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Fast Wavelet Transform and Its Application to Detecting Detonation

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SUMMARY Fast wavelet transform is presented for real-time processing of wavelet transforms. A processor for the fast wavelet transform is of the frequency sampling structure in architectural level. The fast wavelet transform owes its parallelism both to the frequency sampling structure and parallel tapping of a series of delay elements. Computational burden of the fast transform is hence independent of specific scale values in wavelets and the parallel processing of the fast transform is readily implemented for real-time applications. This point is quite different from the computation of wavelet transforms by convolution. We applied the fast wavelet transform to detecting detonation in a vehicle engine for precise real-time control of ignition advancement. The prototype wavelet for this experiment was the Gaussian wavelet (i.e. Gabor function) which is known to have the least spread both in time and in frequency. The number of complex multiplications needed to compute the fast wavelet transform over 51 scales is 714 in this experiment, which is less than one tenth of that required for the convolution method. Experimental results have shown that detonation is successfully detected from the acoustic vibration signal picked up by a single knock sensor embedded in the outer wall of a V/8 engine and is discriminated from other environmental mechanical vibrations.

key words: digital signal processing, wavelets, signals and waves, acoustics

1. Introduction

Ignition advancement control in vehicle engines is indispensable to high power-weight ratio and clean emission. Control failure causes detonation in the chamber of an engine. Detonation is an abnormal phenomenon: after ignition by a spark plug, before-burning air-fuel explodes by itself around the inner wall of an engine vessel and the self-explosions happen suddenly with irregular time intervals. Detonation creates strong pressure waves and can destroy an engine body, if it successively happens to a heavy degree. Detecting detonation is thus of critical importance for ignition advancement control.

Detonation is usually observed as acoustic vibration of the engine body in commercial vehicles. This vibration unfortunately contains many other mechani-

cal noises produced by the rapid motion of piston heads, gear system, and valve system. Detonation needs to be discriminated from other noise components and has to be detected through analysis in the frequency domain as well as in the time domain. Of course each individual burning process of air-fuel exhibits quite different modes from the others, every time depending on cylinders, rotation speed, acceleration, air-fuel ratio, road environment, temperature, humidity, and so on. Detonation signals are highly nonstationary, hence statistical analysis does not always give correct results for the best control of each particular ignition. Detonation signal analysis essentially requires real-time analysis of time-frequency instantaneous distribution under a low signal-to-noise environment.

On the other hand, wavelet transform⁽¹⁾⁻⁽¹³⁾ is a new tool for time-frequency analysis, exactly speaking for time-scale analysis. The wavelet transform of a signal is an expansion of the signal into a special family of functions called as *wavelets*. One can extract some information about a signal with respect to time and frequency, since wavelets exist locally both in the time domain and the frequency domain. A wavelet spans on a segment along with the time axis and also does the same along with the frequency axis. This is completely different from the periodic basis functions such as exponential and Walsh functions. A family of wavelets is generated from a single prototype wavelet by dilation of time and by translation along the time axis.

Owing to their good localization and dilation/translation, wavelets are capable of detecting discontinuities in a signal waveform and its derivatives, hence are suitable to explore abrupt changes in transient signals^{(1)-(7),(9)-(11)}. Also, wavelets provide robust multi-resolution representation⁽²⁾⁻⁽¹¹⁾ (MRR); the wavelet transform offers sharp time-resolution for rapidly changing signal components and fine frequency resolution for slowly varying components. It is difficult to find these desirable properties of singularity detection, MRR, and numerical stability^{(2),(3),(6)} in other existing methods including short-time Fourier transforms and several distributions^{(14),(15)} affiliated to the Cohen's class. The above statements are the major

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reasons why we use wavelets to analyze detonation⁽¹²⁾.

Regarding detecting detonation we have another reason. Statistical analysis is quite different from the analysis by transforms and was actually experimented. Sometimes it was successful but sometimes it failed in detecting the nonstationary detonation at every explosion. In addition, the statistical method was used to be too time-consuming to control every the next explosion.

By definition, the wavelet transform with respect to non-orthogonal wavelets can be computed by convolving a signal with every wavelet. However the larger the dilation scale goes, the more multiplications and additions are needed. It is thus difficult to apply the convolution scheme for wavelet transforms to those problems which impose fast and compact processing such as detonation detection.

For real-time processing, this paper presents a scheme for *fast wavelet transforms*⁽¹³⁾. It can be identified with a wavelet transform processor. The processor is capable of computing non-orthogonal wavelet transforms with a constant number of multiplications independent of specific scales. If the fast wavelet transform is implemented in a parallel processing system, the computation time per sample can be reduced to that spent for two complex multiplications. The processing speed is thus much faster than that of the conventional implementation of convolution.

