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Characteristics of Dynamic Channel Assignment in Cellular Systems with Reuse Partitioning

Keisuke NAKANO[†], *Member*, Naoyuki KARASAWA[††], *Student Member*,
Masakazu SENGOKU[††], Shoji SHINODA[†††], *and* Takeo ABE[††††], *Members*

SUMMARY This paper describes communication traffic characteristics in cellular systems employing the concept of reuse partitioning and Dynamic Channel Assignment. Such systems have a problem of the spatial unbalance of blocking probability. The objective of this paper is overcoming this problem. To accomplish this objective, we use a method for analyzing communication traffic characteristics. We also show results on traffic characteristics in the systems.

key words: *cellular, reuse partitioning, dynamic channel assignment, communication traffic analysis*

1. Introduction

In mobile communication systems, the efficient use of frequency bands is one of the most important issues. To improve the efficiency, mobile communication systems employ the concept of cellular systems [1]. A cellular system permits simultaneous use of a channel in different cells to increase system capacity. Keeping a sufficient distance between cochannel cells is important in order to avoid interference caused by the channel reuse.

The concept of reuse partitioning, proposed by Halpern [2], also contributes to increasing system capacity. Reuse partitioning divides a cell into several smaller cells. Then, a base station controls communication in some small cells. In the reuse partitioning cellular system, distance from a base station to each of the divided small cells is different. Therefore, calls in an inner small cell receive stronger carrier waves than those of an outer small cell. This enables the reuse distance for the inner cells to be shorter than that of outer cells because the reuse distance is determined by carrier to interference ratio. Consequently, reuse partitioning increases system capacity.

Halpern evaluated characteristics of Fixed Channel Assignment (FCA) in the reuse partitioning cellular system. Some other articles [3]-[10] evaluated characteristics of Dynamic Channel Assignment (DCA) in the reuse partitioning cellular system. Some of the articles say that the distributed DCA algorithms autonomously realize reuse partitioning [4]-[9]. In the cellular systems employing DCA and reuse partitioning, blocking probability in outer small cells is higher than that of inner small cells when all channels are common to all small cells. Sallberg et al. pointed out this problem of the unequal blocking probability [3]. This causes a problem of unfairness because the system gives different grades of service to subscribers. Giving the same grade of service to subscribers is an important system objective.

The problem is caused by simple reasons: first, the channel reuse distance for outer small cells is longer than that of inner small cells as mentioned above; and secondly, if a channel is available for an outer small cell of a base station, this channel is also available for an inner small cell of the base station. The reverse is not always true. This causes a situation where inner small cells hold a lot of channels which are also available for outer small cells. Consequently, the blocking rate in the outer small cells is higher than that in the inner small cells.

The objective of this paper is unifying the blocking rate in each of the small cells in cellular systems with reuse partitioning. To do that, we divide the system channels into some groups and reserve each of the groups for specific small cells. Each of the groups for specific small cells is not available for other cells. Under such a restriction, the inner cells never hold the channels reserved for the outer cells. This prevents the spatial unbalance of the blocking probability.

Sallberg et al. first proposed to make such a restriction as mentioned above. In [9], the author discussed the influence of a similar restriction in fully distributed systems. For the exact decision of the number of channels to be reserved for inner cells and outer cells, we must know the relation between the blocking rate in each of the small cells and the number of channels to be reserved. The articles above, however, roughly decided the number of channels to be reserved and did not discuss procedure to do this in detail.

To decide the number of channels to be reserved, this paper applies a simple method [11], [12] to obtaining the relation between the blocking rate of DCA and

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[†] The author is with Niigata College of Technology, Niigata-shi, 950-21 Japan.
[††] The authors are with the Faculty of Engineering, Niigata University, Niigata-shi, 950-21 Japan.
[†††] The author is with the Faculty of Science and Technology, Chuo University, Tokyo, 112 Japan.
[††††] The author is with Niigata Institute of Technology, Kashiwazaki-shi, 945 Japan.

traffic load in the system as mentioned above. This method is conceptually simple, quick in computation and applicable to several DCA methods in the same way. By using this method, we can decide the number of channels of each of the groups according to the number of system channels and the traffic intensity. Section 2 proposes a model of the system and the method. Section 3 shows results and discussion. In Sect. 4, we conclude this paper.

