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Some Covering Problems in Location Theory on Flow Networks

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SUMMARY Location theory on networks is concerned with the problem of selecting the best location in a specified network for facilities. Many studies for the theory have been done. However, few studies treat location problems on networks from the standpoint of measuring the closeness between two vertices by the capacity (maximum flow value) between two vertices. This paper concerns location problems, called covering problems on flow networks. We define two types of covering problems on flow networks. We show that covering problems on undirected flow networks and a covering problem on directed flow networks are solved in polynomial times.
key words: *graphs and networks, flow network, location theory, covering problem, maximum flow*

1. Introduction

Location theory[(1)] on networks is concerned with the problem of selecting the best location in a specified network for facilities. Many studies for the theory have been done. Most of these studies treat location problems on networks from the standpoint of measuring the closeness between two vertices by the distance between two vertices. On the other hand, few studies treat location problems on networks from the standpoint of measuring the closeness between two vertices by the capacity (maximum flow value) between two vertices.

This paper concerns location problems, called covering problems on flow networks. In Ref. (2), we defined a covering problem on flow networks and proposed an algorithm to solve the covering problem on undirected flow networks. In this paper, we define another covering problem on flow networks. We propose algorithms to solve covering problems on undirected flow networks and a covering problem on directed flow networks. We show that these problem can be solved in polynomial times. These problems are applicable to assigning files to some computers in a computer network. For general terminology in graph theory, we refer the reader to Refs. (3) and (4).

2. Definitions

Let us consider an undirected flow network $N=(V, E, w_N)$ such that V, E and w_N are the vertex set, the edge set and the function assigning a positive real number $w_N(e)$, called edge-capacity, to each edge $e \in E$, respectively. The maximum flow value between two vertices u and v in N is called the capacity between u and v, denoted by $g_N(u, v)$. Especially, we define $g_N(v, v)=\infty$.

Next, we define the capacity $g_N(X, Y)$ between two vertex sets X and Y, as follows. If $X \cap Y=\phi$, we

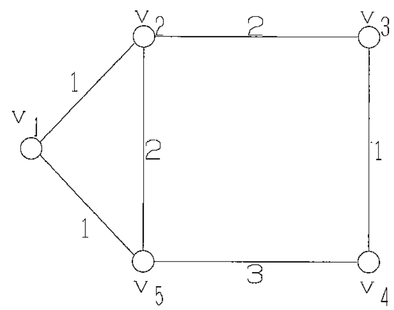

Fig. 1 An undirected flow network N.

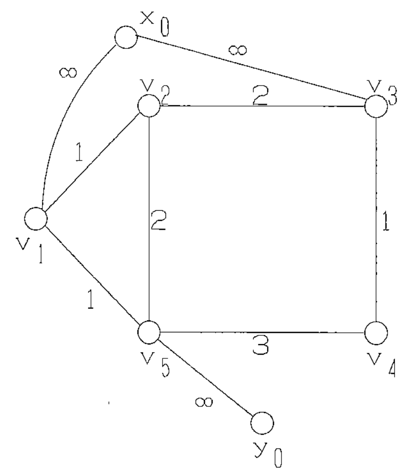

Fig. 2 An undirected flow network N'.

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construct an undirected flow network N' from N. The vertex set of N' is $V \cup \{x_0, y_0\}$. The edge set of N' is $E \cup \{(x_0, u) | u \in X\} \cup \{(v, y_0) | v \in Y\}$. For $u \in X$, $w_{N'}(x_0, u) = \infty$ and for $v \in Y$, $w_{N'}(v, y_0) = \infty$.

Let

$$g_N(X, Y) = g_{N'}(x_0, y_0).$$

If $X \cap Y \neq \phi$, we define $g_N(X, Y) = \infty$. $g_N(X, Y)$ is called the capacity between X and Y of N. Especially, if $Y = \{v\}$, the capacity between X and Y is also denoted by $g_N(X, v)$. Obviously, if $X = \{u\}$ and $Y = \{v\}$, $g_N(X, Y) = g_N(u, v)$.

