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Transient Characteristics of Mobile Communication Traffic in a Band-Shaped Service Area

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SUMMARY In a cellular system for mobile communications, every service area is divided into a number of cells for utilizing the frequency spectrum efficiently. Service areas for such systems are two dimensional, however, the analysis of the characteristics of the communication traffic for the areas are quite complicated, since the motion of the vehicles in the area can not be predicted precisely. For making the analysis easily, the areas are assumed to be band-shaped like a highway. Furthermore, in the analysis, the traffic offered to a cell is assumed to be stationary. In actual systems, the density of vehicles and the offered communication traffic is not stationary, so that many differences exist between the analysis and the actual systems. This paper presents an analysis method using state equations. The equations represent the transient characteristics of mobile communication traffic when a band-shaped service area is assumed. The transition is made by accidents or congestion, and causes the rapid offered traffic change in a communication system. In the method, numerical analysis is made under the consideration of "handoff" operation. The operation consists of surrendering the channel used in the previous cell and reassigning a new channel when the vehicle crosses the cell boundary. The analytical results are compared with the simulations, and the two results show good agreement. The method presented in this paper can be used for designing the switching system when the offered traffic changes rapidly due to accidents or congestion.

key words: *cellular system, band-shaped service area, state equations, handoff*

1. Introduction

A cellular system is used in mobile communications for utilizing the frequency spectrum efficiently.[1],[2] In the cellular system, service areas are divided into a number of cells, and the areas are generally two dimensional. However, the analysis for the characteristics of the communication traffic in the two-dimensional service area is quite difficult since the motion of the vehicles in the area can not be predicted precisely. For making the analysis easily, the areas are assumed to be band-shaped like a highway, and traffic offered to a cell is stationary. Under these assumptions, the characteristics can be analyzed easily using Erlang B formula.[3] In actual systems, however, the density of vehicles in a cell changes with time and so does the offered traffic. When the traffic offered to a cell increases rapidly, the blocking probability in the cell also increases, and there exist many differences between analysis and real systems. For designing switching systems, it is important to analyze the transient characteristics of mobile communication traffic accurately.

The time-dependent analysis of mobile communication traffic has been investigated for making an accurate analysis.[4] In Ref. (4), the state equations including the probability of handoff are presented. Handoff is the operation which consists of surrendering the channel used in the previous cell and reassigning a new channel when the vehicle crosses the cell boundary. However, the equations are not solved for analyzing the characteristics, and simulations are used for the analysis since it is difficult to obtain the probability of handoff from the previous cell to the next one.

In this paper, we analyze the transient characteristics under the consideration of the movement of vehicles in a band-shaped service area when the offered traffic changes rapidly. The rapid traffic changes are made by accidents or congestion in real systems. In the analysis, the state equations including the probability of handoff are used. We derive the probability of handoff at first, and then solve the state equations numerically using the probability. The analytical results are compared with the simulations for verifying the analytical results.

2. Analytical Model

Figure 1 shows the model of a band-shaped service area like a highway. The model has the infinite length. Let the length of a cell be L and let the cell numbers be $\#j-1$, $\#j$, $\#j+1\cdots$.

We make the following assumptions to analyze the transient characteristics in the band-shaped service area:

(1) all vehicles move at the same velocity, V, in the

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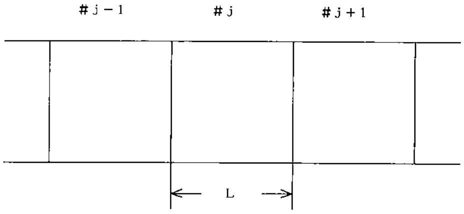

Fig. 1 Band-shaped service area.

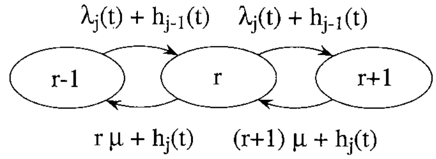

Fig. 2 State transition diagram.

same direction to which the cell number increases,

(2) calls occur according to a Poisson process,
(3) the holding time is an exponential random variable with mean h_0,
(4) using the fixed channel assignment algorithm and the number of channels is S,
(5) the traffic, $a_j(t)$, offered to the cell $\#j$ changes transiently and the cells $\cdots\#j-2$, $\#j-1$ are in steady state.

With regard to assumption (1), if the density of vehicles changes, it is considered that the velocity of vehicles also changes. However, in this paper, because of the simplicity of analysis we assume that the velocity is constant.