After wavelet transforms are overviewed in the next section, we describe the fast wavelet transform in Sect. 3. An application of the fast wavelet transform is given in Sect. 4 where a fast wavelet transform processor is designed for the Gaussian wavelet^{(3),(6)} as a basic wavelet and a successful experiment for detonation detection is shown.

2. Wavelet Transforms

The wavelet transform⁽¹⁾ of a signal is computed by expanding the signal onto a basis of wavelets. The expansion coefficients describe the distribution of the signal on the time-scale plane: roughly speaking, on the time-frequency plane. A given basic wavelet $g(t)$ is scaled by a in the time domain and is translated by b to generate a basis family

$$g_{a,b}(t) = \frac{1}{\sqrt{a}} g\left(\frac{t-b}{a}\right) \tag{1}$$

The basic wavelet $g(t)$ is selected as a function such that $g(t)$ spans its amplitude around the origin and vanishes apart from the origin. This must be true in both domains of time and frequency. In fact, a wavelet decays at least faster than the reciprocal of time^{(2)-(7),(9),(10)}. Hence all the wavelets $g_{a,b}(t)$ are also localized functions both in the time domain and in the frequency domain. Typical modes of wavelets in the time domain and in the frequency domain are illus-

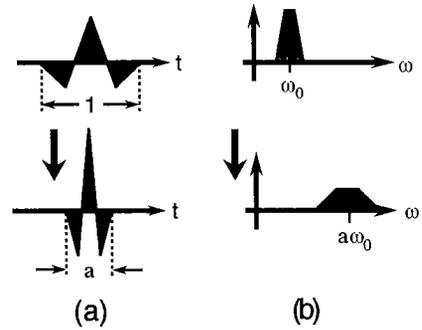


Fig. 1 Dilation of wavelets, (a) in time, (b) in frequency.

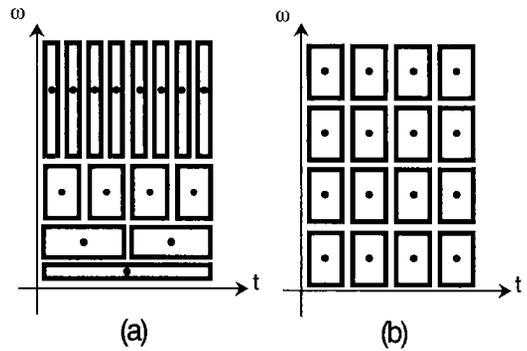


Fig. 2 Resolution in time and frequency, (a) of wavelet transforms, (b) of Fourier transforms.

trated in Fig. 1. The discrete-time wavelet transform of a signal $s(t)$ is defined by

$$S(a, b) = \sum_{n=-\infty}^{\infty} \bar{g}_{a,b}(nT) s(nT) \tag{2}$$

where T is a sampling period and the top bar denotes complex conjugate. $S(a, b)$ represents an expansion coefficient of the signal with respect to the wavelet $g_{a,b}(nT)$. Strictly speaking, we are considering the discrete-time version of discrete wavelet transform (i.e. the wavelet frame expansion^{(2),(3)}) that may be orthonormal and non-orthogonal.

Wavelet transforms are well compared with Fourier transforms in resolution space with respect to time and frequency^{(3),(4)}. The time resolution and frequency resolution of the short-time Fourier transform are uniformly fixed by the uncertainty relationship between time and frequency, as shown in part (b) of Fig. 2. By contrast, the wavelet transform has fine frequency resolution for low frequency components, because these components are analyzed over long periods. Higher frequency components are observed during shorter periods, and hence the frequency resolution is degraded but the time resolution is improved, as illustrated in part (a) of Fig. 2.

Most random signals and transient signals contain rapid changes (higher frequency components) and the duration periods of important changes are short but

background noises might be extended over longer periods with strong intensity. One of the objectives in analyzing those signals is to extract when some characteristic changes happen and how long period they remain before vanishing. The short-time Fourier transform with a particular window can analyze an appropriate frequency component during a particular period. However, if a signal varies discontinuously, this transform produces a too-much regularly-smoothed⁽¹¹⁾ spectrum and the occurrence of the discontinuous change will be smeared. The Wigner-Ville distribution analysis suffers from ghosts due to the cross term effect^{(14),(15)} in both time and frequency. Furthermore, if signal-to-noise ratio is low, there is no hope for these two types of analysis to detect abrupt changing components.

