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Scale Factor of Resolution Conversion Based on Orthogonal Transforms

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SUMMARY It is known that the resolution conversion based on orthogonal transform has a problem that is difference of luminance between the converted image and the original. In this paper, the scale factor of the system employing various orthogonal transforms is generally formulated by considering the DC gain, and the condition of alias free for DC component is indicated. If the condition is satisfied, then the scale factor is determined by only the basis functions.

key words: *digital image processing, filter banks, resolution conversion, orthogonal transform*

1. Introduction

Orthogonal transform, which is typified discrete cosine transform (DCT),[1] has been developed as a key technique to compress an amount of digital image data.[2] It is expected to be used in the area of visual communication and storage. These applications generally contain various resolutions of displays. Therefore, "scalability" among different resolutions or "compatibility" between standard TV and HDTV has been becoming one of the most important features.[3],[4]

For these requirements, techniques of decimation in orthogonal transform domain have attracted a great deal of attention.[5] These decimation techniques are compared against time domain decimation schemes by K. N. Ngan.[6] He derive a relation among DCT, Hadamard transform (HT) and discreet Fourier transform (DFT). On the other hand, it has been proved that orthogonal transforms can be evaluated as filter banks.[7] However, the decimation employing orthogonal transforms has never been theoretically discussed the property of altering luminance of converted pictures from the original.

In this paper, we purpose to formulate the scale factor, that is necessary to correct the luminance of the converted pictures. In Sect. 2, we summarize a basis method that converts size or resolution of digital image data, including interpolation case. In Sect. 3, we evaluate the conversion system as filter banks. In Sect. 4, by considering the DC gain of the conversion system, we formulate the scale factor. In Sect. 5, we show the result of the simulation of the luminance correction.

2. Resolution Conversion Based on Orthogonal Transforms

In this paper, although we consider the orthogonal transform of one-dimensional (1-D) signals, the results are of interest in image processing, since it can easily be extended to the two-dimensional (2-D) case.

2. 1 Orthogonal Transform[2]

The general 1-D orthogonal transform, and the inverse transform are written as

$$\boldsymbol{X}_{\mathrm{OT}}=\boldsymbol{A}\boldsymbol{x}, \tag{1}$$

$$\boldsymbol{x}=\boldsymbol{A}^t\boldsymbol{X}_{\mathrm{OT}} \tag{2}$$

where $\boldsymbol{x}$ and $\boldsymbol{X}_{\mathrm{OT}}$ are N-dimensional vectors of time and transform domain respectively, $\boldsymbol{A}$ is a $N\times N$ orthogonal matrix, and 't' indicates transposition. Equations (1) and (2) are also expressed as

$$X_{\mathrm{OT}}(k)=\sum_{n=0}^{N-1}a(k,n)x(n) \quad 0\leqq k\leqq N-1, \tag{3}$$

$$x(n)=\sum_{k=0}^{N-1}X_{\mathrm{OT}}(k)a(k,n) \quad 0\leqq n\leqq N-1 \tag{4}$$

where $x(n)$ and $X_{\mathrm{OT}}(k)$ are N-point sequences of time and transform domain respectively, and $a(k,n)$ is an element of matrix $\boldsymbol{A}$, namely basis function.

2. 2 Interpolation and Decimation[6]

Applying above transform, we explain the basic method that converts a N-point sequence $x(n)$ into a M-point sequence $y(m)$ by the rational factor M/N, as shown in Fig. 1.

First, in decimation case $(N>M)$, we make a new M-point sequence $Y_{\mathrm{OT}}(l)$ which is composed of lower M coefficients of the N coefficients $X_{\mathrm{OT}}(k)$ obtained by Eq. (3) as

$$Y_{\mathrm{OT}}(l)=X_{\mathrm{OT}}(l) \quad 0\leqq l\leqq M-1. \tag{5}$$

In interpolation case $(N<M)$, we make a new M-point sequence $Y_{\mathrm{OT}}(l)$ which is composed of M coefficients that are added $M-N-1$ zeros over the N