In the undirected flow network N in Fig. 1, let $X = \{v_1, v_3\}$ and $Y = \{v_5\}$. The undirected flow network N' is shown in Fig. 2. Since $g_{N'}(x_0, y_0) = 4$, $g_N(X, Y) = 4$.

For the vertex sets W_1 and W_2 of N, where $W_1 \cap W_2 = \phi$, let

$$\langle W_1, W_2 \rangle = \{(u, v) \in E | u \in W_1, v \in W_2\}.$$

$\langle W_1, W_2 \rangle$ is called the cut of W_1 and W_2. Let

$$c(W_1, W_2) = \sum_{(u,v) \in \langle W_1, W_2 \rangle} w_N(u, v).$$

$c(W_1, W_2)$ is called the cut capacity of the cut $\langle W_1, W_2 \rangle$. If $W_2 = V - W_1$, $u \in W_1$ and $v \in W_2$, then $\langle W_1, W_2 \rangle$ is also called the cut separating u and v. If $W_2 = V - W_1$, $X \subset W_1$ and $Y \subset W_2$, then $\langle W_1, W_2 \rangle$ is also called the cut separating X and Y. If $\langle W_1, W_2 \rangle$ is a cut separating u and v and $c(W_1, W_2)$ is the minimum cut capacity of all cuts separating u and v, then $\langle W_1, W_2 \rangle$ is called the minimum cut separating u and v. The minimum cut separating two vertex sets X and Y is defined similarly.

The following theorems[5] are the fundamental theorems concerning maximum flow.

Theorem 1: Let N be an undirected flow network and let x, y and z be vertices. Then the following expressions hold.

(i) $g_N(x, y) = g_N(y, x)$,

(ii) $g_N(x, y) \geqq \min\{g_N(x, z), g_N(z, y)\}$. □

The following theorem is called the max-flow min-cut theorem.

Theorem 2: In any undirected flow network, the capacity between two vertices x and y is equal to the minimum cut capacity of all cuts separating x and y. □

Obviously, the similar result holds in the case of the capacity between two vertex sets.

Theorem 3: In any undirected flow network, the capacity between two vertex sets X and Y where $X \cap Y = \phi$ is equal to the minimum cut capacity of all cuts separating X and Y. □

In a communication network, a vertex represents a terminal computer and an edge represents a link between computers. We assign a file to computers of the network. The file is copied and assigned to some computers. How do we assign these files? We assume that the delay time to transport the data of a file can be ignored in this network. In this case, for each terminal computer pair, the number of links between the two computers is the measure representing the closeness between the two computers. Location theory on flow networks is applicable to the above case.

There exist various conditions to assign the files. Therefore various assignment problems are considered. Here, we consider the following two cases in this paper.

(a) For each computer, the file which can be used by the computer is only one.

(b) For each computer, the computer can use any file.

The assignment problem in the case (a) is the following.

Definition 1: Let N be an undirected flow network and r be a positive real number. A subset U of V is called a single cover with r if for any $v \in V$, there exists $u \in U$ such that $g_N(u, v) \geqq r$. If U is a single cover with r and $|U|$ is the minimum cardinal number of all single covers with r, then U is called an r-single cover. We simply call the problem of finding an r-single cover the r-single cover problem. □

The single cover problem was defined in Ref. (2). The assignment problem in the case (b) is the following.

Definition 2: Let N be an undirected flow network and r be a positive real number. A subset U of V is called a plural cover with r if $g_N(U, v) \geqq r$ for any $v \in V$. If U is a plural cover with r and $|U|$ is the minimum cardinal number of all plural covers with r, then U is called an r-plural cover. We simply call the problem of finding an r-plural cover the r-plural cover problem. □

3. Covering Problems on Undirected Flow Networks

3.1 Single Cover Problem

Definition 3: Let N be an undirected flow network. If W is a nonempty subset of V where $W = V$ or $g_N(v, w) < r$ for any $v \in V - W$, $w \in W$, then W is called an r-insufficient set with single cover. If W is an r-insufficient set with single cover and there does not exist a proper subset W' of W such that W' is an r-insufficient set with single cover, then W is called a minimal r-insufficient set with single cover. □

In Fig. 3, Let $r=5$. $W = \{v_2, v_3, v_5\}$ is an r-insufficient set with single cover. However, W is not minimal. $\{v_3\}$ and $\{v_2, v_5\}$ are minimal r-insufficient sets with single cover included in W.