By contrast, wavelets respond quickly to abrupt changes and this is due to the essential property of singularity detection. This point is a distinct difference from the others. There exists another advantage in wavelets: robust reconstruction from the transform. Even for non-orthogonal wavelets, reconstruction is still possible, if the discrete grid in the a - b plane has a fine mesh, that is, if the frame is *snug*⁽³⁾. Wavelet transforms are thus qualified for analyzing random or transient signals, and the information gained through wavelet analysis is no other than what we hope to catch.

3. Fast Wavelet Transforms^{(12),(13)}

The wavelet defined by Eq.(1) has a finite duration period on the time axis. Hence the expansion coefficients into translated wavelets with a particular scale correspond to a sequence of the output of a finite-duration impulse response (FIR) filter. There is a well known frequency sampling structure⁽¹⁶⁾ to implement FIR filters, as shown in Fig. 3. It consists of a cascade of a comb filter to form equi-spaced zeros and several resonators to produce poles on the unit circle. It is of benefit to identify the comb filter with a switch to drive the following resonators initially with in-

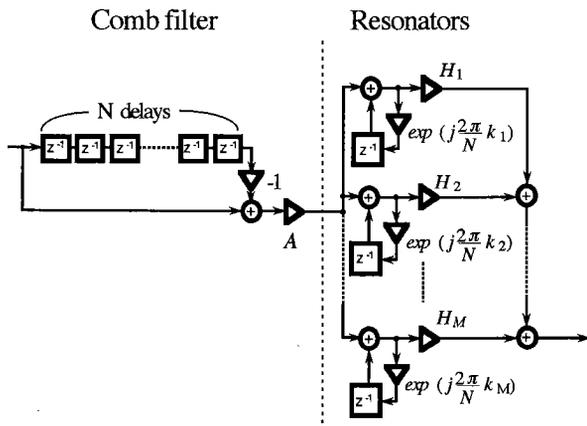


Fig. 3 Frequency sampling structure.

phase and then with anti-phase after some time lag, hence producing finite-duration oscillations⁽¹⁷⁾. The impulse response of the frequency sampling filter in Fig. 3 is of the form

$$h(nT) = A \sum_{i=1}^M H_i e^{jnT \frac{2\pi}{N} k_i} \quad \text{for } 0 \leq n < N$$

$$= 0 \quad \text{elsewhere} \tag{3}$$

where M and N stand for the numbers of resonators and the serial delay elements, respectively. A is a constant factor to adjust to normalization for the present, but it can be removed by combining it with the following multiplies H_i in actual implementations. M, N and resonant frequencies specified by k_i are designed to give an impulse response such as

$$h(nT) = \bar{g}(-nT) \tag{4}$$

where for simplicity $h(nT)$ is considered to be zero-phase. Then the output sequence of the frequency sampling system

$$y(nT) = \sum_{m=-\infty}^{\infty} \bar{g}(mT - nT) s(mT) \tag{5}$$

is equal to the expansion coefficients $S(1, nT)$ with respect to the wavelets with the dilation scale of $a=1$ and the translation shift of $b=nT$. In this paper, a frequency sampling filter of which impulse response is described by Eq.(4) will be referred to as a basic filter.

Filters corresponding to all the other wavelets with different scales are derived from the basic filter by varying the number of the serial delays, N , and the normalization factor A . To design such a filter for the wavelet with the scale of

$$a_i = \frac{N_i}{N} \tag{6}$$

it is enough to replace N with N_i and to change A with

$$A_i = \sqrt{\frac{N}{N_i}} \tag{7}$$

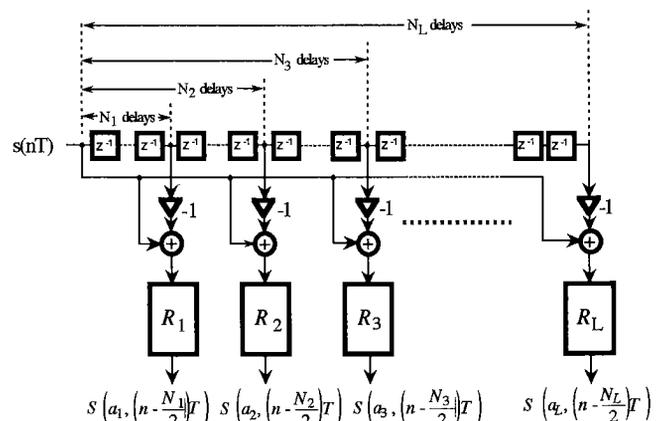


Fig. 4 A processor for the fast wavelet transform.

where N_i is an integer. As N is replaced with N_i and k_i is left to be unchanged, so resonant frequencies are shifted by a factor of the scale a_i .