3. State Transition Diagram and State Equations

In order to analyze traffic characteristics of the cell $\#j$ in the band shaped service area, when the offered traffic $a_j(t)$ changes, we must obtain $P_{j,r}(t)$, where $P_{j,r}(t)$ is the probability that r channels are used in the cell $\#j$ at time t. Therefore, we must consider the events that may occur from time t to $t+\Delta t$, where Δt is a very short time. If r channels are used in the cell $\#j$ at time $t+\Delta t$, the events that may lead from time t into the status at time $t+\Delta t$ are limited to the following three mutually exclusive events:

(1) the state is r at time t and no events occur during Δt,
(2) the state is $r-1$ at time t and during Δt one call is born in the cell $\#j$, or one call enters the cell $\#j$ from the cell $\#j-1$ by handoff,
(3) the state is $r+1$ at time t and during Δt one call ends in the cell $\#j$, or leaves the cell $\#j$ to the cell $\#j+1$ by handoff.

The state transition diagram of the cell $\#j$ from time t to $t+\Delta t$ is shown in Fig. 2, where $\lambda_j(t)$ is the probability of gaining one call in the cell $\#j$ at time t, μ is the probability of losing one call in the cell $\#j$ and $h_j(t)$ is the probability of losing one call in the cell $\#j$ by handoff.

From Fig. 2 we can write down the state equations as follows:

$$P'_{j,r}(t)=[\lambda_j(t)+h_{j-1}(t)]P_{j,r-1}(t)$$
$$+[(r+1)\mu+h_j(t)]P_{j,r+1}(t)$$
$$-[\lambda_j(t)+r\mu+h_{j-1}(t)+h_j(t)]P_{j,r}(t),$$
$$(\text{when} \quad 0\leqq r<S) \qquad (1a)$$

$$P'_{j,s}(t)=[\lambda_j(t)+h_{j-1}(t)]P_{j,s-1}(t)$$
$$-[S\mu+h_j(t)]P_{j,s}(t), \qquad (1b)$$

where "′" denotes the derivative with respect to t. And then,

$$P_{j,-1}(t)=0, \qquad (2a)$$

$$\sum_{r=0}^{S}P_{j,r}(t)=1. \qquad (2b)$$

4. Probability of Handoff

In order to solve the state equations given by Eqs. (1a) and (1b), we must obtain the probability of handoff. Therefore, at first when r calls exist at time t in the cell $\#j$, we obtain the mean number of calls that leave the cell $\#j$ from time t_0 to t_0+T, where T is constant time, which is denoted by $H_{r,t_0}(T)$. As shown in Fig. 3, let the position of the call in the cell $\#j$ be x. Then, the call must exist in the range, $L-VT \leqq x\leqq L$, to leave the cell during T at time t_0. Furthermore, the holding time of a call must be more than $(L-x)/V$, which is the time that the call will move to the boundary of the cell. Let the density of vehicles of the cell $\#j$ at x and t be $d_j(x, t)$. Then, the density of calls may be equal to $d_j(x, t)$. We assume that the holding time of the call is an exponential random variable with mean h_0, so $H_{r,t_0}(T)$ can be written as follows:

$$H_{r,t_0}(T)=\int_{t_0}^{t_0+T}\int_{L-VT}^{L} r\exp[-(L-x)/(h_0V)]$$
$$\cdot d_j(x, t)\,dxdt. \qquad (3)$$

Then, in order to solve the state equations we must obtain the probability of handoff during Δt denoted by $h_j(t)$. So, we give the mean number of calls that leave the cell $\#j$ from time t to $t+\Delta t$. As shown in Fig. 4, to leave the cell $\#j$ from t to $t+\Delta t$, a call must exist in the range, $L-V\Delta t\leqq x\leqq L$, at time t. That probability can be expressed as:

$$p_x\{L-V\Delta t\leqq X\leqq L\}=d_j(L, t)\,V\Delta t, \qquad (4)$$

where X is the random variable of the position of the call.

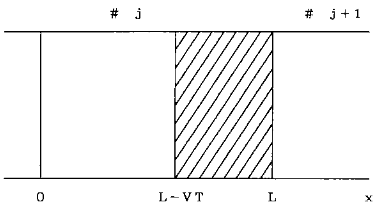

Fig. 3 Probability of handoff.

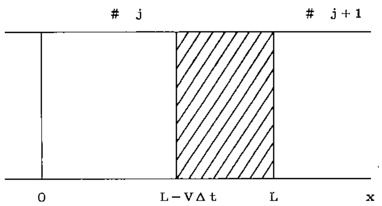

Fig. 4 Probability of handoff during Δt.