Lemma 1: Let N be an undirected flow network and $v_0 \in V$. Let

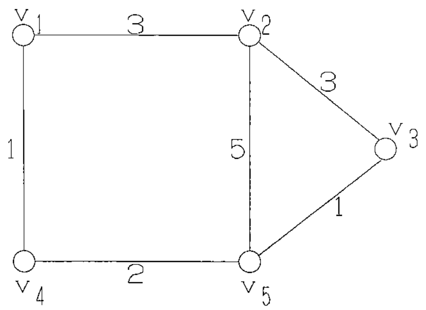

Fig. 3 An undirected flow network N illustrating an r-insufficient set with single cover.

$$V_0=\{u\in V|g_N(u, v_0)\geqq r\}.$$

Then, V_0 is a minimal r-insufficient set with single cover.

Proof: First, we show that V_0 is an r-insufficient set with single cover. If $V_0=V$, then V_0 is an r-insufficient set with single cover from the definition. If $V_0\neq V$, then $g_N(v', v'')<r$ for any $v'\in V-V_0$, $v''\in V_0$. The reason is the following. We assume that $g_N(v', v'')\geqq r$. Since $g_N(v'', v_0)\geqq r$, $g_N(v', v_0)\geqq r$ from Theorem 1. This contradicts the fact that $v'\in V-V_0$. Therefore V_0 is an r-insufficient set with single cover.

Next, we show that V_0 is minimal. Since $g_N(v, v_0)\geqq r$ and $g_N(v', v_0)\geqq r$ for any vertex v, $v'\in V_0$, $g_N(v, v')\geqq r$ from Theorem 1. Let V' be a proper subset of V_0, and let $u'\in V'$ and $u\in V_0-V'$. Since $g_N(u, u')\geqq r$, V' is not an r-insufficient set with single cover. Therefore, V_0 is a minimal r-insufficient set with single cover. □

From Lemma 1, we obtain a necessary and sufficient condition for U to be a single cover with r.

Lemma 2: Let N be an undirected flow network and U be a subset of V. A necessary and sufficient condition for U to be a single cover with r is the following.

For any r-insufficient set with single cover W, there holds $W\cap U\neq\phi$.

Proof: Let U be a single cover with r and W be an r-insufficient set with single cover. We assume that $W\cap U=\phi$. Let $w\in W$. For any $u\in U$, $u\in V-W$. Since W is an r-insufficient set with single cover, $g_N(u, w)<r$. Therefore U is not a single cover with r. This is a contradiction. Thus $W\cap U\neq\phi$.

Conversely, we assume that for any r-insufficient set with single cover W, there holds $W\cap U\neq\phi$. Let $v_0\in V$ and $V_0=\{u\in V|g_N(u, v_0)\geqq r\}$. From Lemma 1, V_0 is an r-insufficient set with single cover. From the above assumption, $V_0\cap U\neq\phi$. Let $u\in V_0\cap U$. $g_N(u, v_0)\geqq r$ from the definition of V_0. Therefore, U is a single cover with r. □

From Lemma 2, we obtain the following corollary, immediately.

Corollary 2: Let N be an undirected flow network and U be a subset of V. A necessary and sufficient condition for U to be a single cover with r is the following.

For any minimal r-insufficient set with single cover W, there holds $W\cap U\neq\phi$. □

Lemma 3: Let N be an undirected flow network and W be a minimal r-insufficient set with single cover.

Then $g_N(u, v)\geqq r$ for any u, $v\in W$.