Since the scale change is performed by changing the length of the serial delays in a comb filter, an appropriate tapping of the delay line can do the same effect as the scale change. Hence we get a wavelet transform processor as shown in Fig. 4. Every resonant block consists of the same number of parallel resonators and is labeled in R_i where $i=1, 2, 3, \dots, L$. A cascade of the comb filter prepared by the proper tapping and the resonant block R_i generates the wavelet with the scale of a_i as its impulse response.

This processor is capable of concurrently computing the wavelet transform at every scale. The processor involves the same number of arithmetic operations for every scale: the computational complexity of the fast wavelet transform presented here is independent of particular scales in wavelets. In fact, the number of complex multiplications needed to compute the fast transform is given by $2ML$ per sample: twice the number of resonators for a single wavelet times the number of scales.

In the convolution method for computing wavelet transforms, the required arithmetic operations increase in the number as the scale takes a larger value. Hence the fast wavelet transform offers much faster computation than the convolution method. It may be of significance to compare the complexity between the fast transform and subband coding scheme, though the subband coding is only applicable to the octave scale (i.e. 2^n), while the fast transform is capable of computing any number of scales per octave. If we make the comparison under the restriction of the octave scale, say the number of scales be L , and if the sum of the durations of two band-split filters is equal to $2M$, then the complexity would be the same for the subband coding and the fast transform.

Furthermore it is possible to implement a hardware system for the fast wavelet transform through easier VLSI layout because of the simplicity and regularity of the fast wavelet transform. Parallel processing of the fast wavelet transform can be readily performed in a multi-processor system or in a dedicated hardware. Because of the rich parallelism shown in Figs. 3 and 4, the fast wavelet transform per sample can be computed within the time spent on two multiplications.

4. Detonation Detection

The fast wavelet transform described in the previous section is designed for providing a time-scale analysis tool, and then it is applied to detecting detonation in vehicle engines.

4.1 Design of the Fast Wavelet Transform

The Gaussian Wavelet⁽³⁾ (Gabor function)

$$g(t) = e^{-t^2/2} (e^{j\Omega t} - e^{-\Omega^2/2}) \tag{8}$$

is employed as a basic wavelet, and the constant Ω is given by 2π that is greater than $\pi\sqrt{2/\ln 2}$ ⁽³⁾. The subtraction term in Eq.(8) ensures the *admissibility* and is negligible in practice. The reasons for this choice are threefold: because /1/ the Gaussian function has the least spread in both domains of time and frequency, /2/ it is not yet thoroughly investigated what extent is the best for analyzing detonation in both time and frequency, and /3/ the non-orthogonal wavelets, redundant frames, can offer numerically-stable reconstruction^{(2),(3)}, if the scale factor and the shift constant are close to one and zero, respectively. Especially the Gaussian wavelet is used to be preferred in singularity detection^{(1)-(3),(5),(6)}. For comparison, several results obtained by the other types of wavelets including orthogonal wavelets are added in Appendix.

Detonation is perceived by the human hearing sense, and the audible frequencies should be analyzed under 50 kHz sampling. Hence our analysis is targeted at the 5-octave band from 0.0125 Hz up to 0.4 Hz in the normalized frequency. As seen by simple calculation, the 128-point rectangular time-window suffices to approximate the Gaussian wavelet into the unity-scale basic wavelet of which center frequency is at 0.05 Hz. The windowed wavelet is shown in Fig. 5 where heavy and light curves display the real and imaginary parts, respectively. Then it is approximated in the frequency domain with a frequency sampling structure consisting of a series of delays and seven resonators. Resonant frequencies are specified by the coefficients k_i by selecting 7 frequencies at which frequencies Fourier coefficients have larger values. The weighting coefficients H_i are assigned to be equal to the 7 Fourier coefficients.