2. Model and Method

This section gives a simple model of a reuse partitioning cellular system and proposes a method to accomplish our objective as mentioned above. The following is assumptions made in this paper:

(A1) A service area is constructed with regular hexagonal cells and has no bounds as shown in Fig. 1. This assumption is made for ignoring the influence of the bounds on traffic characteristics.

(A2) A base station is installed in the center of a cell. A cell is divided into 3 small cells as represented in Fig. 2.

(A3) The system is a loss system, and the call arriving rate forms a Poisson process.

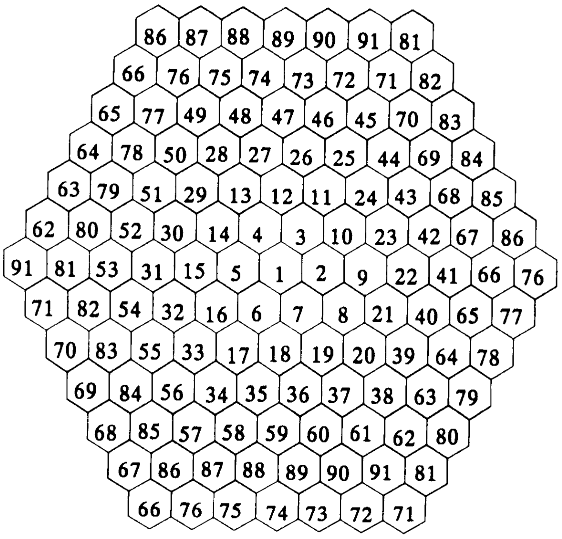


Fig. 1 The structure of the service area which is considered in this paper. In this figure, some cells which have the same number are the same cell, and this service area has no bound.

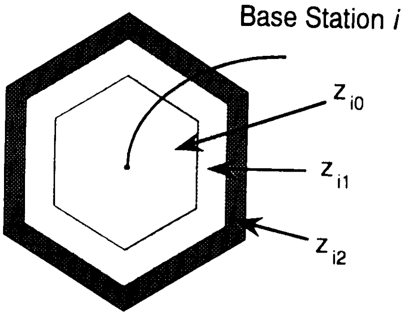


Fig. 2 The divided small cells. Each of the cells in the service area in Fig. 1 is divided into 3 small cells as shown in this figure.

(A4) The area of each of the 3 small cells is the same.

(A5) The average traffic intensity in each of the 3 small cells is with the mean value a.

(A6) The holding time is an exponential random variable. The average holding time is 1.5 minutes.

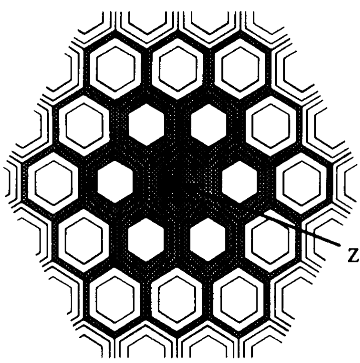
(A7) Subscribers never get out of the cells where they originated.

We make some definitions as follows: Consider base station i. Let z_{i0} be the small cell which is nearest to its own base station i. z_{i1} is the small cell which is adjacent to z_{i0}. z_{i2} is the rest of the small cells. Base station i controls communications in z_{i0}, z_{i1} and z_{i2}. z_{i0}, z_{i1} and z_{i2} are shown in Fig. 2. Define that $Z_j=\{z_{1j}, z_{2j}, \cdots, z_{n_z j}\}$, where n_z is the number of base stations in the system. Let B_0, B_1 and B_2 be the blocking probabilities of DCA in cells of Z_0, Z_1 and Z_2, respectively.

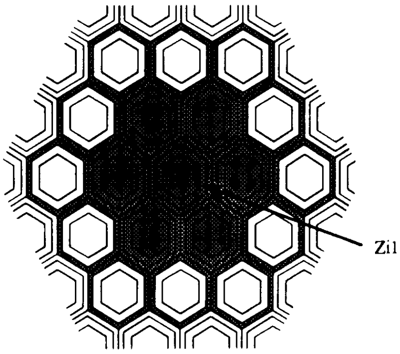
Let us consider channel reuse constraint in the system. Assume that the cochannel reuse constraint is based only on the distance from a small cell to its base station and the distance from the small cell to other small cells. This paper assumes that z_{ij} has interference cells as illustrated in Fig. 3 when all the channels are available for all cells. This interference model is basically the same as the model in [10]. When a new call arrives at a small cell and a channel is already used in interference cells of the small cell, we can not assign the channel for the new call.