Proof: We assume that there exist vertices u_0 and v_0 such that u_0, $v_0\in W$ and $g_N(u_0, v_0)<r$. Let $V_0=\{u\in V|g_N(u, v_0)\geqq r\}$. Since $v_0\in W$ and W is an r-insufficient set with single cover, V_0 is a subset of W. Since $u_0\in W-V_0$, V_0 is a proper subset of W. From Lemma 1, V_0 is an r-insufficient set with single cover, contradicting the fact that W is minimal. Therefore $g_N(u, v)\geqq r$ for any u, $v\in W$. □

Lemma 4: Let N be an undirected flow network. If W_1 and W_2 are minimal r-insufficient sets with single cover where $W_1\neq W_2$, then $W_1\cap W_2=\phi$.

Proof: We assume that $v\in W_1\cap W_2$. Since W_2 is minimal, there exists a vertex u such that $u\in W_1-W_2$. Since u, $v\in W_1$ and W_1 is minimal, $g_N(u, v)\geqq r$ from Lemma 3. However, $g_N(u, v)<r$ since $u\in V-W_2$ and $v\in W_2$. This is a contradiction. Therefore $W_1\cap W_2=\phi$. □

From Lemma 2, 4 and Corollary 2, we obtain the following theorem.

Theorem 4: Let N be an undirected flow network and U be a single cover with r. If there does not exist a proper subset U' of U such that U' is a single cover with r, then U is an r-single cover.

Proof: Let $W_1, \cdots$, and W_t be all the minimal r-insufficient sets with single cover. Let U^* be an r-single cover. From Corollary 2 and Lemma 4, $|U^*|=t$. Since there does not exist a proper subset U' of U such that U' is a single cover with r, $|W_i\cap U|=1$ for any i. Therefore $|U|=t$. U is an r-single cover from the definition. □

From Theorem 4, an r-single cover on an undirected flow network N can be obtained by the following simple algorithm.

procedure SINGLE_COVER(r)
  begin
    $U:=V$; (* $V=\{v_1, \cdots, v_n\}$ *)
    for $i=1$ to n do
      begin
        if $U-\{v_i\}$ is a single cover with r
        then $U:=U-\{v_i\}$
      end
  end. (* U is an r-single cover of N *)

We obtain the capacities between every vertex pairs in $O(|V|s(|V|, |E|))$ time[(6)], where $s(|V|, |E|)$ is the time required to solve a maximum flow problem in N (the best time bound for $s(|V|, |E|)$ known to date is $O(|V||E|\log(|V|^2/|E|))$[(7)]). Since judging whether $U-\{v_i\}$ is a single cover with r or not can be obtained in $O(|V|^2)$ time, the time complexity of SINGLE_COVER(r) is $O(|V|s(|V|, |E|))$. This algorithm is very simple and applicable to the covering problem on directed flow networks (see the next section). In Ref. (2), the other algorithm to obtain an r-single cover on an undirected flow network was proposed. For any undirected flow network N, there exists a tree flow network T such that $g_N(u, v) = g_T(u, v)$ for any vertex pair. The algorithm uses this property. The time complexity of the algorithm is also $O(|V|s(|V|, |E|))$.

3.2 Plural Cover Problem

Definition 4: Let N be an undirected flow network. If W is a nonempty subset of V where $W = V$ or $c(W, V-W) < r$, then W is called an r-insufficient set with plural cover. If W is an r-insufficient set with plural cover and there does not exist a proper subset W' of W such that W' is an r-insufficient set with plural cover, then W is called a minimal r-insufficient set with plural cover. □

In Fig. 4, Let $r=5$. $W=\{v_1, v_2, v_5\}$ is an r-insufficient set with plural cover. However, W is not minimal. $\{v_1\}$ is a minimal r-insufficient set with plural cover included in W. There does not exist a minimal r-insufficient set with plural cover including $\{v_2\}$. For any vertex v, there does not always exist a minimal r-insufficient set with plural cover including $\{v\}$.

We obtain similar results in the case of the single cover problem.

Lemma 5: Let N be an undirected flow network and U be a subset of V. A necessary and sufficient condition for U to be a plural cover with r is the following.

For any r-insufficient set with plural cover W, there holds $W \cap U \neq \phi$.

