-vertex fault detectable, if $\min\{|V_{mi}|, |V_{zi}|\}=k$. □

[Theorem 9] Let N be a network in which only the vertices with the maximum increments and the minimum increments of intransmission number are distinguishable. Then for any network N, N is 1-edge fault detectable, if and only if $|V_{zi}|=1$. □

Next, let us consider a network in which both outtransmission number and intransmission number are measurable.

[Theorem 10] A network N in which outtransmission number and intransmission number are all measurable is 1-edge fault detectable.

(proof) Let v_i be an element of V_{mo}. Let v_j be an element of V_{mi}. Since $d'(v_p, v_q)>d(v_p, v_q)$, $\Delta t_o(v_p)=\Delta t_o(v_i)$ and $\Delta t_i(v_q)=\Delta t_i(v_j)$, increment of the distance from v_i to v_j by the fault is not zero, that is $d'(v_i, v_j)>d(v_i, v_j)$. Let

$$V_{sj}=\{v_k|d(v_k, v_j)\leqq d(v_i, v_j), v_k, v_i\in V_{mo}\} \qquad (59)$$

Then, $V_{sj}=\{v_p\}$, since $|V_{sj}|=1$ from Theorem 1.

Similarly, let

$$V_{si}=\{v_k|d(v_i, v_k)\leqq d(v_i, v_j), v_k, v_j\in V_{mi}\} \qquad (60)$$

Then we get $V_{si}=\{v_q\}$. The fault edge is $e=(v_p, v_q)$. If N has more than one edge associated with a pair of vertices v_p and v_q, the edge with minimum length in the edges is the fault edge. □

In Theorem 10, both V_{mo} and V_{mi} are used for detecting a fault edge. However, we can detect a fault edge without using all vertices of V_{mo} or V_{mi}.

[Corollary 1] A fault edge is detectable if either V_{mo} and a vertex v_j with $\Delta t_i(v_j)>0$ or V_{mi} and a vertex v_j with $\Delta t_o(v_j)>0$ are given.

(Proof) Suppose that V_{mo} and a vertex v_j with $\Delta t_i(v_j)>0$ are given. Let

$$V_s=\{v_k|d(v_k, v_j)\leqq d(v_i, v_j), v_k, v_i\in V_{mo}\} \qquad (61)$$

Since $\Delta t_i(v_j)>0$,

$$d'(v_p, v_j)>d(v_p, v_j) \qquad (62)$$

Therefore, $V_s=\{v_p\}$ from Theorem 1.

Let $E_p=\{e_1, e_2, \cdots, e_m\}$ such that $e_1=(v_p, v_{k1})$, $e_2=(v_p, v_{k2})$, $\cdots$ and an edge $e_i\in E_p$ is contained in a shortest path from v_p to v_j. Assume that $m>1$. This means that there are more than one shortest path from v_p to v_j. Then, the distance $d(v_p, v_j)$ does not change by increasing the length of an edge $e_i\in E_p$. This is contradictory to Eq. (62). Hence, $m=1$ and $E_p=\{e_k\}$, that is, e_k is the fault edge. □

Then, we obtain a following algorithm detecting a

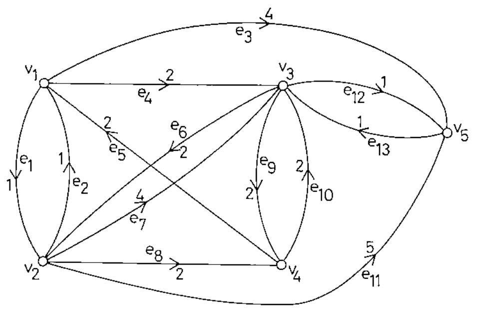


Fig. 3 A Network N.

fault edge.

[An algorithm of detection of a fault edge] $N, \Delta T_o$ and ΔT_i are given.

Step 0: Set $V_{mo}=\{v_i|\Delta t_o(v_i)\geqq \Delta t_o(v_j),\ v_i,\ v_j\in V\}$

Step 1: Select a vertex v_j such that $\Delta t_i(v_j)>0$.

Step 2: Set $V_S=\{v_k|d(v_k,\ v_j)\leqq d(v_i,\ v_j),\ \ v_k,\ v_i\in V_{mo}\}$ Then, $V_S=\{v_p\}$.