The number of interference cells of an inner small cell is smaller than that of an outer small cell in the system. Also, interference cells of an outer small cell include those of an inner small cell if the small cells belong to the same base station. For example, interference cells of z_{i2} include those of z_{i1} and those of z_{i0}. Similarly, interference cells of z_{i1} include those of z_{i0}. Then, if a channel is with no interference in an outer small cell of a base station, this channel is also with no interference in an inner small cell of the base station. The reverse is not always true. Therefore, the inner cells hold a lot of channels which are also desired in outer cells. This causes the problem of the blocking rate in an outer cell being higher than that in an inner cell. In the system as represented in Fig. 3, it is expected that $B_0 < B_1 < B_2$.

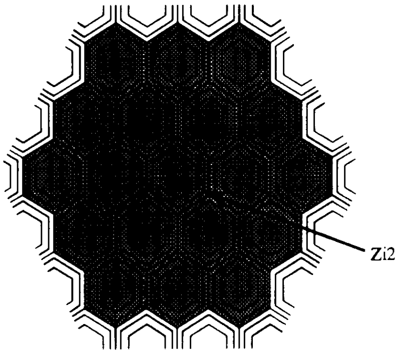
For preventing the unequal blocking probability, we divide channels into groups and reserve each of the groups for specific cells. This indicates the restriction on the channel usage. Considering the system in Fig. 3, we divide channels into 3 groups. The first group is only for Z_0. The second and the third groups are only for Z_1 and only for Z_2, respectively. Therefore, a cell of Z_0 cannot use a channel which is reserved for cells of Z_1 or Z_2 even if the channel is with no interference. Let n_j be the number of channels which are reserved for cells of Z_j. If we have n_d system channels in the system, then $n_d=n_0+n_1+n_2$. Our purpose is to make



(a) Interference cells of z_{i0}.

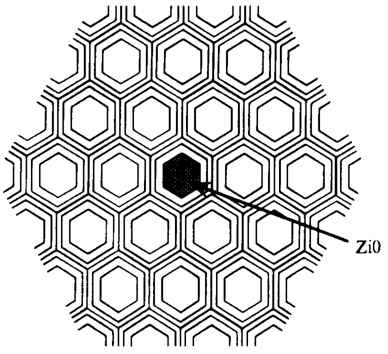


(b) Interference cells of z_{i1}.

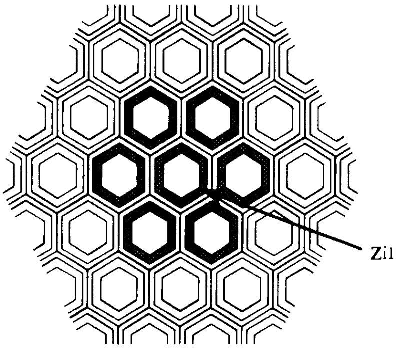


(c) Interference cells of z_{i2}.

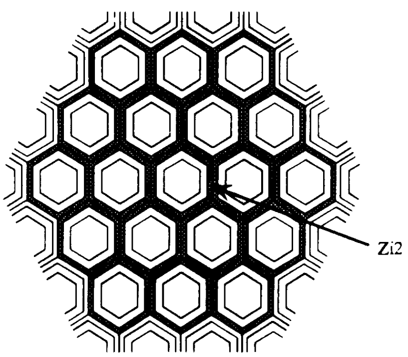
Fig. 3 Interference cells of z_{i0}, z_{i1} and z_{i2} when all of the system channels are available for Z_0, Z_1 and Z_2.



(a) Interference cells of z_{i0}.



(b) Interference cells of z_{i1}.



(c) Interference cells of z_{i2}.

Fig. 4 Interference cells of z_{i0}, z_{i1} and z_{i2} when the system channels are divided into 3 sets, each of which is reserved for Z_0, Z_1 and Z_2, respectively. In this case, channel assignments in Z_0, Z_1 and Z_2 are independent. Therefore, interference cells of z_{ij} consist of only the cells of Z_j.

$B_0=B_1=B_2$. Then, we must determine the values of n_0, n_1 and n_2 which result $B_0=B_1=B_2$.