We have prepared 51 wavelets with each scale of

$$a_i = (10\sqrt{2})^i \tag{9}$$

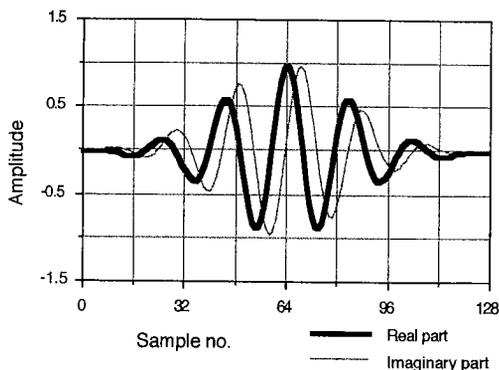


Fig. 5 The basic wavelet.

where i runs through -30 to 20 . Integer N_i , which gives the number of delays required for the dilation to the i th wavelet, is approximated by

$$N_i = Q[a_i N] \tag{10}$$

where Q denotes truncation. Parts (a) and (b) in Fig. 6 display the frequency responses of the minimum-scale wavelet and the maximum-scale wavelet in this application, respectively, each being compared with the frequency response of the basic wavelet. Figure 6 well illustrates that the frequency resolution of the wavelet transform is fine in low frequencies and is coarse in high frequencies.

The 51-scale wavelet transform is carried out with $2 \times 7 \times 51 = 714$ multiplications and additions per sample (see the previous section). In contrast, the direct convolution by the definition of wavelet transforms requires the computational complexity in proportion to the dilation scale. The basic wavelet spans its duration over 128 samples, and thus the convolution with the dilated wavelet of scale a_i needs $128a_i$ multiplications. The summation of these terms over i from -30 to 20 gives the total number of multiplications required for this application and it amounts to 7,398. The computational burden of this fast wavelet transform is less than one tenth of that for the convolution scheme.

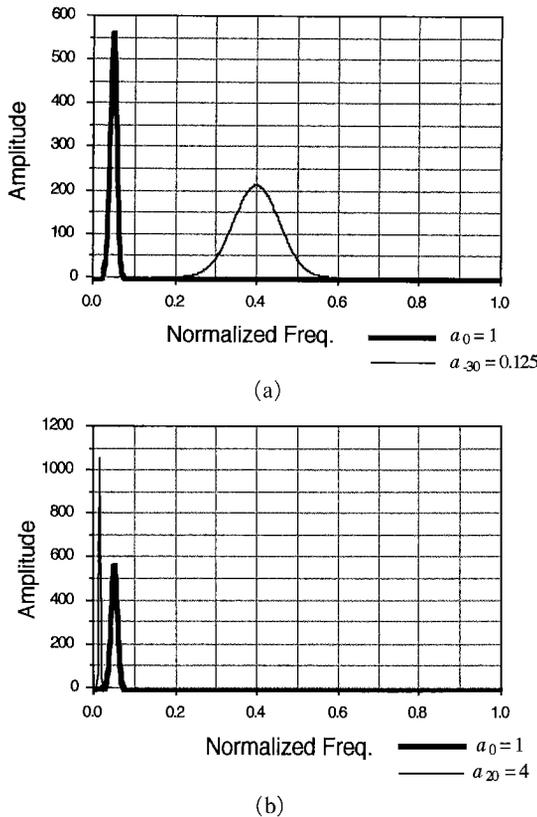


Fig. 6 Frequency responses of the wavelets, (a) with the scale of $a=0.125$, (b) with the scale of $a=4$, each being compared with that of the basic wavelet.

4.2 Experiment

The acoustic vibration observed on an engine body is picked up by a single pressure-voltage sensor embedded in the outer shell of a V/8 vehicle engine. An A/D converter samples the vibration signal at the rate of 50 kHz. Detonation is caused by misadjusting the ignition advancement of the first cylinder, which is most far from the sensor, under 1,200 rpm.

The detonation-free vibration signal after A/D conversion is shown in Fig. 7(a), where the duration of 50 samples corresponds to 1 ms. The amplitude and phase of the fast wavelet transform are shown in (b) and (c) of the figure, respectively. Figure 8 shows the