Proof: Let U be a plural cover with r and W be an r-insufficient set with plural cover. We assume that $W \cap U = \phi$. Since $c(W, V-W) < r$ and $W \cap U = \phi$, $g_N(U, W) < r$. Then $g_N(U, v) < r$ where $v \in W$. U is not a plural cover with r. This is a contradiction. Therefore $W \cap U \neq \phi$.

Conversely, we assume that for any r-insufficient set with plural cover W, there holds $W \cap U \neq \phi$. We call the assumption Assumption (A). We assume that U is not a plural cover with r. There exists a vertex v such that $g_N(U, v) < r$. From Theorem 3, the capacity $g_N(U, v)$ is equal to the minimum cut capacity $c(W_1, W_2)$ of all cuts separating U and v. Namely, $g_N(U, v) = c(W_1, W_2)$. W_2 is an r-insufficient set with plural cover. Since $U \subset W_1$, $U \cap W_2 = \phi$. This contradicts Assumption (A). Therefore, U is a plural cover with r. □

From Lemma 5, we obtain the following corollary, immediately.

Corollary 5: Let N be an undirected flow network and U be a subset of V. A necessary and sufficient condition for U to be a plural cover with r is the following.

For any minimal r-insufficient set with plural cover W, there holds $W \cap U \neq \phi$. □

Lemma 6: Let N be an undirected flow network. If W_1 and W_2 are minimal r-insufficient sets with plural cover where $W_1 \neq W_2$, then $W_1 \cap W_2 = \phi$.

Proof: We assume that $W_1 \cap W_2 \neq \phi$. Since W_1 and W_2 are minimal r-insufficient sets with plural cover, $W_1 - W_2 \neq \phi$ and $W_2 - W_1 \neq \phi$. Let

$$a_1 = c(W_1 - W_2, W_2 - W_1),$$
$$a_2 = c(W_1 - W_2, W_1 \cap W_2),$$
$$a_3 = c(W_2 - W_1, W_1 \cap W_2),$$
$$a_4 = c(W_1 - W_2, V - W_1 - W_2),$$
$$a_5 = c(W_1 \cap W_2, V - W_1 - W_2),$$
$$a_6 = c(W_2 - W_1, V - W_1 - W_2).$$

On the network in Fig. 5, the vertex set is $\{W_1 - W_2, W_2 - W_1, W_1 \cap W_2, V - W_1 - W_2\}$ and each edge weight represents the cut capacity. For example, the edge weight of $(W_1 - W_2, W_2 - W_1)$ is $c(W_1 - W_2, W_2 - W_1)$. Since W_1 and W_2 are r-insufficient sets with plural cover,

$$a_1 + a_3 + a_4 + a_5 < r, \tag{1}$$

$$a_1 + a_2 + a_5 + a_6 < r. \tag{2}$$

We can see from the definitions of W_1 and W_2 that neither $W_1 - W_2$ nor $W_2 - W_1$ are r-insufficient sets with

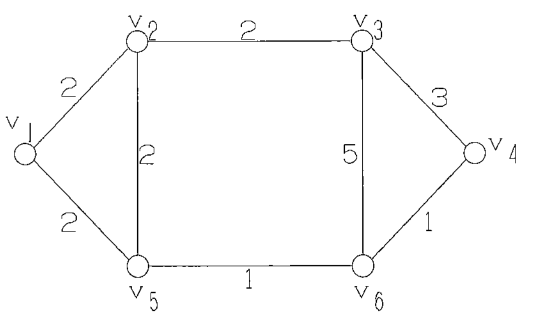

Fig. 4 An undirected flow network N illustrating an r-insufficient set with plural cover.