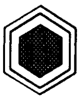
For the determination of the values of n_0, n_1 and n_2, we must solve simultaneous equations for n_0, n_1 and n_2 in the following expressions, where offered load a and the number of channels n_d are given:

$$\begin{cases} n_d=n_0+n_1+n_2, \\ B_0=B_1=B_2. \end{cases} \qquad (1)$$

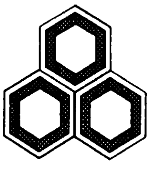
To solve the equations, we must obtain the values of B_0, B_1 and B_2 as functions of n_0, n_1 and n_2, respectively. The blocking probabilities are usually obtained by computer simulation; however, computer simulation is not suitable to this case because a computer simulation requires a long time to be executed. It is, however, difficult to obtain the exact value of the blocking probability of DCA analytically even if the reuse partitioning is not applied to the system. Therefore, we should use an approximate method for calculating the blocking probabilities of DCA in the reuse partitioning cellular system with the restriction on the channel usage.

Before we explain this method, we must again consider the system as mentioned above. In this system, we divide channels into 3 groups. A channel of a channel group is reused dynamically only in small cells for which the channel group is reserved. Therefore, it is possible to consider that interference cells of a cell of Z_i consist of only cells of Z_i. Then, in this case, interference cells of z_{ij} are as shown in Fig. 4. The p-belt buffering system means the system where interference cells of a cell consist of all cells closer than p-cells away from it. As shown in Fig. 4, Z_0 is equal to the 0-belt buffering system and interference cells of a cell of Z_0 are only the cell itself. By concentrating on cells of Z_1 in Fig. 3(b), we can confirm that Z_1 is equivalent to the 1-belt buffering system. In the same manner, Z_2 is equal to the 2-belt buffering system. Consequently, B_i is the blocking probability in the p_i-belt buffering system with n_i channels, where p_i means the channel reuse separation in Z_i.

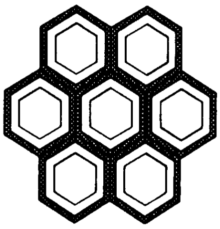
A method, we have proposed in [11] and [12], is just suitable to the calculation of blocking probability of DCA in the p-belt buffering system. This method is conceptually simple and quick in computation. The output of this method agrees well with simulation results. This method is based on a property of cliques. A clique means a set of cells which interfere with each other. A maximum clique is a clique which has the largest number of cells. For example, the maximum cliques in Z_0, Z_1 and Z_2 are represented in Fig. 5, where interference cells are as illustrated in Fig. 4. From the definition of a clique, channels are never reused in cells of a clique. Therefore, any DCA methods cannot accept more calls than the number of channels in a clique. Furthermore, to accept the same number of calls as channels in a clique, DCA methods must optimize allocation of channels completely; however, almost all practical DCA methods do not have the ability of full optimization.



(a) A maximum clique in cells of Z_0.



(b) A maximum clique in cells of Z_1.



(c) A maximum clique in cells of Z_2.

Fig. 5 Maximum cliques in cells of Z_0, Z_1 and Z_2 when the system channels are divided into three sets, each of which is for Z_0, Z_1 and Z_2 respectively.

Clique Packing [11], [12], [14] can always accept up to the same number of calls as channels opposing to the practical DCA methods. Hence it is considered that Clique Packing is a model of an idealized DCA. Channel assignment operation of Clique Packing at a call arrival is considered as follows: (1) Count the number of calls except the new call in each of the cliques which includes the new call; (2) Reject the new call if some of the cliques already have the same number of calls as channels, respectively; Accept the new call in other cases. We give an approximate formula for calculating the blocking rate of Clique Packing in [11]. The numerical results of the formula agree well with computer simulation results. From the results of [11], traffic characteristics of Clique Packing depend on only maximum cliques under uniform traffic load.

Also, in [11] and [12], we estimate the mean value of the upper limit of the number of calls in a maximum clique for some DCA methods. The value is denoted by βn_d, where β is the characteristic value of each of the DCA methods. Using the estimation results, we characterize some practical DCA methods by Clique Packing in [11] and [12]. As a result of the characterization, we can replace each of the DCA methods which has n_d channels by Clique Packing which has βn_d channels. It also results that β is almost constant even if the number of channels and the offered load change. These results indicate that a DCA method can accept up to βn_d calls in a maximum clique in average. Consequently, we can calculate the blocking rate of DCA with n_d channels by using the formula for Clique Packing with βn_d channels. The following is the procedure for calculating the blocking probability of DCA with n_d channels in the p-belt buffering system: (1) Estimate β of the DCA method [11], [12]; (2) Substitute βn_d and the offered load in each cell into the formula for Clique Packing.