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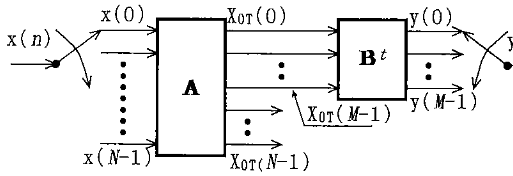
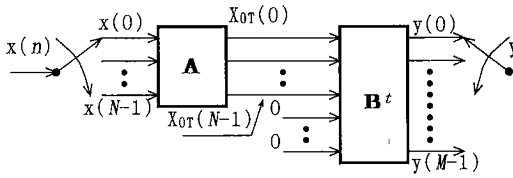


(a)Decimation



(b)Interpolation

Fig. 1 Methods for conversion with orthogonal transform.

coefficients $X_{OT}(k)$. The sequence $Y_{OT}(l)$ is presented as

$$Y_{OT}(l)=\begin{cases}X_{OT}(l) & 0\leqq l\leqq N-1\\ 0 & N\leqq l\leqq M-1.\end{cases} \tag{6}$$

Next, by employing a $M\times M$ inverse transform matrix $\boldsymbol{B}^t$, such as inverse DCT (IDCT) matrix against DCT matrix, we perform an inverse transformation of the vector $\boldsymbol{Y}_{OT}$ as

$$\boldsymbol{y}=\boldsymbol{B}^t\boldsymbol{Y}_{OT} \tag{7}$$

or

$$y(m)=\sum_{l=0}^{M-1} Y_{OT}(l)\,b(l,\ m) \quad 0\leqq m\leqq M-1 \tag{8}$$

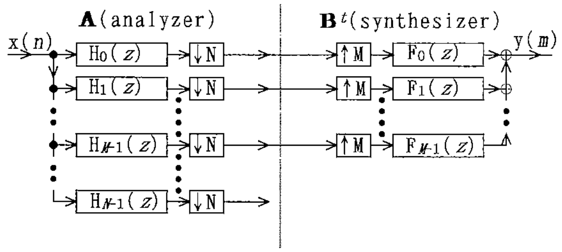
where $\boldsymbol{y}$ and $\boldsymbol{Y}_{OT}$ are M-dimensional vectors of time and transform domain respectively, $y(m)$ and $Y_{OT}(k)$ are M-point sequences of time and transform domain respectively, and $b(l,\ m)$ is an element of matrix $\boldsymbol{B}$, namely basis function. Thus, we can obtain the desired sequence by above procedure.

3. Evaluation as Filter Banks

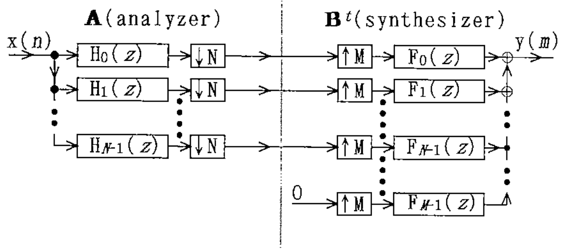
For considering the problem of the luminance, we evaluate orthogonal transforms as operation of filter banks. Here, we also discuss a case of the orthogonal matrix of which basis functions are longer than the transfer size, such as lapped orthogonal transform (LOT).[8]

3.1 Relation between Transfer Function and Basis Function[7]

An orthogonal transform using $N\times L$ matrix corresponds to N-band critically decimated filter bank, in which the impulse response of the synthesis filters

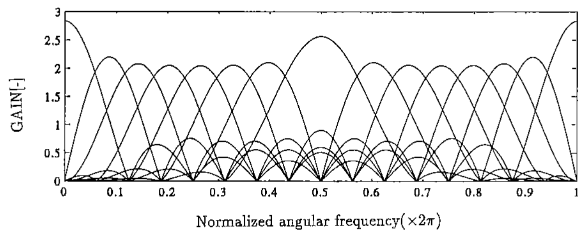


(a)Decimation

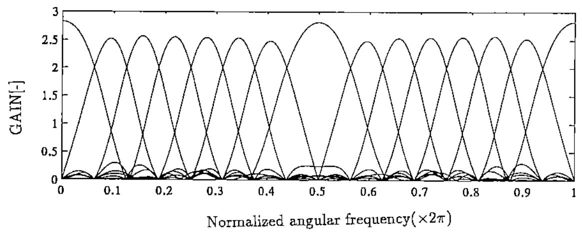


(b)Interpolation

Fig. 2 Evaluation as filter banks.