Furthermore, since the holding time must be more than Δt for handoff, the probability can be written by:

$$p_h\{T_h \geqq \Delta t\} = \exp(-\Delta t/h_0), \tag{5}$$

where T_h is the random variable of the holding time. We express the right side of Eq. (5) by the Taylor expansion and ignore the terms more than the second order, then Eq. (5) can be expressed as:

$$p_h\{T_h \geqq \Delta t\} = 1 - \Delta t/h_0 + (\Delta t)^2/(2!h_0) + \cdots$$
$$\fallingdotseq 1 - \Delta t/h_0. \tag{6}$$

Therefore, $h_j(t)$ can be expressed by the product of Eqs. (4) and (6). We assume that the density of vehicles is the uniform distribution in all cells. Then, the density of vehicles of the cell $\# j$ is given by Eq. (7) and $h_j(t)$ can be written as Eq. (8).

$$d_j(x, t) = 1/L. \tag{7}$$

$$h_j(t)\Delta t = rp_x\{L - V\Delta t \leqq X \leqq L\} \cdot p_h\{T_h \geqq \Delta t\}$$
$$= r/L \cdot V\Delta t(1 - \Delta t/h_0)$$
$$\fallingdotseq r/L \cdot V\Delta t. \tag{8}$$

As presented by assumption (5), the offered traffic $a_j(t)$ changes and the previous cells remain in steady state. Therefore, we must obtain $h_{j-1}(t)$, which is the probability that a call leaves the cell $\# j-1$ to $\# j$ during Δt. The mean number of calls that leave the cell $\# j-1$ to $\# j$ during Δt is constant. So, as shown in Fig. 5, the mean number of calls that exist in the range, $L - V\Delta t \leqq x \leqq L$, at time t is equal to $r_m/L \cdot V\Delta t$, where r_m is the mean number of channels used in the cell in steady state and does not depend on time. Therefore, $h_{j-1}(t)$ can be expressed as Eq. (9) by replacing r in Eq. (7) with r_m.

$$h_{j-1}(t)\Delta t = r_m/L \cdot V\Delta t. \tag{9}$$

r_m is given by replacing the left side of Eqs. (1a) and

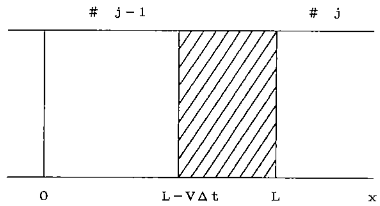

Fig. 5 Probability of handoff during Δt (cell $\# j-1$).

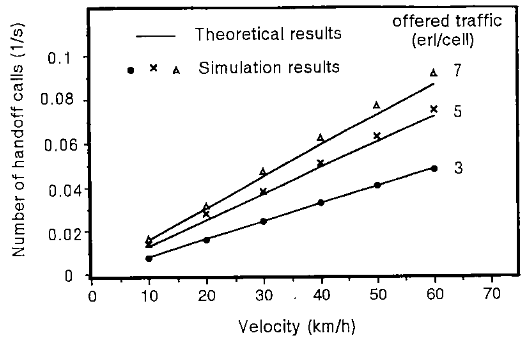

Fig. 6 Number of handoff calls vs. velocity.

(1b) with zero as:

$$r_m = (1 - P_s)/(\mu + V/L \cdot P_s), \tag{10}$$

where λ is the mean arrival rate of call and P_s is the blocking probability of the cell in steady state. P_s is given by solving the non-linear equation given from Eqs. (1a) and (1b), as follows:

$$P_s = \frac{1}{S!}\left(\frac{\lambda}{\mu + V/L \cdot P_s}\right)^s \Bigg/ \left\{\sum_{r=0}^{s} \frac{1}{r!}\left(\frac{\lambda}{\mu + V/L \cdot P_s}\right)^r\right\}. \tag{11}$$

In order to confirm Eq. (9), we obtain the mean number of calls, $H_{j-1}(T)$, that leave the cell $\# j-1$ to $\# j$ during the constant time T and compare it with the simulation results. $H_{j-1}(t)$ is given by replacing r with r_m in Eq. (3) as follows:

$$H_{j-1}(T) = \frac{Vh_0}{L} r_m[1 - \exp(-T/h_0)]. \tag{12}$$

The theoretical results of Eq. (12) are shown in Fig. 6, where $h_0 = 90$[s], $L = 1$[km], $S = 8$, $T = 1$[s]. The simulations results are also shown in Fig. 6, where the number of cells is 10. Good agreement with the theoretical results and the simulation ones show the validity of Eq. (9).