Let us consider B_0, B_1 and B_2 in the system employing First Available method [13], which is one of the simplest DCA methods and assigns a channel which is found first and is without no interference for an arriving call. In cells of Z_0, no interference from other cells exists. Each cell of Z_0 can always use n_0 channels. This means the group of the channels is the full-available system. The blocking rate of cells of Z_0 can be obtained by Erlang B formula directly. Then,

$$B_0 = E_{n0}(a), \qquad (2)$$

where $E_S(a)=\dfrac{\dfrac{a^S}{S!}}{\sum_{r=0}^{S}\dfrac{a^r}{r!}}$, which is Erlang B formula.

Consider B_1 and B_2. From the results of [11] and [12], β_1, which is β of First Available method in the 1-belt buffering system, is about 0.83. As mentioned above, Z_1 is the 1-belt buffering systems; therefore, B_1 is considered as the blocking rate of Clique Packing with $\beta_1 n_1$ channels at offered load a in the 1-belt buffering system, which is denoted by $B_{M1\text{-}belt}(a, \beta_1 n_1)$. $B_{M1\text{-}belt}(a, \beta_1 n_1)$ is given as follows [11]:

$$B_{M1\text{-}belt}(a, \beta_1 n_1)=6E_{3n_f}(3a)-6E_{4n_f}(4a)$$
$$-3E_{5n_f}(5a)+6E_{6n_f}(6a)-2E_{7n_f}(7a), \quad (3)$$

where $E_S(a)$ means Erlang B formula as represented above and $n_f=$ (The number of acceptable calls in a maximum clique) / (The number of cells in a maximum clique); therefore, in this case, $n_f=\beta_1 n_1/3$ because First Available method accepts up to $\beta_1 n_1$ calls in a maximum clique, which has 3 cells, in the 1-belt buffering system.

β_2, which is β of First Available method in the 2-belt buffering system, is about 0.78 [11], [12]. Then, B_2 is considered as the blocking probability of Clique Packing with $\beta_2 n_2$ channels at offered load a in the 2-belt buffering system, which is denoted by $B_{M2\text{-}belt}(a, \beta_2 n_2)$. $B_{M2\text{-}belt}(a, \beta_2 n_2)$ is given as follows [11]:

$$B_{M2\text{-}belt}(a, \beta_2 n_2)=7E_{7n_f}(7a)-12E_{10n_f}(10a)$$
$$+12E_{13n_f}(13a)-6E_{14n_f}(14a), \quad (4)$$

where $n_f=\beta_2 n_2/7$ because First Available method accepts up to $\beta_2 n_2$ calls in a maximum clique, which has 7 cells, in the 2-belt buffering system. Consequently, Eqs.(2),(3) and (4) enables to solve Eq.(1).

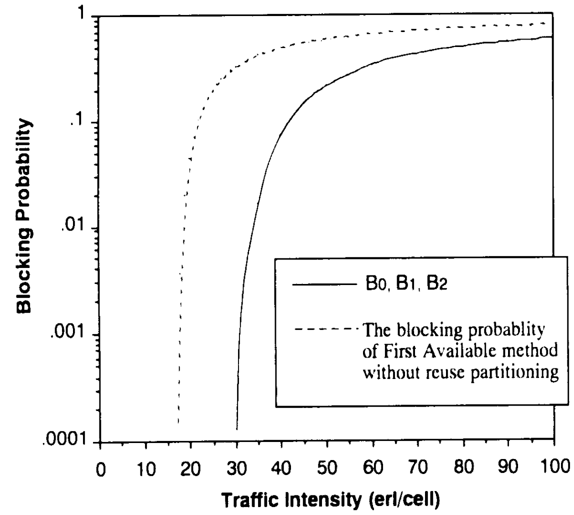
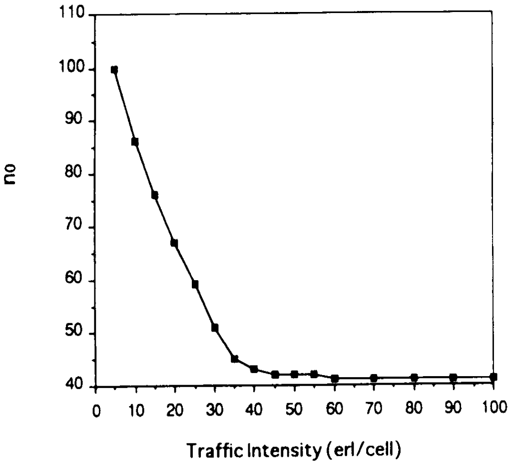