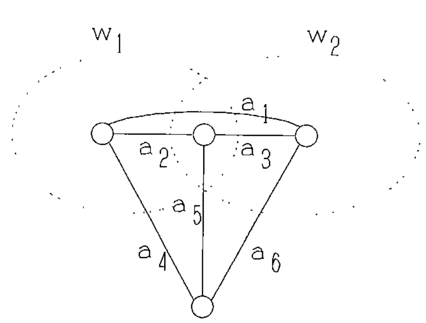

Fig. 5 Explanation for the proof of Lemma 6.

plural cover. Therefore,

$$a_1+a_2+a_4 \geqq r, \quad (3)$$

$$a_1+a_3+a_6 \geqq r. \quad (4)$$

(1)+(2)

$$2a_1+a_2+a_3+a_4+2a_5+a_6<2r. \quad (5)$$

(3)+(4)

$$2a_1+a_2+a_3+a_4+a_6 \geqq 2r. \quad (6)$$

From Eqs. (5) and (6), $2a_5<0$. This contradicts the fact that $a_5 \geqq 0$. Therefore $W_1 \cap W_2=\phi$. □

From Lemma 5, 6 and Corollary 5, we obtain the following theorem. Since the proof of Theorem 5 is similar to the proof of Theorem 4, we omit the proof.

Theorem 5: Let N be an undirected flow network and U be a plural cover with r. If there does not exist a proper subset U' of U such that U' is a plural cover with r, then U is an r-plural cover. □

From Theorem 5, an r-plural cover on an undirected flow network N can be obtained by the following simple algorithm.

procedure PLURAL_COVER(r)
  begin
    $U:=V$; (* $V=\{v_1, \cdots, v_n\}$ *)
    for $i=1$ to n do
      begin
        if $U-\{v_i\}$ is a plural cover with r
        then $U:=U-\{v_i\}$
      end
  end. (* U is an r-plural cover of N *)

Judging whether $U-\{v_i\}$ is a plural cover with r or not can be obtained in $O(|V|s(|V|, |E|))$ time. Therefore, the time complexity of PLURAL_COVER (r) is $O(|V|^2 s(|V|, |E|))$.

4. Covering Problems on Directed Flow Networks

We have defined covering problems on undirected flow networks in Sect. 3. We define covering problems on directed flow networks similarly and we consider these covering problems. In this section, $g_N(u, v)$ denotes the capacity from u to v in a directed flow network N.

4.1 Single Cover Problem

Definition 5: Let N be a directed flow network and r be a positive real number. A subset U of V is called a single cover with r if for any $v \in V$, there exists $u \in U$ such that $g_N(u, v) \geqq r$ and $g_N(v, u) \geqq r$. If U is a single cover with r and $|U|$ is the minimum cardinal number of all single covers with r, then U is called an r-single cover. We simply call the problem of finding an r-single cover the r-single cover problem. □

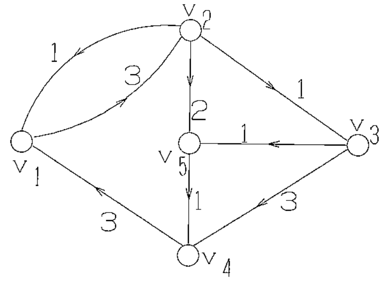

Fig. 6 A directed flow network N illustrating an r-insufficient set with single cover.

Definition 6: Let N be a directed flow network. A nonempty subset W of V is satisfied the following condition.

$$W=V;$$

or for any $v \in V-W$, $w \in W$,

$$g_N(v, w)<r \text{ or } g_N(w, v)<r.$$

Then, W is called an r-insufficient set with single cover. If W is an r-insufficient set with single cover and there does not exist a proper subset W' of W such that W' is an r-insufficient set with single cover, then W is called a minimal r-insufficient set with single cover. □

In Fig. 6, let $r=3$. $W=\{v_1, v_2, v_4\}$ is an r-insufficient set with single cover. However, W is not minimal. $\{v_1, v_2\}$ and $\{v_4\}$ are minimal r-insufficient sets with single cover included in W.

We obtain the following results in the single cover problem on directed flow networks. These proofs are similar to the proofs of Lemma 2, 4 and Theorem 4. Therefore we omit the proofs.

Lemma 7: Let N be a directed flow network and U be a subset of V. A necessary and sufficient condition for U to be a single cover with r is the following.