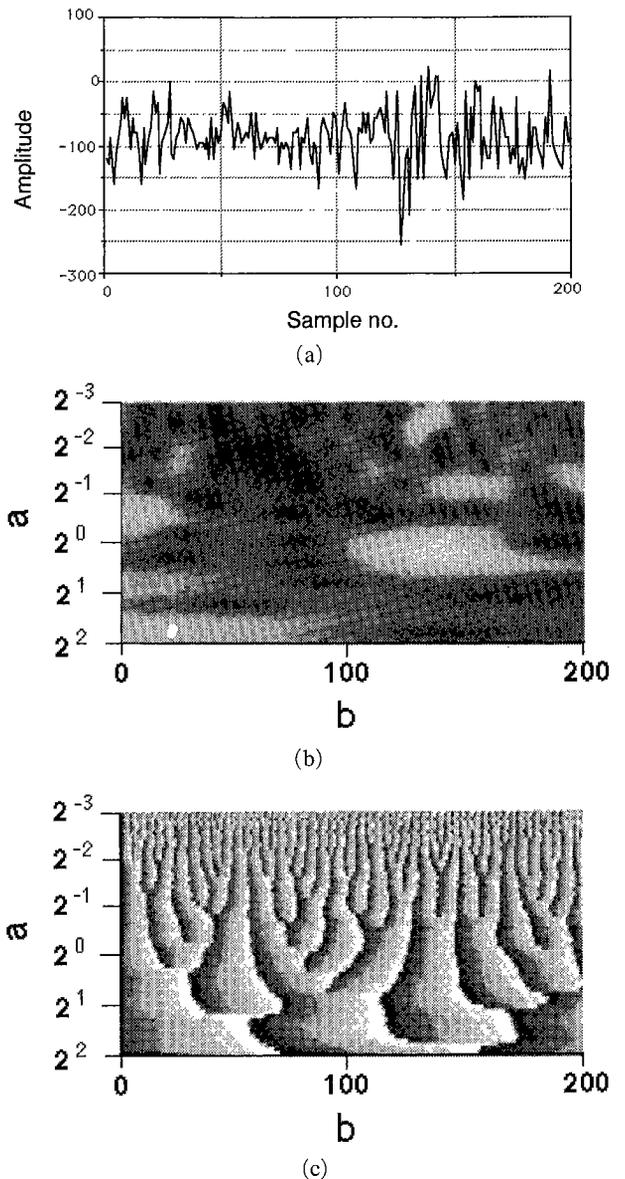


Fig. 7 Wavelet analysis of the detonation-free signal. (a) Original signal, (b) amplitude of the fast wavelet transform, (c) phase of the fast wavelet transform.

corresponding signal and transforms in the case of detonation. Amplitude in the transformed domain is represented in gray through dark to bright. The largest amplitude is displayed in white and the smallest is in black. The phase of the wavelet expansion coefficients is represented in the gradations of gray and the change from black to white corresponds to the phase shift of 2π . Figures 7 and 8 display the interval from 1.4 ms to 5.4 ms after ignition.

Comparing the amplitudes in Figs. 7(b) and 8(b), it is found that the components with the scales smaller than 2 grows high during the period from $b=120$ to $b=160$. These components are independent of the presence of detonation. Taking into account of the

additional information about engine motion, it is found that these are due to the mechanical vibration produced by slapping of piston against cylinder wall. Special behavior is found during the period through $b=50$ to $b=100$. As shown in Fig. 8(b), the wavelet expansion coefficients with scales of 1 through 2 grow high only under detonation. Also it is found in the same period that the wavelet expansions grow over the scales of 2^{-3} to 2^{-1} .

Concerning the phase of the wavelet expansion over $b=50$ to $b=100$, the detonation signal exhibits the coherent phase shifts of 2π over a wide range of scales around 1 in Fig. 8(c), whereas no remarkable phase shift other than disordered behavior is found in Fig. 7(c).

It is concluded that the detonation under 1,200 rpm of the experimented engine is successfully detected by the wavelet analysis with the dilation scales of 1 to 2 and of 2^{-3} to 2^{-1} during the period of 2.4 ms to 3.5 ms after each ignition.

5. Conclusions

An advantageous scheme for fast wavelet transforms is presented. It is also of benefit to identify the scheme with an architectural processor for computing discrete-time wavelet transforms. The fast wavelet transform has been applied to the detection of detonation in vehicle engines. Detonation was successfully detected by the fast wavelet analysis of the vibration signal on the engine body, and it was discriminated from other mechanical noise components over some dilation scales and during a particular period.

Through the experiment it has been demonstrated that the wavelet analysis provides a useful tool for visualizing transient signals. This is because of the fact that engineers involved with the experiment could see the process of appearance, growth and decay of particular detonations by looking at the behavior of the wavelet transforms.

Further investigation should be followed, which includes the comparison between the wavelet analysis and in-cylinder pressure analysis and statistical reasoning for the agreement between the wavelet analysis and human hearing sense for listening to detonation.