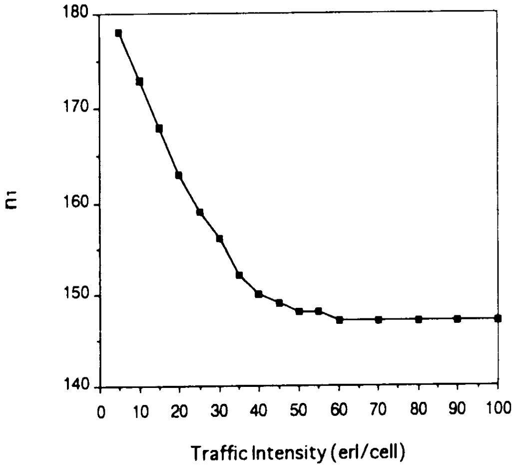
Fig. 6 The results of B_0, B_1 and B_2. The blocking probability of First Available method in the 2-belt buffering system without reuse partitioning is also represented. The number of system channels is 550. The values under the horizontal axis mean the traffic intensity in one of the small cells, as denoted by a in this paper.

3. Results and Discussion

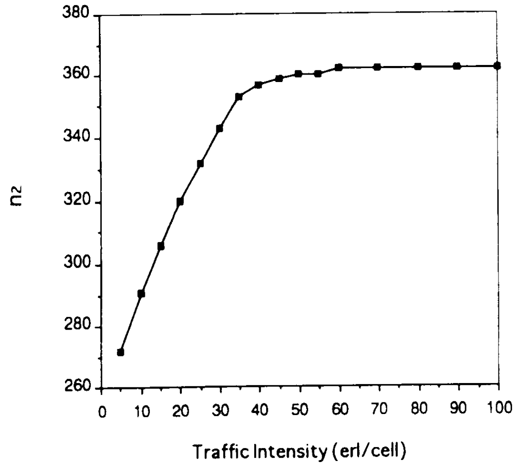
We show characteristics of First-Available method in Fig. 6 and Fig. 7. Figure 6 shows the characteristics of B_0, B_1 and B_2 satisfying Eq.(1). Figure 7 shows the characteristics of n_0, n_1 and n_2 which satisfy the condi-



(a) Characteristics of no.



(b) Characteristics of n1.



(c) Characteristics of n2.

Fig. 7 Results of n_0, n_1 and n_2 which are obtained by solving Eq. (1), where $n_a=550$.

tion such that $B_0=B_1=B_2$. For obtaining these results, we solve Eq.(1). In this case, $n_d=550$. We cannot guarantee to prove existence of the exact solution of the simultaneous equations. Therefore, we use an approximate method for solving the equations. We assume that n_0, n_1 and n_2 are integers, and employ, as a solution of the equations, n_0, n_1 and n_2 which give the minimum value of $|B_0-B_1|+|B_1-B_2|+|B_2-B_0|$.

In Fig. 6, B_0, B_1 and B_2 are represented with the blocking probability of First Available method in the 2-belt buffering system without reuse partitioning. B_0, B_1 and B_2 are lower than the blocking probability in the system without reuse partitioning though we restrict the channel usage in the reuse partitioning system. It seems, in this result, that increase of the system capacity by the effect of reuse partitioning overcomes the decrease of the system capacity by the restriction on the channel usage.