5. Analytical Results

Substituting $h_j(t)$, the probability of handoff from the cell $\# j$ given by Eq. (8), and $h_{j-1}(t)$, the probability of handoff from the cell $\# j-1$ given by Eq. (9), into Eqs. (1a) and (1b), we obtain the simultaneous differential equations as follows:

$$P'_{j,r}(t)=[\lambda_j(t)+r_m V/L]P_{j,r-1}(t)$$
$$+(r+1)(\mu+V/L)P_{j,r+1}(t)$$
$$-[\lambda_j(t)+r\mu+(r+r_m)V/L]P_{j,r}(t),$$
$$(\text{when } 0\leqq r<S) \qquad (13a)$$

$$P'_{j,s}(t)=[\lambda_j(t)+r_m V/L]P_{j,s-1}(t)$$
$$-S[\mu+V/L]P_{j,s}(t). \qquad (13b)$$

When the offered traffic $a_j(t)$ changes like the step function shown in Fig. 7, we show the solution of Eqs. (13a) and (13b) by the Runge-Kutta-Gill method in Fig. 8, where $h_0=90$ [s], $L=1$ [km], and $S=8$. The horizontal axis is time t normalized by h_0 and the vertical axis is blocking probability. We also show the simulation results in Fig. 8, where the number of cells is 10. When the velocity becomes bigger, there is a little difference between theoretical and simulation results. However, as a whole, good agreement in both

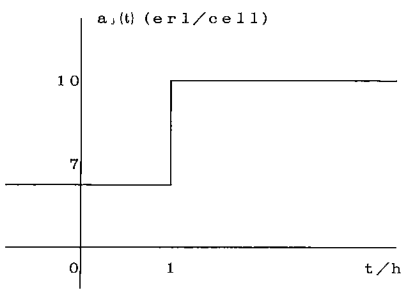

Fig. 7 Change of offered traffic. (Step function).

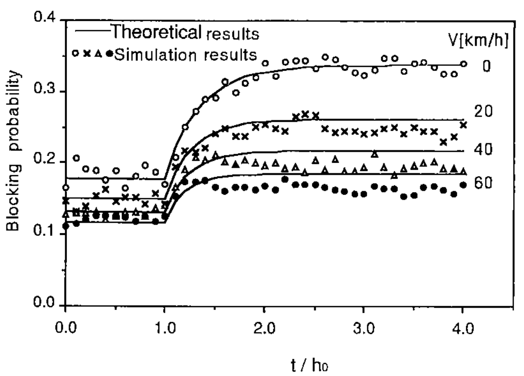

Fig. 8 Blocking probability vs. time.

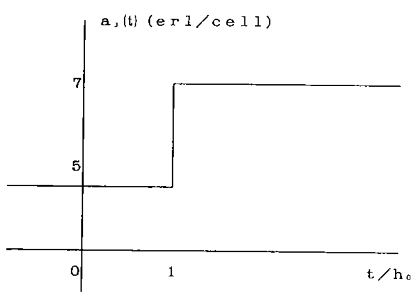

Fig. 9 Change of offered traffic. (Step function).

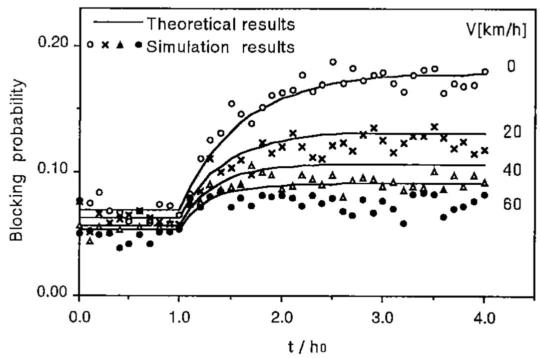

Fig. 10 Blocking probability vs. time.

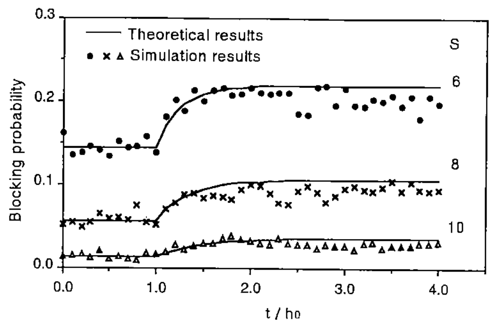

Fig. 11 Blocking probability vs. time.