(a)DCT(N=8,L=8)



(b)LOT(N=8,L=16)

Fig. 3 Amplitude responses of analysis filter $H_k(z)$.

are the transform basis functions, the impulse response of analysis filters are equal to the time-reversed basis functions, namely the tap length of those filters is L. Therefore, the M/N rate conversion operator can be shown in Fig. 2.

The transfer function of the filters $H_k(z)$ and $F_k(z)$ in Fig. 2 are written as

$$H_k(z)=\sum_{i=0}^{L_a-1} a(k,\ L_a-1-i)\,z^{-i} \quad 0\leqq k\leqq N-1, \tag{9}$$

$$F_k(z)=\sum_{i=0}^{L_b-1} b(i,\ k)\,z^{-i} \qquad 0\leqq k\leqq M-1 \tag{10}$$

where L_a and L_b are the length of the basis vectors of matrix $\boldsymbol{A}$ and $\boldsymbol{B}$, respectively.

In Fig. 3, the amplitude responses of 8×8 DCT and 8×16 LOT filter bank are shown by calculating Eq. (9).

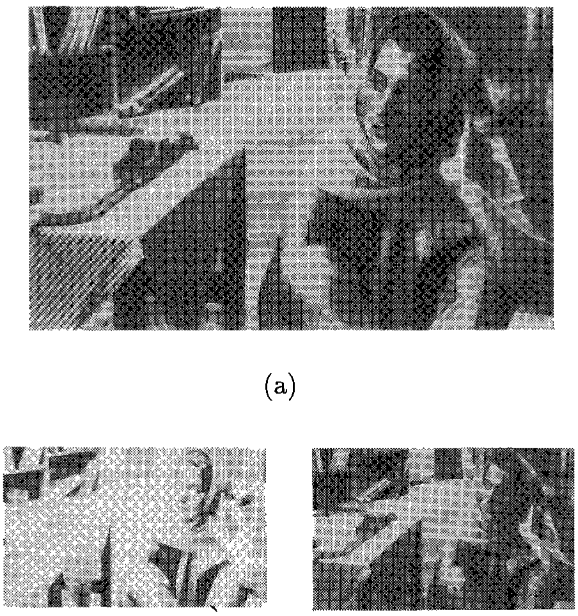
3. 2 Relation between Input and Output Signals[9]

The relation between input and output spectrums of filter bank shown in Fig. 2 is expressed as

$$Y_{FT}(e^{j\omega})=\frac{1}{N}\sum_{k=0}^{K-1} F_k(e^{j\omega})\sum_{n=0}^{N-1} H_k\left(e^{j\left(\frac{M}{N}\omega-\frac{2\pi}{N}n\right)}\right) X_{FT}\left(e^{j\left(\frac{M}{N}\omega-\frac{2\pi}{N}n\right)}\right) \tag{11}$$

where K is minimum value among M and N.

For example, a result of the decimation employing LOT is displayed for $N=8$ and $M=4$ in Fig. 4(b). Figure 4(a) is the original image consisting of 640×400 pixels. Note that the luminance of the result must be corrected.



(a)

(b) (c)

Fig. 4 Result of Simulation.
(a) Original image consisting of 640×400 pixels, encoded with 8bits/pixel.
(b) Decimated image with LOT, $N=8$, $M=4$, consisting of 320×200 pixels.
(c) Image corrected Fig. 4: (b) by scale factor $\alpha^2=M/N=1/2$.

4. Scale Factor for Luminance Correction

Executing resolution conversion based on orthogonal transform, it generates a problem that is difference of amplitude scale between input and output signals. After all, it influences the luminance of the converted image.

4. 1 Gain of System for Sampling Rate Conversion[9]

Relation between input and output spectrums of ideal system for sampling rate conversion by a rational factor M/N is expressed as

$$Y'_{FT}(e^{j\omega})=\begin{cases}\dfrac{M}{N}X_{FT}(e^{j\frac{M}{N}\omega}), & \text{for } |\omega|\leqq\min\left[\pi,\ \dfrac{N}{M}\pi\right]\\ 0, & \text{otherwise.}\end{cases} \tag{12}$$

Here, since the operator shown in Fig. 2 is a kind of the multirate system, the gain of the system must be settled M/N for $|\omega|\leqq\min[\pi,\ N\pi/M]$, namely pass band for input spectrum. However, the conversion employing orthogonal transform is not always enough to that conditions.