Consider the results represented in Fig. 7. The results show the relations such that $n_0<n_1<n_2$. The reuse distance of a channel in cells of Z_0 is smaller than that of Z_1. The reuse distance in cells of Z_1 is smaller than that of Z_2. Therefore, the relations are intuitively expected. We consider the ratios $n_0:n_1:n_2$ from the results in Fig. 7. Figure 7 shows the following results: At high traffic load, n_0, n_1 and n_2 converge to their own value, respectively. After the convergence, it results that $n_0:n_1:n_2$ are about $1:3.6:8.9$. We consider the meaning of these ratios. Assume that all maximum cliques are saturated with calls at high traffic load. In this case, it is considered that the number of calls in a maximum clique in Z_1 is equal to $\beta_1 n_1$. Because the offered load per cell is uniform and the maximum clique in Z_1 has 3 cells, it is considered that each cell of Z_1 has $\beta_1 n_1/3$ calls. In the same manner, each cell of Z_0 and Z_2 has n_0 calls and $\beta_2 n_2/7$ calls, respectively. In this system, the blocking rate and the offered traffic in each cell are uniform. Then, the carried traffic in each cell is also uniform; therefore, it is expected that $n_0 \fallingdotseq \beta_1 n_1/3 \fallingdotseq \beta_2 n_2/7$, where β_1 is 0.83 and β_2 is 0.78. From these relations, we can derive the ratios $n_0:n_1:n_2=1:3.6:9.0$. These ratios agree well with the results shown in Fig. 7. This indicates that we can determine n_0, n_1 and n_2 by using β and the number of acceptable calls in a maximum clique when each small cell has uniform high traffic load. At low traffic load, the ratios $n_0:n_1:n_2$ are not constant. This result indicates that we cannot easily determine the ratios $n_0:n_1:n_2$ at low traffic. In this case, therefore, our method, which requires to solve Eq.(1), is effective on determination of n_0, n_1 and n_2.

This paper considers only First Available method; however, as shown in [12], we can calculate the blocking probabilities of some other DCA methods in the same way as used in this paper; therefore, it is possible to evaluate characteristics of other DCA methods in such a system as considered in this paper.

Let a_0, a_1 and a_2 be the traffic loads in z_{i0}, in z_{i1} and in z_{i2}, respectively. This paper assumes that the area of each small cell is uniform, for simplicity. Hence, this paper also assumes that $a_0=a_1=a_2=a$. Consider the situation where the area of each small cell of a cell is different. In this case, a_0, a_1 and a_2 may become different; however, our method is also applicable to this case by replacing a in Eqs.(2), (3) and (4) with a_0, a_1 and a_2, respectively.

4. Conclusions

We will conclude by listing important results:
(1) This paper describes the problem of the unequal blocking probability in the reuse partitioning cellular system and proposes a countermeasure against the problem. The method, which is based on the simple traffic analysis technique, is effective on preventing the unequal blocking rate.
(2) The size of a maximum clique and the characteristic parameter of DCA methods, which is denoted by β, influence the ratios of the number of channels to be reserved for each small cell.

Future problems are as follows:
(1) Traffic analysis in the reuse partitioning system under non uniform traffic.
(2) Analyzing the effect of handoffs on the traffic characteristics.

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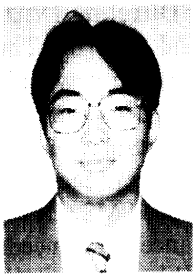
Keisuke Nakano was born in Niigata, Japan, on April 22, 1966. He received the B.E. and M.E. degrees from Niigata University in 1989 and 1991, respectively. He received the Ph.D. degree from Niigata University in 1994. He joined Niigata College of Technology in 1994. His research interests include the development and performance evaluation of mobile communication networks.



Naoyuki Karasawa was born in Niigata, Japan, in 1973. He is now working at Niigata University toward the B.E. degree. His research interests include performance evaluation of mobile information networks.

Masakazu Sengoku received the B.E. degree in electrical engineering from Niigata University in 1967, and M.E. and Dr.Eng. degrees from Hokkaido University in 1969 and 1972, respectively. In 1972, he joined the Faculty of Engineering, Hokkaido University, where he is now a Professor at the Department of Information Engineering, Niigata University. His interests are in the field of network theory, transmission of information and mobile communications.

Shoji Shinoda received the B.E., M.E., and Dr.Eng. degrees, all in electrical engineering from Chuo University in 1964, 1966 and 1973, respectively. Since 1965, he has been with the Faculty of Science and Engineering, Chuo University, where he is coherently a Professor at the Department of Electrical and Electronics Engineering, Chuo University. His interests are in the field of fault-diagnosis, analysis and design of circuits, networks and discrete systems.

Takeo Abe received the B.E. and Dr.Eng. degrees from Tokyo Institute of Technology in 1949 and 1966, respectively. From 1950 to 1959, he was a Research Scientist at the Electrotechnical Laboratory, Tokyo. From 1962 to 1966 he was an Associate Professor at Tokyo Institute of Technology. From 1966 to 1991 he was a Professor at Niigata University. From 1991 to 1995 he was a Professor at Chiba Institute of Technology. He is now President of Niigata Institute of Technology. His interests are in the field of electromagnetic theory, transmission of information and network theory.