For any r-insufficient set with single cover W, there holds $W \cap U \neq \phi$. □

Lemma 8: Let N be a directed flow network. If W_1 and W_2 are minimal r-insufficient sets with single cover where $W_1 \neq W_2$, then $W_1 \cap W_2=\phi$. □

Theorem 6: Let N be a directed flow network and U be a single cover with r. If there does not exist a proper subset U' of U such that U' is a single cover with r, then U is an r-single cover. □

From Theorem 6, an r-single cover on a directed flow network can be obtained by using SINGLE_COVER(r). Since we obtain the capacities between every ordered vertex pair in $O(|V|^2 s(|V|, |E|))$ time, the time complexity of SINGLE_COVER(r) is $O(|V|^2 s(|V|, |E|))$ in the case of directed flow networks.

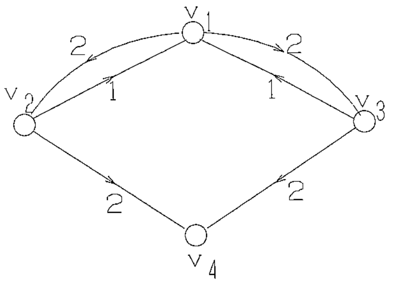

Fig. 7 A directed flow network N.

4. 2 Plural Cover Problem

First, we define the capacity $g_N(X, Y)$ from a vertex set X to a vertex set Y, as follows. If $X \cap Y = \phi$, we construct a directed flow network N' from N. The vertex set of N' is $V \cup \{x_0, y_0\}$. The (directed) edge set of N' is $E \cup \{(x_0, u) \mid u \in X\} \cup \{(v, y_0) \mid v \in Y\}$. For $u \in X$, $w_{N'}(x_0, u) = \infty$ and for $v \in Y$, $w_{N'}(v, y_0) = \infty$.

Let

$$g_N(X, Y) = g_{N'}(x_0, y_0).$$

If $X \cap Y \neq \phi$, we define $g_N(X, Y) = \infty$. $g_N(X, Y)$ is called the capacity from X to Y of N. Especially, if $Y = \{v\}$, the capacity from X to Y is also denoted by $g_N(X, v)$. Obviously, if $X = \{u\}$ and $Y = \{v\}$, $g_N(X, Y) = g_N(u, v)$.

Definition 7: Let N be a directed flow network and r be a positive real number. A subset U of V is called a plural cover with r if $g_N(U, v) \geqq r$ and $g_N(v, U) \geqq r$ for any $v \in V$. If U is a plural cover with r and $|U|$ is the minimum cardinal number of all plural covers with r, then U is called an r-plural cover. We simply call the problem of finding an r-plural cover the r-plural cover problem. □

We show that an r-plural cover on a directed flow network is not always obtained by using PLURAL_COVER(r).

In Fig. 7, let $r=2$. $U = \{v_2, v_3, v_4\}$ is a plural cover with r. There dose not exist a subset U' of U such that U' is a plural cover with r. However, U is not an r-plural cover. $\{v_1, v_4\}$ is a unique r-plural cover. Therefore, in directed flow networks, Theorem 5 does not hold, namely, an r-plural cover on a directed flow network is not always obtained by using PLURAL_COVER (r). Actually, the output of PLURAL_COVER(r) in $U = \{v_2, v_3, v_4\}$ in N in Fig. 7.

5. Conclusion

We have given the definitions of covering problems called the single cover problem and the plural cover problem on flow networks. These problems are applicable to assigning files to some computers in a computer network. We have shown that these covering problems on undirected flow networks and a single cover problem on directed flow networks are solved in polynomial times. In this paper, we treat only a single cover problem on directed flow networks. There are some other covering problems on directed flow networks to be solved in future.

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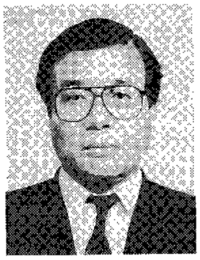

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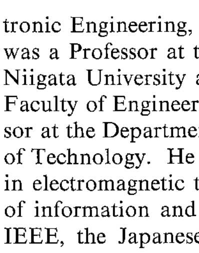

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