4. 2 Selection of Scale Factor

For normalization of the output signal, let us consider to adjust the gain of the whole of the filter banks. However, it is mostly impossible for all ω, since each filter of the system possesses not an ideal amplitude characteristic and the system performs decimation before interpolation. Thus, we investigate only DC gain, because the luminance of image is mostly determined the DC component. Then, we suggest to perform the gain correction by only scale factor for DC component. Here, from Eq. (12), we define the scale factor as

$$\boldsymbol{y}'=\alpha\boldsymbol{y}, \tag{13}$$

$$\alpha=\frac{M}{N}\times\left|\frac{X_{FT}(e^{j0})}{Y_{FT}(e^{j0})}\right| \tag{14}$$

where $\boldsymbol{y}'$ is the output vector corrected its amplitude, and α is a scale factor for luminance correction. However, by Eq. (11), we see that α depends on the input signal. It is problem for digital image processing, because the value of the scale factor alter on each block in block processing.

But, if the filter banks shown in Fig. 2 satisfies the next conditions

$$F_k(e^{j0})=0 \quad k\neq 0\,(k=1,\ 2,\ \cdots,\ K-1), \tag{15}$$

$$H_0(e^{-j\frac{2\pi}{N}n})=0 \quad n\neq 0(n=1, 2, \cdots, N-1), \tag{16}$$

then α is expressed as Eq. (17) by Eqs. (11) and (14). We see that it independents on the input signal.

$$\alpha=\frac{M}{|H_0(e^{j0})F_0(e^{j0})|} \tag{17}$$

The combination of Eqs. (15) and (16) is a sufficient condition that the filter banks shown in Fig. 2 is alias free for DC component of any input signals.

The basic idea expanded before can be extended for subband coding (SBC).

4.3 Examples

All of HT, DCT and LOT satisfy both of conditions Eqs. (15) and (16). Therefore, in this case, scale factor α is obtained by Eq. (17). Now, for example, let us investigate the α of the system of resolution conversion employing LOT by rational factor M/N in 1-D case.

Example: LOT

The DC gain of the filter $H_0(z)$ is obtained by evaluating Eq. (9) on unit circle as

$$\begin{aligned} H_0(e^{j0}) &= \sum_{i=0}^{2N-1} a(0, 2N-1-i)\, e^{-j0} \\ &= \sum_{i=0}^{N-1} \frac{1}{2\sqrt{N}}\left\{1-\sqrt{2}\cos\left(\frac{(2i+1)\pi}{2N}\right)\right\} \\ &\quad + \sum_{i=N}^{2N-1} \frac{1}{2\sqrt{N}}\left\{1-\sqrt{2}\cos\left(\frac{(2(2N-1-i)+1)\pi}{2N}\right)\right\} \\ &= 2N\times\frac{1}{2\sqrt{N}}=\sqrt{N}. \end{aligned} \tag{18}$$

Similarly, DC gain of filter $F_0(z)$ is obtained as

$$F_0(e^{j0})=\sqrt{M}. \tag{19}$$

Hence, by Eqs. (17), (18) and (19),

$$\alpha=\sqrt{\frac{M}{N}}. \tag{20}$$

Therefore, in 2-D case, namely image processing, it is better that we select M/N as scale factor.

The scale factor of the system employing DCT or HT can be obtained similarly, and both their result are also $\sqrt{M/N}$.

5. Simulation

According to Fig. 4(b), we find that the luminance correction is necessary. In Fig. 4(c), the decimated image with scale factor α is shown. From the result, we see that the correction is accomplished subjectively.

6. Conclusion

In this paper, we formulated the scale factor of the system employing various orthogonal transforms by considering the DC gain, and indicated that the scale factor is determined by only the basis functions, if the condition of alias free for DC component is satisfied.

Actually, we confirmed that the solution is adequate to correct the luminance in both interpolation and decimation case by employing various orthogonal transforms, such as DCT and HT.

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