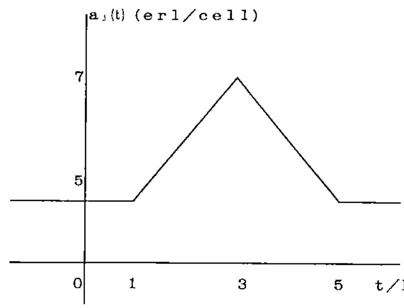

Fig. 12 Change of offered traffic. (Triangle function).

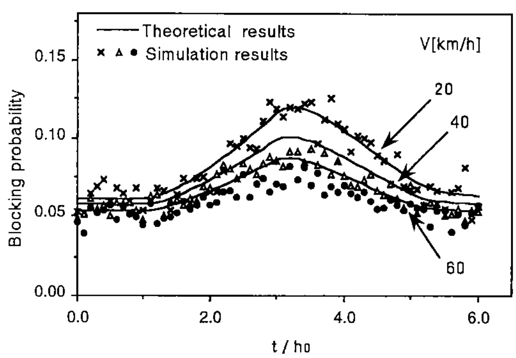

Fig. 13 Blocking probability vs. time.

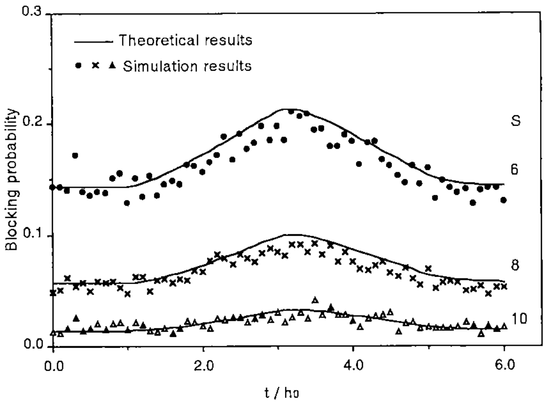

Fig. 14 Blocking probability vs. time.

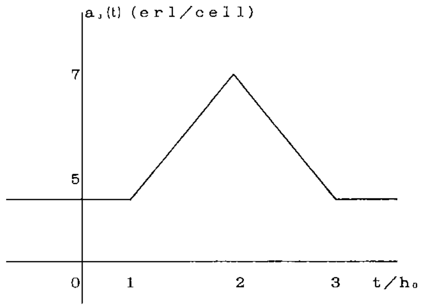

Fig. 15 Change of offered traffic. (Triangle function).

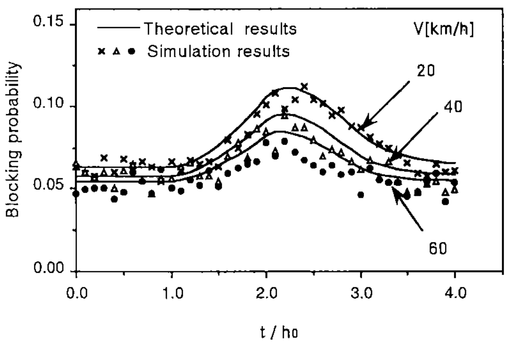

Fig. 16 Blocking probability vs. time.

results is obtained. We also show the analytical results in Fig. 10, when $a_j(t)$ changes like the step function shown in Fig. 9 and show the analytical results in Fig. 11, when let the parameter be S and $V=40$ [km/h]. Furthermore, the analytical results are shown in Figs. 13 and 14, when $a_j(t)$ changes like the triangle step function shown in Fig. 12. We also show the analytical results in Fig. 16, when $a_j(t)$ changes like the triangle step function shown in Fig. 15. In all cases, good agreement about theoretical and simulation results is obtained.

6. Conclusion

In the cellular mobile communication systems, transient characteristics of mobile communication traffic in a band-shaped service area are presented in this paper. We have obtained the state equations considering the movement of the vehicle. Then, we have presented the probability of handoff. It is expressed as the mean number of vehicles that cross the cell boundary during a very short time. We have solved the state equations numerically. The numerical results have been compared with the simulation ones, and good agreement about both results has been obtained. The analytical results can be used for designing switching systems, when the offered traffic changes rapidly due to accidents or congestion.

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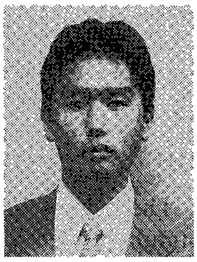

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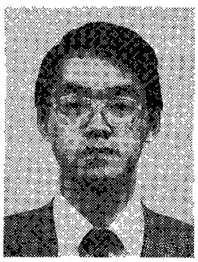

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