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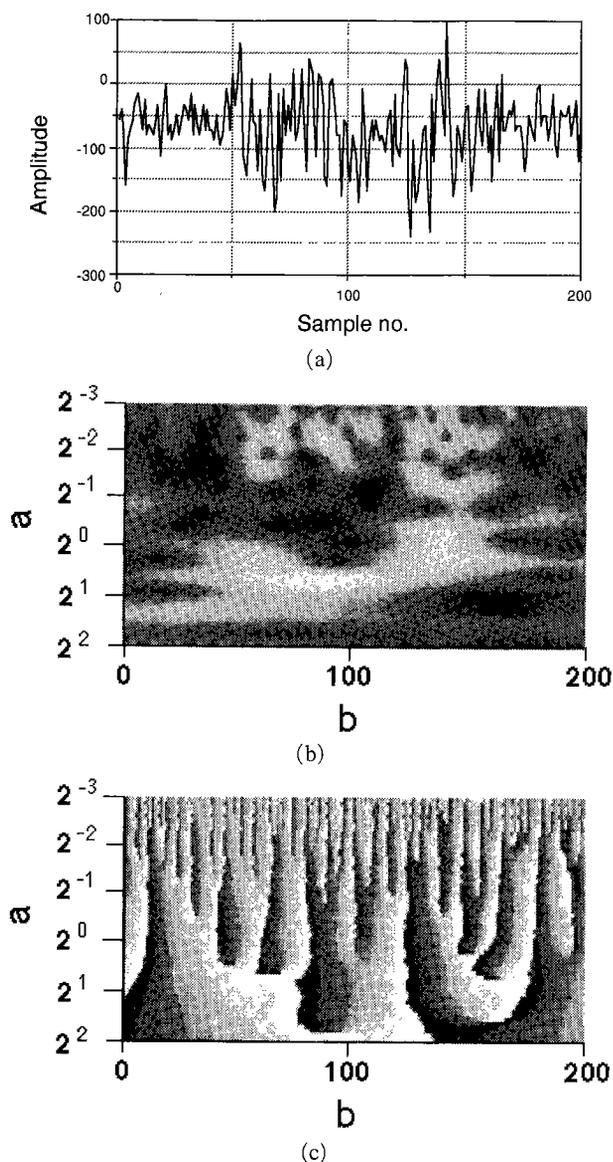


Fig. 8 Wavelet analysis of the abnormal signal under detonation. (a) Original signal, (b) amplitude of the fast wavelet transform, (c) phase of the fast wavelet transform.

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Appendix : More Examples

Three types of wavelets other than the Gaussian wavelet in Sect. 4 are applied to the same pair of the signals. Parts (a) and (b) in the following figures display the wavelet transforms of the detonation-free signal and the abnormal signal, respectively.

A.1 Another Gaussian Wavelet

Another Gaussian wavelet is designed as a Gaussian-windowed sinusoid expressed by

$$g(t) = e^{-t^2/2\sigma^2} e^{j2\pi ft}.$$

The duration, σ^2 , is fixed at 50 so that the Gaussian part may be sufficiently close to zero at the ends of the tails, $t = \pm 64$. The center frequency, f , is selected to be 6/128 so that the lower and upper normalized-frequencies, $2^{-2}f$ and 2^3f , may fit the audible frequency band under 50 kHz sampling. The resulting function corresponds to the Gaussian wavelet with $Q = 2.08$ in Eq.(8). Figure A·1 shows a significant improvement in time resolution compared with Figs. 7

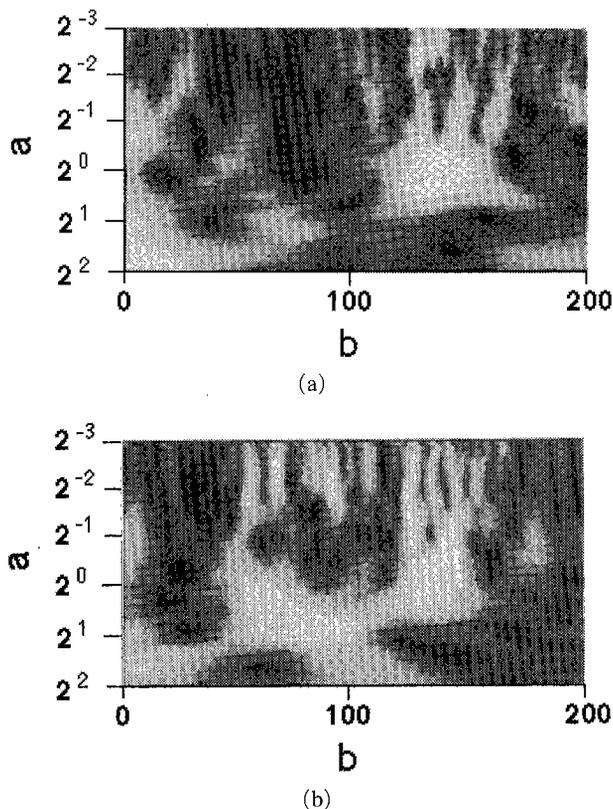


Fig. A·1 Wavelet analysis via another Gaussian wavelet.