Step 3: Find a shortest path from v_p to v_j. Then, an edge $e_m=(v_p,\ v_q)$ on the shortest path is a fault edge. □

We show two examples for this algorithm.

[Examples]

(I) Consider the example in the proof of Theorem 7, in which the network was not 1-edge fault detectable by the measurements of only outtransmission numbers of all vertices.

In the network of Fig. 2(a), N and

$$\Delta T_o=\begin{bmatrix}1\\1\\0\\0\end{bmatrix} \tag{63}$$

$$\Delta T_i=\begin{bmatrix}0\\0\\2\\0\end{bmatrix} \tag{64}$$

are given.

Step 0: $V_{mo}=\{v_1,\ v_2\}$

Step 1: $v_j=v_3$, since $\Delta t_i(v_3)=2>0$

Step 2: $V_S=\{v_k|d(v_k,\ v_3)\leqq d(v_j,\ v_3),\ \ v_k,\ v_j\in V_{mo}\}$ $=\{v_1\}$

$\therefore v_p=v_1$

Step 3: Since the shortest path from v_1 to v_3 consists of e_1, $v_q=v_3$. The fault edge is e_1.

Note that in this example, we do not need Step 3 because we can see $v_q=v_3$ in step 1 since $V_{mi}=\{v_3\}=\{v_j|\Delta t_i(v_j)>0,\ \ v_j\in V\}$.

(II) Consider a network of Fig. 3 in which the real number given to each edge indicates the edge length. Let $w(v_i)=1$ for every vertex $v_i\in V$. D of this network is as follows.

$$D=\begin{bmatrix}0&1&2&3&3\\1&0&3&2&4\\3&2&0&2&1\\2&3&2&0&3\\4&3&1&3&0\end{bmatrix} \tag{65}$$

$N, \Delta T_o$ and ΔT_i are given.

$$\Delta T_o=\begin{bmatrix}2\\2\\0\\0\\0\end{bmatrix} \tag{66}$$

$$\Delta T_i=\begin{bmatrix}0\\0\\2\\0\\2\end{bmatrix} \tag{67}$$

Step 0: $V_{mo}=\{v_1,\ v_2\}$

Step 1: Let v_j be v_5, since $\Delta t_i(v_5)=2>0$.

Step 2: $V_S=\{v_k|d(v_k,\ v_5)\leqq d(v_j,\ v_5),\ \ v_k,\ v_j\in V_{mo}\}$ $=\{v_1\}$

$\therefore v_p=v_1$

Step 3: The shortest path from v_1 to v_5 consists of e_4 and e_{12}. Since the initial vertex of e_4 is $v_p=v_1$, the fault edge is $e_4=(v_p,\ v_q)=(v_1,\ v_3)$. □

6. Conclusion

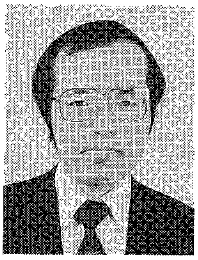
A theory and an algorithm for fault diagnosis of a network have been developed in relation to outtransmission and intransmission numbers, in the case of regarding the increase of the length of an edge as a fault. As similar discussions are also possible, in the case of regarding the decrease of the length of an edge as a fault, we can develop a theory and algorithm for fault diagnosis in the case of regarding the increase or decrease of the length of an edge as a fault.

At present we do not have any concrete idea of applying our theory to practical problems, but we hope that some practical applications will be motivated by this paper.

Future problems to be solved include a fault diagnosis in a directed network in which the outtransmission and intransmission numbers are not always measurable in all vertices, and the number of fault edges is more than one.

References

(1) Christofides N. : "Graph Theory", Academic Press (1975).
(2) Handler G. Y. and Mirchandani P. B. : "Location on Networks", MIT Press (1979).
(3) Sengoku M. and Shinoda S. : "A Theory and an Algorithm for Fault Diagnosis of Directed Networks", IEEE, Proc ISCAS'86 (San Jose, CA), pp. 639-642 (May 1986).
(4) Minieka E. : "Optimization Algorithms for Networks and Graphs", Marcel Dekkers (1978).



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