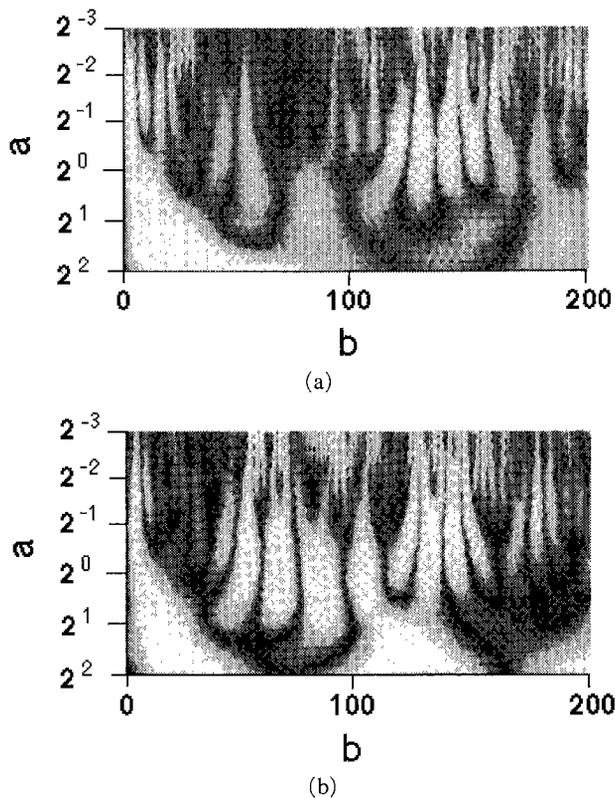


Fig. A·2 Wavelet analysis via the Mexican hat.

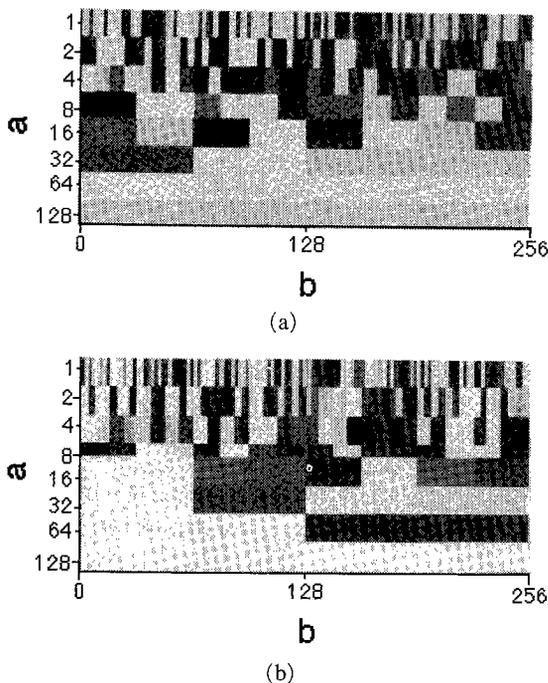


Fig. A-3 Wavelet analysis via the Haar wavelet.

and 8 at the cost of frequency resolution.

A.2 The Mexican Hat Wavelet

The Mexican hat wavelet⁽³⁾ is the second derivative of the Gaussian function, hence the Laplacian-Gaussian. The basic wavelet for the Mexican hat is also approximated by a 128-point FIR filter. Figure A-2 shows clear corn-like patterns in the vertical direction which are due to the sharp time-resolution in the Mexican hat. Frequency-resolution is inferior to the modulated Gaussian wavelets.

A.3 The Haar Wavelet: An Orthogonal Basis

Figure A-3 shows the discrete wavelet transforms based on the Haar function, which forms an orthogonal wavelet basis⁽³⁾. The mosaic patterns are too rich to understand the variation of the signals. This is because the orthogonal wavelet basis should be constructed with the strict scale factor of 2, and hence the shift step should grow in proportion to 2^n that is the reciprocal of the Nyquist scale. This example demonstrates that orthogonal wavelets are not always preferable more than non-orthogonal wavelets. Subband coding is well carried out by orthogonal wavelets^{(4),(8)}, but non-orthogonal wavelets will find a wide range of application because of their finer scales and denser time shifts.



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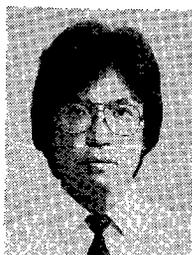
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